平成27年度　土木学会北海道支部　論文報告集　第72号

Fatigue analysis of RC slabs under moving load based on the bridging stress degradation concept

Hokkaido University　Student member　○Ahmed Attia M. DRAR
Hokkaido University　Member　Takashi MATSUMOTO

1. INTRODUCTION

Recently, there is an increasing request to estimate the fatigue behaviors of reinforced concrete (RC) bridge slabs under a moving load. Therefore, many numerical and experimental studies have been conducted to predict the fatigue life of these slabs. Previous experimental studies[1, 2] found that the fatigue life of RC slabs under a moving load is lower than that under a fixed pulsating load. Maekawa et al.[3] presented a numerical fatigue simulation of RC slabs under a moving load. This numerical method used a direct path-integral scheme with fatigue constitutive models for concrete tension, compression and crack surfaces shear. The influence of tension fatigue modeling is more effective than that of shear transfer and compression modeling. Therefore, understanding a mechanism of fatigue crack propagation is essential to evaluate concrete fatigue representation in tension. The bridging stress degradation concept was introduced for the first time by Li and Matsumoto[4] as a principal cause of fatigue crack propagation in concrete and fiber reinforced concrete beams. Suthiwarapirak and Matsumoto[5] used this concept in their numerical model to predict the fatigue behaviors of RC slabs. This study successfully provided a fatigue analysis of these slabs under moving and fixed pulsating loads.

Most of the previous numerical studies focused on the modeling of fatigue behaviors of RC slabs reinforced with deformed bars. However, many RC slabs in use today are reinforced with plain bars, and they are suffering from fatigue damages. This study presents a numerical method based on bridging stress degradation concept to simulate the fatigue behaviors of RC slabs reinforced with plain bars under a moving load. The bond-slip effect between a plain reinforcing bar and its surrounding concrete is carefully taken into consideration. Furthermore, fatigue life and slab center displacement evolution are focused in this study. The comparison of fatigue behaviors between numerical and experiential results by Shakushiro et al[6] is also provided.

2. METHOD

A finite element method based on the Newton-Raphson iteration scheme is employed for getting a solution of nonlinear constitutive laws[7] of concrete as shown in Table 1. For concrete, the stress-strain relation in tension is linear until cracking. After cracking, the bridging stress is introduced to represent the cracking behavior of concrete. The bridging stress can be defined as a transferred stress between crack surfaces through aggregates. A crack starts with length, a, and width, w, due to the first loading as shown in Fig. 1 (a). Repetitive loading leads to a reduction of bridging stress, because the cracks are subjected to a process of opening and closing. Therefore, this crack propagates with additional length, da, and additional width, dw, as shown in Fig. 1 (b). This reduction is defined as the bridging stress degradation concept, and it is assumed to depend on two parameters: maximum tensile strain, $\varepsilon_{t\max}$, and number of cycles, N[4]. The equation of bridging stress degradation can be expressed as[8]:

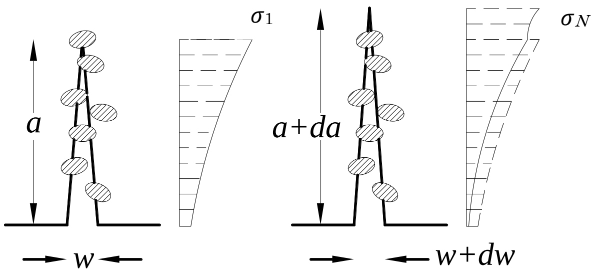
$$\frac{\sigma_N}{\sigma_1} = 1 - (0.08 + 4\varepsilon_{t\max} l)\log(N), \tag{1}$$

where l is cracked element size. σ_N and σ_1 are bridging stress at the Nth and the first cycle, respectively.

Table 1 Constitutive laws for concrete[7]

Compression		Tension	
$0 \le \varepsilon \le \varepsilon_m$	$\sigma = f_c \dfrac{\varepsilon}{\varepsilon_m}\left(2 - \dfrac{\varepsilon}{\varepsilon_m}\right)$	$0 \le \varepsilon \le \varepsilon_t$	$\sigma = E_c \varepsilon$
$\varepsilon_m \le \varepsilon \le \varepsilon_u$	$\sigma = f_c \dfrac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_m}$	$\varepsilon > \varepsilon_t$	$\sigma = f_t \left(\dfrac{\varepsilon_t}{\varepsilon}\right)^{0.4}$

E_c = the modulus of elasticity of concrete, f_c = concrete compressive strength, $\varepsilon_m = f_c/2E_c$ = concrete strain corresponding f_c, f_t = tensile strength, and $\varepsilon_t = f_t/E_c$ = strain at tensile strength.



(a) 1st Cycle　(b) Nth Cycle

Fig. 1 Crack propagation due to bridging stress degradation

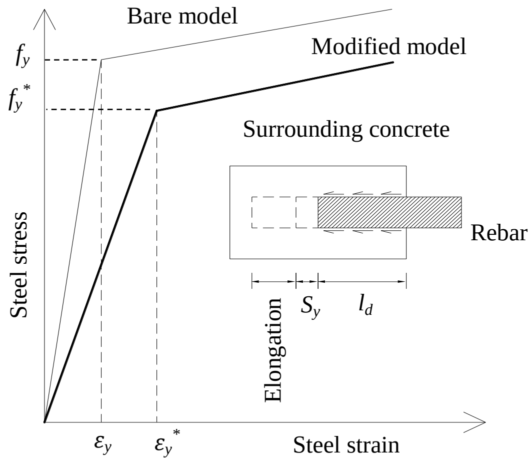


Fig. 2 Reinforcing bar model

The reinforced concrete elements are modeled as a smeared rebar according to the reinforcement ratio distributed in any desired direction. The stress-strain relationship of reinforcing bar is described as a bilinear curve with explicit yield stress, f_y. The effect of bond between a plain bar and its surrounding concrete is considered as the most important factor to simulate a reliable behavior of RC elements reinforced with plain bars. Therefore, considering this effect is essential during analysis. The modified reinforcing bar model by Dehestani and

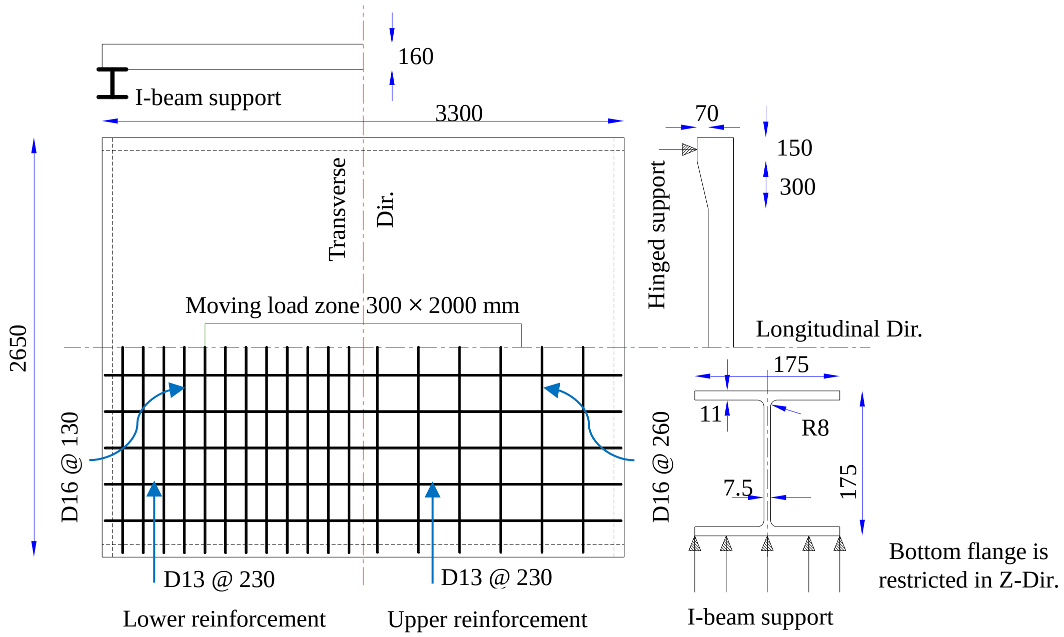


Fig. 3 RC slab geometry (all dimensions in mm)

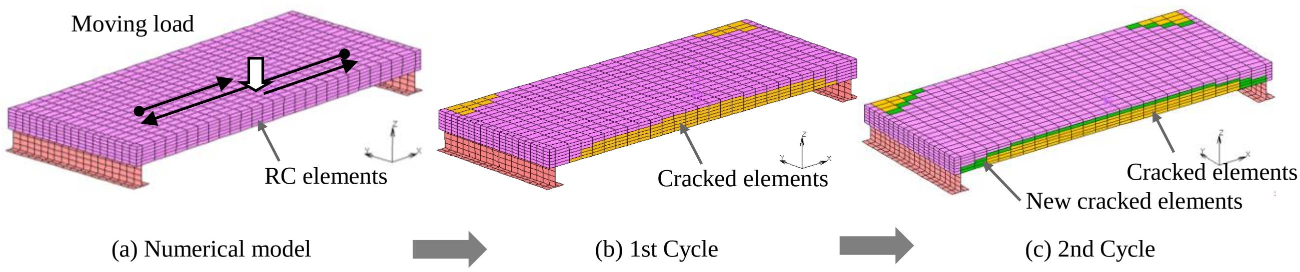


Fig. 4 Analytical procedures

Mousavi[9] is used in this method to simulate the bond-slip effect on the stress-strain (σ-ε) relationship under monotonic loading. According to this modification, an equivalent bond strain is added to reinforcing bar strain as shown in Fig. 2. This leads to a decrease in the rebar effective stiffness and an increase in its deformations. This method can be employed for plain bars to obtain the effective stiffness, E_s^*, as follows.

$$E_s^* = \frac{f_y^*}{\varepsilon_y^*}, \tag{2}$$

$$\varepsilon_y^* = \varepsilon_y + \frac{S_y}{l_d}, \tag{3}$$

$$l_d = \frac{d_b \cdot f_y}{4 \cdot \tau_b}, \tag{4}$$

$$S_y = 0.4\left(\frac{d_b}{4} \cdot \frac{f_y}{\sqrt{f_c'}} \cdot (2\alpha + 1)\right)^{1/\alpha} + 0.34, \tag{5}$$

where f_y^* is the effective yield stress[10], S_y is the slip displacement of a steel bar at yield point[11], d_b is the rebar diameter, f_c' is the concrete compressive strength, α is a tuning parameter used for adjusting the local bond stress-slip relationship equaling 0.5 for plain bar, τ_b is the average bond stress, ε_y and ε_y^* are the explicit and effective yield strain, respectively.

At first cycle, the reduction of rebar stiffness due to an applied load is the essential effect on the RC element behaviors. In other cycles, the bridging stress degradation in concrete plays the most important role. For RC elements reinforced with deformed bars, the bond between a deformed bar and its surrounding concrete is assumed to be perfect. Giuffré-Menegotto-Pinto model[12] is used for the hysteretic behavior of a rebar under repetitive load.

Table 2 RC slabs details[6]

Slab ID	Load (kN)	Yield strength of rebar (MPa)	Concrete (MPa)	
			Strength	Stiffness
P110	110	235 (Plain bar)	43.0	25500
P150	150		41.7	25400
P190	190		36.6	26000
D150	150	345 (Deformed)	38.6	23900

An RC slab model of smeared crack elements is solved to verify this numerical method. The loading pattern and load levels are the same as those used in the experiments as shown in Table 2[6]. All slabs were supported by steel I-beams and hinged supports along its longitudinal and transverse directions, respectively. The concrete and reinforcement properties of these slabs are the same as those used in the experimental study. Dimensions and reinforcing arrangement of the tested RC slab are shown in Fig. 3.

The fatigue analysis procedures are shown in Fig. 4. A moving load starts at the center of the slab elements. These elements are unloaded, while other elements, adjacent to the right side of the loaded elements, are loaded simultaneously with equal increments. In this technique, a constant moving load is moved along longitudinal directions. Due to the

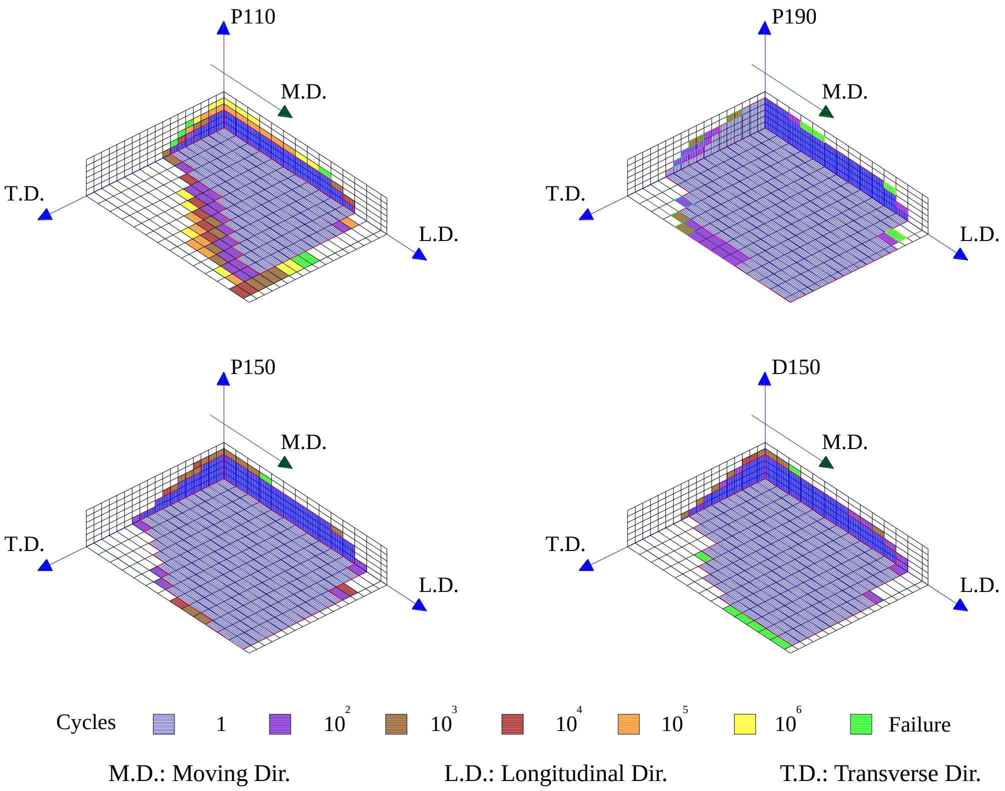


Fig. 5 Propagation of cracked elements

almost symmetry in longitudinal direction, half of the model is analyzed. As shown in Fig. 4 (b), this loading results in the propagation of cracked elements in the first cycle. By increasing the number of cycles, the concrete tensile strength of cracked elements are decreased according to the bridging stress degradation concept. The load capacity cannot reach the moving load level with these existing cracked elements. To exceed the load level, new cracked elements are required as shown in Fig. 4 (c). According to this propagation and the bridging stress degradation of these cracked elements, the overall RC slab stiffness decreases with an increasing number of cycles. The cracked elements were stored to be used in updating the numerical model for the next cycle step at different moving load locations. In each cycle, this procedure is repeated until fatigue failure occurs, while the numerical results are recorded in each cycle.

3. RESULTS AND DISCUSSION

Fatigue analyses are conducted on four full-scale RC slabs to verify this numerical method with the experimental results[6)] as follows.

3.1 Propagation of cracked elements

Fig. 5 shows the propagation of cracked elements of the RC slabs under a moving load at different numbers of cycles. Due to the geometrical symmetry in both directions, the one fourth of a slab model is shown. Uncolored elements point to a non-cracked zone as shown in this figure. The cracked zone caused by the first cycle of the moving load is indicated in blue, while the cracked zones caused by further cycles are indicated by other colored elements. The transverse direction demonstrates the propagation of cracked elements that expands from the center of the loading area, whereas the longitudinal direction shows the cracked elements that propagate in the movement zone in the longitudinal direction. The reason is that the effect of load movement was transmitted along the longitudinal direction. The fatigue life of these slabs can be indicated by the cracked zone after the first cycle as following explanation. P110 shows a smaller cracked zone at the first cycle than that of other slabs. Therefore, more cycles are required for this slab to propagate additional cracked zones until fatigue failure occurs compared with the other slabs. Moreover, cracked elements at the first cycle were subjected to crack closing and opening processes more than those at the additional cycles. These cracked elements deteriorated more than the cracked elements at later cycles with an increasing number of cycles according to Eq. (1). Therefore, RC slab with the largest cracked zone at the first cycle, P190, deteriorated more quickly than the other slabs.

The propagation of the cracked elements observed in D150 and P150 show that D150, which was reinforced with deformed bars, resulted in a slightly smaller cracked zone at the first cycle than P150, which was reinforced with plain bars. The reason is that the effect of bond between a plain bar and its surrounding concrete leads to larger slab deformations than that between a deformed bar and its surrounding concrete.

According to cracked elements propagations, moving load repetition results in a decreasing of bridging stress in concrete according to Eq. (1). The cracked elements are vertically propagated with an increasing number of cycles. This results in the loss of sectional moment balance in the end. At this time, the numerical analysis will terminate indicating to a fatigue failure.

3.2 Center displacement evolution

Fig. 6 shows the center displacement evolution versus the number of cycles to compare with the experimental results. The propagation of cracked elements and its degradation leads to a decrease in the RC slab stiffness and an increase in slab center displacement with increasing number of cycles. The slab under a higher moving load level shows a larger slab center displacement than that under a lower moving load level due to the initial displacement at first cycle. At a higher moving load level, the center displacement evolution shows a larger slope

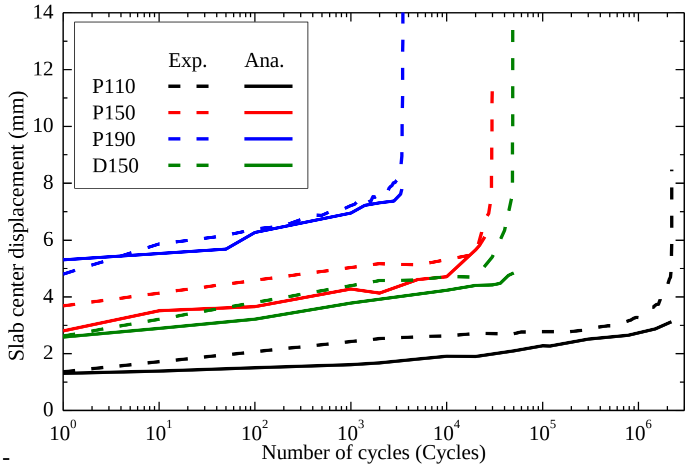


Fig. 6 Center displacement evolutions

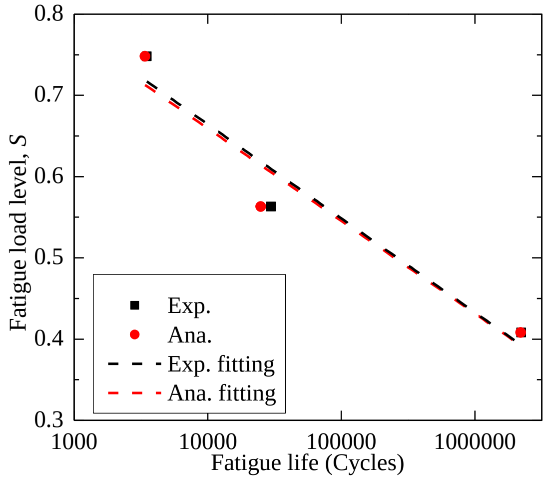


Fig. 7 S-N relationship

Table 3 Fatigue life of RC slabs

Slab ID	Fatigue life (Cycles)	
	Numerical	Experimental[6]
P110	2,200,000	2,160,00
P150	25,500	29,350
P190	3,400	3,400
D150	50,000	48,150

than that at a lower moving load level. The reason is that the slab under a higher moving load level deteriorated more rapidly than that at lower moving load level. According to reinforcing bars modeling, the center displacement evolution of RC slab reinforced with deformed bars, D150, is smaller than that reinforced with plain bars, P150.

The comparison of center displacement evolutions between the experimental and analytical results at different moving load levels can be shown in this figure. It shows similar values for fatigue life and center displacement, indicating an acceptable agreement between them.

3.3 Fatigue life and S-N relationship

The ratio of an applied moving load to the ultimate static load is considered as the fatigue load ratio, *S*. The fatigue life of RC slabs is defined as a relationship between fatigue load ratio and the number of cycles to failure. Fig. 7 shows the S-N relationship of RC slabs under a moving load is plotted with the experimental results for verification. It reveals a good agreement between the analytical results of RC slabs and the fatigue life of those slabs from the experiment. Due to the propagation of cracked elements, the RC slab subjected to a higher fatigue load ratio shows a shorter fatigue life than that at a lower fatigue load ratio. Table 3 shows the fatigue life of theses slabs.

4. CONCLUSIONS

From the numerical results and the experimentally verification, the following can be concluded:

1) The fatigue behaviors of RC slabs reinforced with plain bars under moving load can be analyzed by this numerical method.
2) The propagation of cracked elements due to its bridging stress degradation is considered as the main cause of fatigue failure.
3) The proposed numerical method integrated the bond-slip effect of plain bars. This leads to obtain accurate fatigue life estimations.
4) This method was verified using the experimental results, and it shows a good agreement.

REFERENCES

1) Matsui, S.: Fatigue strength of RC-slabs of highway bridge by wheel running machine and influence of water on fatigue, Proceedings of Japan Concrete Institute, Vol. 9, No. 2, pp. 627-632, 1987.
2) Perdikaris, P. C. and Beim, S.: RC bridge decks under pulsating and moving load, Journal of Structural Division, ASCE, Vol. 114, No. 3, pp. 591-607, 1988.
3) Maekawa, K., Toongoenthong, K., Gebreyouhannes, E. and Kishi, T.: Direct path integral scheme for fatigue simulation of reinforced concrete in shear, Journal of Advanced Concrete Technology, Vol. 4, No. 1, pp.159-177, 2006.
4) Li, V. C. and Matsumoto, T.: Fatigue crack growth analysis of fiber reinforced concrete with effect of interfacial bond degradation, Cement and Concrete Composites, Vol. 20, No. 5, pp. 339-351, 1998.
5) Suthiwarapirak, P. and Matsumoto, T.: Fatigue analysis of RC slabs and repaired RC slabs based on crack bridging degradation concept, Journal of Composites for Construction, Vol. 8, No. 2, pp. 132-140, 2004.
6) Shakushiro, K., Mitamura, H., Watanabe, T., and Kishi, N.: Experimental study on fatigue durability of RC slabs reinforced with round steel bars, Journal of Structural Engineering. Vol. 57A, pp. 1297-1304, 2011 (in Japaness).
7) Maekawa, K., Okamura, H., and Pimanmas, A.: Non-linear mechanics of reinforced concrete, CRC Press.
8) Jun, Z., and Stang, H.: Fatigue performance in flexure of fiber reinforced concrete, ACI Materials Journal, Vol. 95, No. 1, pp. 58-67, 1998.
9) Dehestani, M. and Mousavi, S. S.: Modified steel bar model incorporating bond-slip effects for embedded element method, Construction and Building Materials, No. 81, pp. 284-290, 2015.
10) Belarbi, A. and Hsu, T. T.: Constitutive laws of concrete in tension and reinforcing bars stiffened by concrete, ACI structural Journal, Vol. 91, No. 4, pp. 465-474, 1994.
11) Melo, J., Fernandes, C., Varum, H., Rodrigues, H., Costa, A., and Arêde, A.: Numerical modelling of the cyclic behaviour of RC elements built with plain reinforcing bars, Engineering structures, Vol. 33, No. 2, pp. 273-286, 2011.
12) Menegotto, M. and Pinto, P. E.: Method of analysis for cyclically loaded reinforced concrete frames including changes in geometry and non-elastic behavior of elements under combined normal forces and bending moment, IASBE Proceedings, 1973.