Rebar force estimation using crack mouth opening displacement based on fracture mechanics with bond slip model

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1. INTRODUCTION

For reinforced concrete (RC) structures, rebars determine many structural properties, such as cracking resistance, ductility, impact resistance and fatigue resistance. Therefore, the state of rebars is an important indicator of structural performance, and rebar force estimation plays an important role in structural health monitoring (SHM) and maintenance of RC structures, whereas the rebar force is intrinsic and cannot be measured directly for existing structures. Fortunately, the crack bridging force, which is mainly from rebars for RC structures, has inherent connection with structural surface cracks. This connection provides a possibility of estimating rebar force through indirect approach.

Generally, in concrete structures, the effect of crack bridging forces due to a variety of elements (rebars, aggregates and fibers) is effectively modelled by a continuous distribution of force acting on crack faces. The integral transformation relating crack opening displacements (COD profile) with crack opening stresses due to applied loads and crack bridging forces for various geometries and load conditions have been proposed based on fracture mechanics [1-4]. Inversely, the estimation of crack bridging forces acting on the crack faces using the measured COD profile has been successfully conducted [5]. By assuming rebar forces as a step function and following a weight function method of determining stress intensity factor (SIF), a transformation between the rebar force and COD has been derived [6]. This transformation has been applied in estimating the rebar force in RC beams through inverse analysis of the experimental COD profile [7]. However, for many concrete structures, such as slabs, the COD profile is difficult to be measured, or even unmeasurable. Therefore, a convenient, sophisticated, and reliable technique using easily accessible data deserves great attention in infrastructure maintenance.

In this paper, based on fracture mechanics and local bond slip model, the transformation relating crack mouth opening displacement (CMOD) with rebar forces for RC structures under applied loads is established. Correspondingly, a rebar force estimation method is proposed through using only the easily measurable CMOD.

2. PROBLEM FORMULATION

When an RC structure is subjected to some external loads, the concrete cracking process is governed by two mechanisms: the activation of bond forces at the rebar-concrete interface and the bridging effect of rebars crossing cracks.

To better understand the concrete cracking process, this study interprets this process as consisting of two steps. Step 1: A perfect bond is assumed. The crack opens under applied loads and rebar bridging forces. The resulting crack profile is shown in Fig.1 marked as Line 1. Step 2: Bond slip occurs at rebar location and then transfers to the other positions along the crack. Correspondingly, the COD profile shifts to Line 2 in Fig.1. In Step 2, the rebar bridging force is changed corresponding to the shift due to bond slip.

In previous study of inverse analysis of COD profile, the crack bridging force along the crack can be estimated without considering bond slip because the COD profile shift due to bond slip can be captured and then reflected on the change of crack bridging force estimated through inverse analysis. Therefore, the actual bond slip is implicitly included and the result of inverse analysis of COD profile is the bridging force distribution along the crack regardless of where the bridging force comes from. On the contrary, the COD utilized in this study is only CMOD, which means only the change of COD due to bond slip at crack mouth can be captured, while the bond slip has a more significant contribution to COD profile shift at the region close to rebar.

Therefore, this study establishes the rebar force estimation model by calculating the bond slip related CMOD separately through introducing local bond slip model and then eliminates it from the experimental CMOD. The remaining CMOD is due to rebar bridging forces and applied loads which is calculated based on fracture mechanics of using weight function method in determining stress intensity factor. According to the method of CMOD calculation, the CMOD is divided into two: CMOD due to applied loads and rebars and CMOD due to bond slip, which will be introduced separately.

2.1 CMOD due to applied load and rebar

Based on fracture mechanics, the COD profile of a cracked RC beam can be calculated by

$$u(x)=\frac{4}{E'}\int_x^a\left[\int_0^{a'}G(x',a',b)\left[\sigma(x')-f(x')\right]dx'\right]G(x,a',b)da' \qquad (1)$$

where E' is a combination of elastic constants. x is the distance between the target position and the bottom face of the beam as shown in Fig.1. x' is the dummy variable for x. a is the crack length and a' is the dummy variable for a. b is beam width. G is a weight function which depends on the crack geometry only. For specimens of a variety of important geometries, the weight function can be found in handbooks of crack analysis [8]. $\sigma(x')$ is the stress that would exist on the crack faces in the absence of a crack and $f(x')$ is the stress on crack faces due to rebar force per unit length along the crack.

Correspondingly, defining $u_a(x)$ and $u_b(x)$ as the crack opening and crack closing due to applied load and rebar bridging, separately, then the complete forms of these two components of COD are

$$u_a(x)=\frac{4}{E'}\int_x^a\left[\int_0^{a'}G(x',a',b)\sigma(x')dx'\right]G(x,a',b)da' \qquad (2)$$

$$u_b(x)=-\frac{4}{E'}\int_x^a\left[\int_0^{a'}G(x',a',b)f(x')dx'\right]G(x,a',b)da' \qquad (3)$$

The current study focuses on the mode I fracture problem of RC beams. This fracture mode can be realized if the RC beam is subjected to a uniform bending moment. Under this load condition, linearly distributed bending stress would exist on the crack face considering the widely approved plane cross-section assumption for a linear elastic system. Thus, the stress distribution due to applied load should be:

$$\sigma(x)=\sigma\left(1-\frac{2x}{b}\right) \qquad (4)$$

where σ is the maximum stress at the extreme concrete fibers.

The linear elastic behavior is assumed until rebar yielding. The post-yielding behaviors do not need to be considered because the focus of this study is existing structures under service load.

The term $f(x)$ is assumed as a step function following [7].

$$F = \frac{M}{j \cdot d} \tag{5}$$

$$f(x) = f \cdot [H(x-h) - H(x-h-d_b)] \tag{6}$$

where M is the acting bending moment. jd is the internal lever arm between the tension force of rebars and the resultant compression force of concrete, where d is the effective beam depth. $f=F/d_b$ is a line load converted from the rebar point load. h and d_b are clear distance of rebar from the bottom face and rebar diameter. H is Unit Step Function.

The crack mouth opening displacements (CMOD) due to applied load (um_a), and rebar bridging force (um_b), can be calculated by setting x = 0 in Eq.(2) and Eq.(3), respectively. These two terms are expressed as

$$um_a = u_a(0) = \frac{4}{E'}\int_0^a\left[\int_0^{a'} G(x',a',b)\sigma\left(1-\frac{2x'}{b}\right)dx'\right]G(x,a',b)da' \tag{7}$$

$$\begin{aligned} um_b &= u_b(0) \\ &= -\frac{4}{E'}\int_0^a\left[\int_0^{a'} G(x',a',b)f(x')dx'\right]G(x,a',b)da' \\ &= -f \cdot um_{bunit} \end{aligned} \tag{8}$$

where um_{bunit} is CMOD due to unit linear rebar force.

2.2 CMOD due to bond slip

For RC structures, bond effect is the resistance of relative movement between rebars and concrete. It is one of the fundamental structural behaviors. The bond slip relation is very complicated because it is relevant to many factors, such as concrete strength, rebar surface characteristics and embedment length. Many experimental and theoretical investigations have been carried out on rebar/concrete bond effect under various load conditions, such as monotonic load and cyclic load for various bond conditions. By employing bond slip model for different load and bond conditions, CMOD due to bond slip at the corresponding condition can be obtained. In this paper, the study object is a beam [7], where a newly casted beam was cracked under monotonic load. Therefore, a versatile local bond slip model proposed [9] is employed in calculating the bond slip related CMOD. The local bond slip model can be simply expressed as

$$\tau = 0.9 f_c^{2/3}(1-e^{-40s^{0.6}}) \tag{9}$$

where $s=S/d_b$, τ and S are bond stress and slip at any point along rebar, respectively. f_c is concrete strength in MPa. The unit of slip (S) and rebar diameter (d_b) should be the same.

Defining the x coordinate axis is along rebar longitudinal direction and the point on rebar where the slope of the strain distribution curve is zero as the origin of x coordinate and zero-slip point, the differential equation of the normalized slip (s) with respect to x is obtained through substituting the relations between rebar strain, bond stress and bond slip shown in Fig.2 into Eq.(9).

$$\frac{E_s \cdot d_b^2}{4}\frac{d^2s}{dx^2} = 0.9 f_c^{2/3}(1-e^{-40s^{0.6}}) \tag{10}$$

For long enough embedment condition, the boundary conditions are:

$$x = L_s \Rightarrow \begin{cases} s = S_s / d_b \\ \sigma = f \cdot d_b / A \\ \varepsilon_s = \sigma / E_s \end{cases} \qquad x = 0 \Rightarrow \begin{cases} s = 0 \\ \sigma = 0 \\ \varepsilon_s = 0 \end{cases} \tag{11}$$

where A is rebar sectional area; L_s is the debonding length which is unknown. E_s is rebar modulus.

Obviously, Eq.(10) has no theoretical solution and is solved numerically. To facilitate application, the numerical result is fitted by a polynomial function. The fitting result of the stain to slip relation is simply expressed as

$$S(x) = S(\varepsilon(x)) \tag{12}$$

Both the numerical and fitting strain to slip relations are shown in Fig.5. Regarding the slip as the COD of the center point of rebars, the CMOD can be obtained by assuming the COD due to bond slip increasing linearly from rebars to the crack mouth with a slope of

$$\psi = \frac{a}{a-h-r} \tag{13}$$

where r is the rebar radius. Then the CMOD due to bond slip, $um_s(\varepsilon_s)$, is given as

$$um_s(\varepsilon_s) = S(\varepsilon_s) \cdot \psi \tag{14}$$

where $\varepsilon_s = f \cdot d_b / E_s \cdot A$ is rebar strain at crack location.

2.3 Formula for rebar force estimation

According to previous sections, the calculation formulas for CMOD due to the aforementioned three main contributors have been established. Then, the formula for rebar force estimation using CMOD is derived following the idea that the total CMOD is the summation of CMOD due to each contributor, which is expressed as

$$um = um_a + um_b + um_s = um_s - f \cdot um_{bunit} + um_s(\varepsilon_s) \cdot \psi \tag{15}$$

where um is the total CMOD. The only unknown variable for Eq.(15) is the rebar force f which is obtainable by solving the equation.

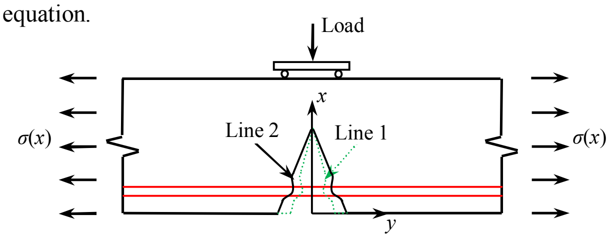

Fig. 1 Simple understanding of cracking process

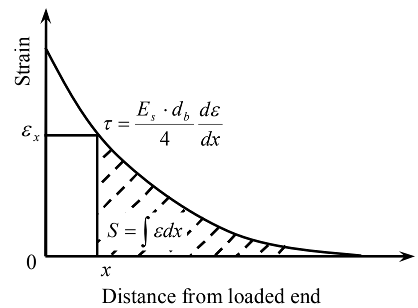

Fig. 2 Relation between rebar strain, bond stress and bond slip

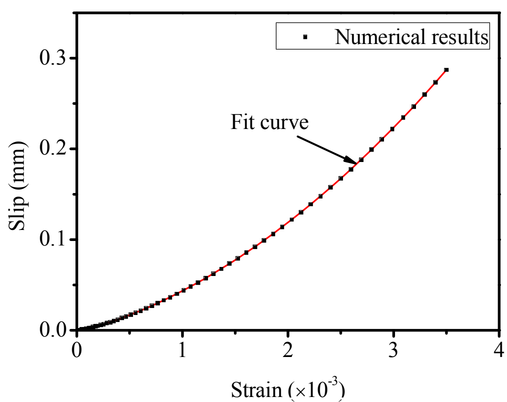

Fig. 3 Strain vs slip relation

3. METHOD APPLICATION

In this study, to facilitate the comparison with the experimental results [7], both analytical and experimental studies are conducted to simulate the crack opening of the tested beam in the reference. Since the focus of this study is mode I fracture problem, a notch of 1 cm depth was set at the mid-span on the bottom face of the beam to ensure the major crack initiates from this position. As a result, under four-point bending tests, the major crack of the RC beam will propagate vertically and almost no shear forces will be transferred crossing the crack because the crack stays in the pure bending region all the time.

The COD data were collected at points with 1 mm spacing along the crack. The experimental COD profiles are drawn by collecting the isolated COD data points with straight line. A typical COD profile for load equaling 19.60 kN is shown in Fig.4. Random fluctuations are observed, which is due to both inherent toughness of the fracture surface, such as aggregates, impurities, voids and heterogeneity in compaction, and error in COD measurement.

In this study, firstly, the maximum COD, COD_{max}, is treated as the experimental CMOD for rebar force estimation. This treatment seems to be more reasonable when considering the adopted assumption that concrete tensile effect is negligible comparing with that of rebars.

However, due to the existence of the notch, the COD_{max} should be still smaller than the actual experimental CMOD. Thus, another experimental CMOD naming as COD_{ext} is employed for rebar force estimation as well. The COD_{ext} is determined by extending the COD profiles to the bottom surface of the beams following the linear polynomial fitting function of the experimental COD profiles. The reasons for using the linear polynomial function are, firstly, to reduce the influence of the fitting curve slope in the immediate vicinity of the notch, because the slope is determined by the COD data closing to the notch; secondly, a strong linear relation is observed in crack faces for all load conditions, which is basically abide to a generally accepted assumption for RC beam behavior under bending, plane cross-section assumption. The fitting curve and function for the typical experimental COD profile are shown in Fig.4. According to the same process, both of these two kinds of CMOD for all load conditions are obtained and listed in Table 1.

Table 2 shows the rebar force estimation results from different approaches: (1) Inverse analysis of COD profile by employing Tikhonov regularization method in [7]; (2) CMOD analysis method using COD_{max}; (3) CMOD analysis method using COD_{ext}; (4) Standard RC cracked beam transformed section analysis as stated in Eq.(5), which is regarded as theoretical rebar force. Comparing with results from approach (4), the estimating errors of each method can be obtained, which are listed in Table 2 as well.

It is found that the accuracy of CMOD analysis method, especially using COD_{ext}, is generally higher than the inverse analysis of COD profile. This can be explained as: assuming the bridging forces as a continuous function $p(x)$, then the COD, $u(x)$, is given by

$$u(x) = \frac{4}{E'}\int_x^a \left[\int_0^{a'} G(x',a',b)\left[\sigma(x') - p(x')\right]dx' \right] G(x',a',b)da' \qquad (16)$$

For a crack on a structure under applied loads, a and $G(x, a, b)$ are invariable. Thus

$$u(x) \propto \int_x^a \int_0^{a'} p(x')dx'da' \qquad (17)$$

and

$$p(x) \propto \frac{d^2u(x)}{dx^2} \qquad (18)$$

which means the bridging force, $p(x)$, is proportional to the curvature of COD profile, $u(x)$. Therefore, the error of the inverse analysis of COD profile is mainly relevant to the ratio of the fluctuate amplitude caused by noise to the depression due to rebars, while the error for the CMOD analysis method is proportional to the ratio of fluctuation amplitude of noise to the total CMOD. As a result, the accuracy of the CMOD analysis model is more stable than the inverse analysis of COD profile.

For load equaling 15.68 kN, the estimating error for inverse analysis of COD profile is very high and Eq.(15) has no solution for using either COD_{max} or COD_{ext}. This is caused by the errors in the experimental COD profile where random fluctuations are observed, especially at the region close to the crack mouth as shown in Fig.1. Assuming

$$g(f) = um - um_a + f \cdot um_{bunit} - um_s(\varepsilon_s) \cdot \psi \qquad (19)$$

The differential equation of Eq.(19) with respect to f is

$$\frac{\partial g(f)}{\partial f} = f \cdot um_{bunit} - \frac{d_b}{E_s A} \cdot \left.\frac{dum_s}{d\varepsilon}\right|_{\varepsilon=\varepsilon_s} \psi \qquad (20)$$

For load equaling 15.68 kN, um_{bunit}=6.39×10^{-5} mm. According to Fig.1, $dum_s/d\varepsilon$ belongs to {80mm, 110mm}, then the second term in the left part of Eq.(19) should belong to {8.9×10^{-5} mm, 1.2×10^{-4} mm}. As a result

$$\frac{\partial g(f)}{\partial f} < 0 \qquad (21)$$

while g(0)<0. Thus, Eq.(15) has no solution.

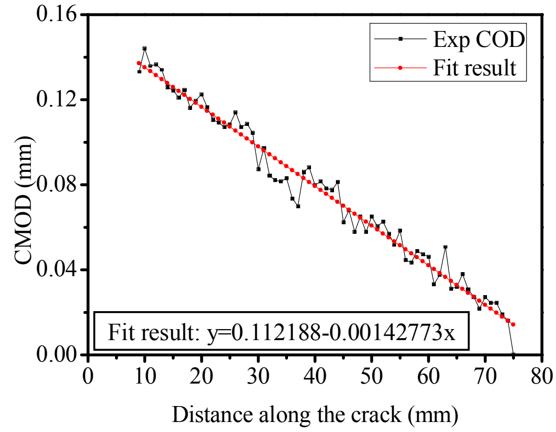

Fig. 4 Typical COD profile at load=19.60kN

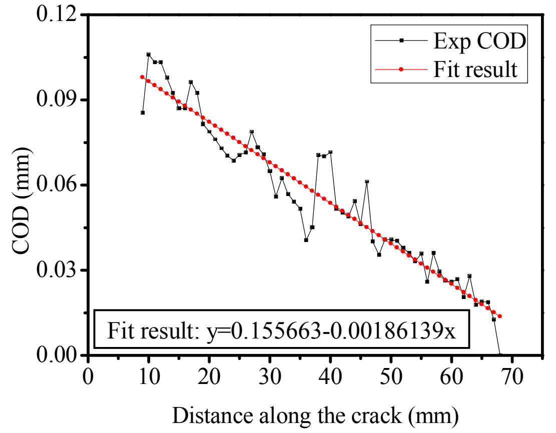

Fig. 5 COD profile at load=15.68kN

Table 1 CMOD under different loads

Load (kN)	15.68	17.64	19.60	21.56	23.52	25.48
$CMOD_{max}$(mm)	0.106	0.125	0.144	0.181	0.201	0.228
$CMOD_{ext}$ (mm)	0.112	0.132	0.156	0.192	0.221	0.257

Table 2 Comparison of results from different approaches

Load (kN)	Rebar force (kN/mm^2)				Estimating error (%)		
	(1) Inverse analysis of COD profile	(2) Analysis of COD_{max}	(3) Analysis of COD_{ext}	(4) Section analysis	[(4)-(1)]/(4)	[(4)-(2)]/(4)	[(4)-(3)]/(4)
15.68	311.090	No solution	No solution	193.763	-60.55	No solution	No solution
17.64	304.202	257.861	244.565	217.919	-39.59	-18.33	-12.23
19.60	257.835	311.024	278.383	242.075	-6.51	-28.48	-15.00
21.56	318.32	286.786	265.069	266.232	-19.56	-7.72	0.44
23.52	290.388	318.368	278.009	280.380	-3.57	-9.64	4.26
24.48	314.544	322.781	287.311	319.266	1.48	-2.62	8.66

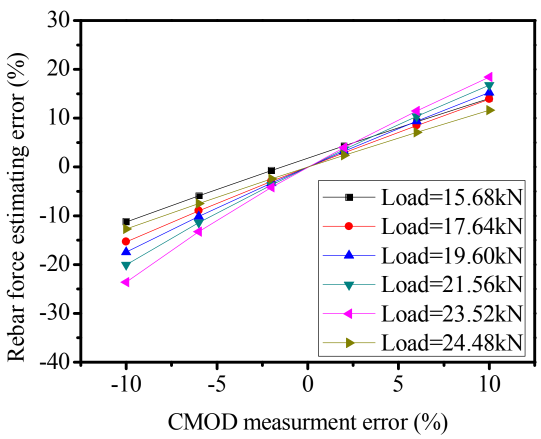

Fig. 6 Sensitivity study of CMOD measurement error

4. SENSITIVITY STUDY

This study aims at developing a maintenance technique for existing structures based on field measurement data. The reliability and anti-interference ability deserve great attention. Since the result error is mainly from the measurement error of CMOD according to previous analysis, result sensitivity study on error in CMOD is conducted in this paper.

In theoretical scale, the CMOD, the left term of Eq.(15), can be calculated through direct analysis. By inputting different levels of error into it and solving Eq.(15), the corresponding rebar force error vs CMOD error relation can be obtained, as shown in Fig.6. The errors of CMOD and rebar force are defined as Eq.(22).

$$\text{PRM error (\%)} = \frac{\text{PRM with - PRM without error}}{\text{PRM without error}} \times 100 \qquad (22)$$

where PRM is the abbreviation of parameter. It is found that the accuracy loses with the increasing of noise level almost linearly. This can be explained by Eq.(19). The positive slope means the estimated rebar force increases with the decrease of CMOD which really makes sense because larger rebar force lead to smaller crack opening and can be explained by Eq.(19) as well.

5. CONCLUSIONS

Following the mechanisms of concrete cracking for RC structures, the relation between crack mouth opening displacement (CMOD) and the rebar forces is developed based on fracture mechanics and bond slip model. Correspondingly, a rebar force estimation method through CMOD analysis is established.

The proposed CMOD analysis method is employed successfully in rebar force estimation for a newly casted RC beam cracked under four-point bending tests. Under almost all load conditions, better accuracy of rebar force estimation is observed for CMOD analysis method than a verified method, the inverse analysis of COD profile, which further verifies the applicability and stability of the proposed method.

The accuracy of rebar force estimation depends on the accuracy of CMOD measurement and the employed bond slip model. More sophisticated techniques of CMOD measurement, such as treating the average value of CMODs on the bottom face of the beams as the experimental CMOD, should be adopted.

Since the primary error source is measurement CMOD, sensitivity study has been conducted on it. It is found that the errors of rebar force estimation increased almost linearly with the amount of error inputted into the CMOD. This result further emphasized the importance of improving the accuracy of obtaining it.

The most important advantage of this method lies in the fact that only CMOD and crack depth are necessary for rebar force estimation, which makes it a potential non-destructive test and evaluation (NDT & E) technique for structure health monitoring of existing RC structures where the COD profile is unmeasurable

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