Developing a numerical model for fatigue life prediction of plain concrete beams

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1. INTRODUCTION

Previously, concrete structures were designed to support both static and dynamic service loads which are appropriately small to cause failure. But, the repetition of these loads in the long term performance dangerous effects on the safety of constructions. So, there is a growing request of organizations and engineers to estimate the life and performance of structures. So, a new concept for structural collapses was appeared as fatigue failure.

Accurate modeling of fatigue failure mechanism is the real key to avoid the mentioned problem, so it is important to develop an effective method for predicting fatigue performance of concrete structures. Namely, its degradation and failure processes have to be characterized by progressive concrete cracking.

Many previous studies of the behavior and modeling of concrete structures at multi-scale levels exist to obtain the improvement of fatigue analysis method. The recently developed fatigue analysis method is proposed for solving the problems of concrete structures depending on the general concept that the propagation characteristics of both flexural and shear cracks are taken into account as an inducement of the failure. The bridging stress degradation relation is proposed for cementitous materials such as plain concrete. The concept of fatigue crack propagation was proposed for predicting fatigue life of fiber reinforced concrete beam by Li and Matsumoto[1]; Matsumoto T.[2]; and Zhang[3] and RC beams by Suthiwarapirak et al.[4]. The bridging stress degradation, the reduction of transferred stress across a crack under fatigue loading, has been introduced for the first time by Li and Matsumoto[1] as a principal cause of fatigue crack propagation in concrete and fiber reinforced concrete beams.

According to these studies and the possible resources for their solutions, the researchers have succeeded in providing a good conception of fatigue failure mechanism. But, dealing with the large structural elements requires using of numerical computational method as finite element method (FEM). So, the authors believe that it is time to improve well established numerical proposal that will be used for static and fatigue analysis.

In the present study, a developed three-dimensional model of FEM is used to simulate the concrete behaviors, concrete fatigue regarding distributed cracks and their bridging stress degradation characteristics. The fatigue life, crack propagation characteristics, and failure mechanism of repeated load of plain concrete beams will be numerically obtained by this study and will be compared with experiments by Zhang et al.[5] to verify the numerical model.

2. ANALYTICAL MODEL APPROACH

The procedure of analysis for predicting fatigue behavior of concrete beams is shown in **Fig. 1**. A FEM is proposed based on the fatigue failure mechanisms. The fatigue cracks propagation due to crack bridging degradation is taken into account as the main mechanism that leads to a beam failure. The concrete material model is established from material tests under both static and fatigue loading as mentioned in the previous studies[4, 6]. With this material model data, the performances and evaluation of concrete beams fatigue, such as *S-N* relationship, and failure mode, can be analyzed with given geometries and boundary conditions.

In this study, the developed 3D FEM is used to solve a smeared crack model. An incremental-iterative method based on the Newton-Raphson iteration scheme is employed for getting the solution of nonlinear constitutive laws for concrete, which adopted from the literature[7] as the following equations in **Table 1**.

Crack propagation characteristics of concrete are represented by 8-node 3D smeared crack elements with multiple fixed crack concepts[8]. First crack is assumed to start perpendicular to the direction of the maximum principal strain in concrete matrix when tensile strain is larger than the cracking strain. After the first crack forms, the second crack can propagate perpendicular to the first crack when the second tensile strain component exceeds the cracking strain. Also, the third crack can form perpendicularly to the existing two cracks (**Fig. 2**). There is one normal stress component, σ, and two shear stress components, τ, on each crack plane. The relationships between stresses, σ, and strains, ε, on the crack plane are shown by the following equation. It is assumed that there is no dilatancy between normal and shear terms.

$$\begin{Bmatrix} \sigma_{11}^{cr} \\ \tau_{12}^{cr} \\ \tau_{13}^{cr} \end{Bmatrix} = \begin{bmatrix} d^{cr} & 0 & 0 \\ 0 & d_{\tau}^{cr} & 0 \\ 0 & 0 & d_{\tau}^{cr} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_{11}^{cr} \\ \varepsilon_{12}^{cr} \\ \varepsilon_{13}^{cr} \end{Bmatrix} \qquad (1)$$

Herein d^{cr} = tangential slope of the normal stress–strain relationship after cracking and d_{τ}^{cr} = tangential slope of shear stress–strain relationship. The overall stress–strain relationship of the smeared cracked elements of concrete with respect to the global coordinate system can be written as follows[9].

$$\sigma = D^{cocr}\varepsilon \qquad (2)$$

$$D^{cocr} = \left[D^{co} - D^{co}N_t(D^{cr} + N_t{}^t D^{co} N)^{-1} N_t{}^t D^{co}\right] \qquad (3)$$

where D^{cocr} = stiffness matrix of cracked concrete element formulated from the stiffness matrix of crack, D^{cr}, and stiffness matrix of concrete in elastic, D^{co}. N_t is a transformation matrix reflecting the orientation of the crack in **Fig. 2** (b). The off-diagonal terms of the crack stiffness matrix, D^{cr}, are negligible.

For introducing a cracking behavior under static loading, the normal stress-strain relationship or the bridging stress relation was presented.

The concrete crack bridging stress degradation characteristic under fatigue loading is the most important part in the fatigue analysis. When the concrete bridging stress degradation relation describing the degradation characteristics of transferring stress across a crack under fatigue loading was prescribed, the crack propagation rate can be predicted so that

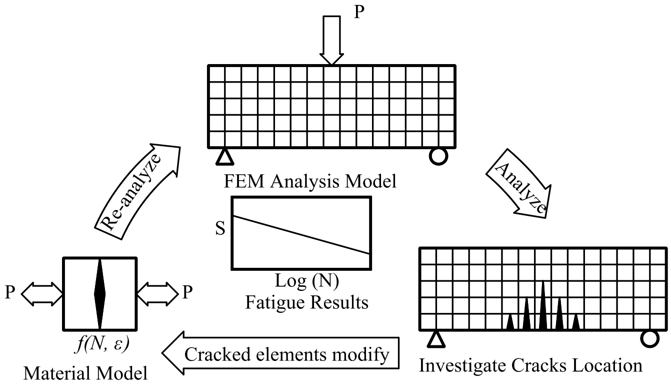

Fig. 1 Scheme of fatigue analysis

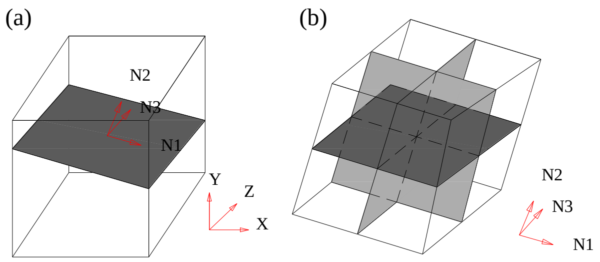

Fig. 2 Crack formation: (a) initiation of first crack; (b) three perpendicular crack formations

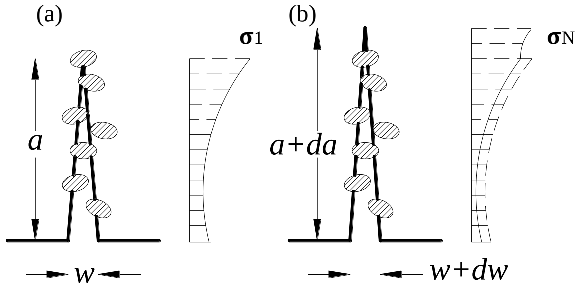

Fig. 3 Crack propagation due to bridging stress: (a) first cycle; (b) after N cycles

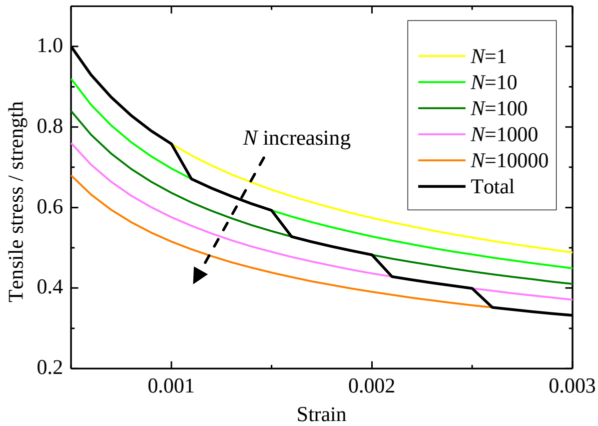

Fig. 4 Crack bridging degradation

Table 1. Constitutive laws for concrete[8)]

	Compression		Tension
$0 \geq \varepsilon > \varepsilon_m$	$E = 2\dfrac{f_c}{\varepsilon_m}\left(1-\dfrac{\varepsilon}{\varepsilon_m}\right)$ $\sigma = f_c\dfrac{\varepsilon}{\varepsilon_m}\left(2-\dfrac{\varepsilon}{\varepsilon_m}\right)$	$\varepsilon_t \geq \varepsilon \geq 0$	$E = E_c$ $\sigma = E\varepsilon$
$\varepsilon < \varepsilon_m$	$E = \dfrac{f_c}{\varepsilon_m - \varepsilon_u}$ $\sigma = \dfrac{f_c \cdot \varepsilon_u}{\varepsilon_u - \varepsilon_m} + E \cdot \varepsilon$	$\varepsilon > \varepsilon_t$	$E = -0.4 \cdot \dfrac{f_t}{\varepsilon} \cdot \left(\dfrac{\varepsilon_t}{\varepsilon}\right)^{0.4}$ $\sigma = f_t\left(\dfrac{\varepsilon_t}{\varepsilon}\right)^{0.4}$

E_c = the modulus of elasticity of concrete, E = tangential modulus of concrete, f_c = concrete compressive strength, $\varepsilon_m = f_c/2E_c$ = concrete strain corresponding f_c, f_t = tensile strength, and $\varepsilon_t = f_t/E_c$ = strain at tensile strength.

the required loading cycles until failure can be estimated. The bridging stress degradation occurs to the deterioration of material constituents on the crack plane under repetitive load. It is mainly caused by the deterioration of aggregate bridging. The relation of concrete bridging stress degradation is assumed to be contingent on two main parameters corresponding to tensile strain, ε_t, and number of cycles, N, and it can be expressed as [3, 10)].

$$\frac{\sigma_N}{\sigma_1} = f(N, \varepsilon_t) = 1 - (0.08 + 4\varepsilon_t l)\log(N) \qquad (4)$$

where $f(N,\varepsilon_t)$ = bridging stress degradation ratio, which is less than 1. σ_N and σ_1 are bridging stress at the Nth and first cycle, respectively.

The process of bridging stress degradation leads to the fatigue crack propagation of concrete as shown in **Fig. 3** (a). A crack begins with a length, a, and width, w, due to aggregate bridging under loading. When the crack experiences repetitive loading, transferred stress across the crack decreases due to the gradual deterioration of concrete on the crack plane. Here, the aggregates lose their bonds. The equilibrium cannot be retained with the degraded bridging stress distribution of the cracked area. Therefore, the existing crack propagates with additional length, da, **Fig. 3** (b). This procedure will be continued until the load capacity initiates to drop with the increasing crack length. According to this procedure, for the Nth load cycle, the fracture zone will be separated into N sections with different fatigue histories, ranging from 1 to N cycles as showing in **Fig. 4**.

3. VERIFICATION OF ANALYTICAL MODEL

3.1 One-element analysis

In order to verify the stress-strain relationship with regard to the used equations, static analysis for one element made from plain concrete with 100 x 100 x 100 mm dimensions will be used.

This element was loaded on its top face and supported its bottom. **Fig. 5** shows the normalized stress-strain relationships for the analysis results compared with the used equations. The figure shows good matching between them. The sequence of bridging stress after cracking point is almost similar to the one observed in the used tension equation curve.

3.2 Modeling of plain concrete beams

In this study, the 3-point bending static and fatigue analysis

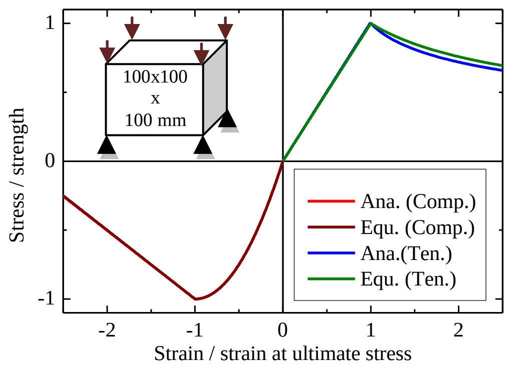

Fig. 5 Normalized stress-strain relationship

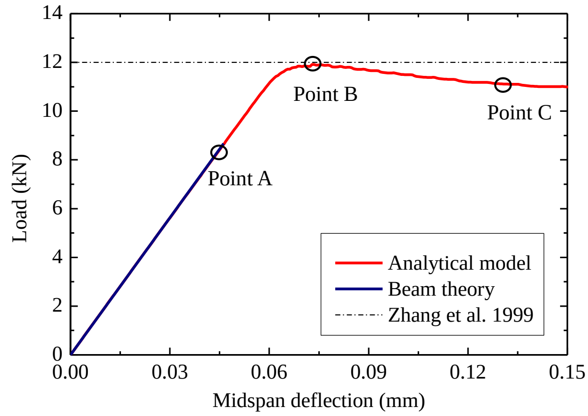

Fig. 8 Load-midspan deflection curves

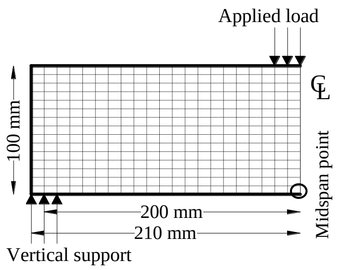

Fig. 6 Developed model

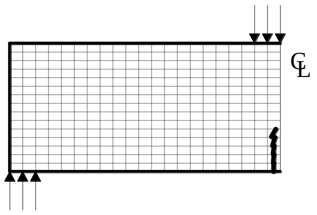

Fig. 7 Crack pattern

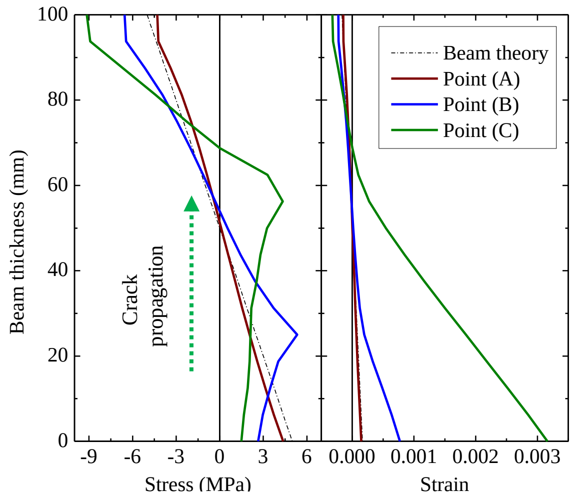

Fig. 9 Stress and strain distribution curves

of the plain concrete beams are carried out for a beam dimension of 100 x 100 x 420 mm and their span is 400 mm to compare with experimental results[5)]. The concrete proportions of these beams are the same as those used in Zhang et al.[5)] study. The tensile strength, f_t, compressive strength, f_c, and modulus of elasticity, E_c, of used concrete are 5.20 N/mm^2, 53.22 N/mm^2 and 30000 N/mm^2 respectively.

Fig. 6 shows the developed analysis model with its boundary conditions and dimensions. 8-Nodes 3D solid elements were utilized for concrete.

3.3 Static analysis results and discussions

As shown in **Fig. 7**, the crack pattern of the developed analytical model at maximum load shows that vertical crack formed in the maximum moment region. **Fig. 8** shows the load-midspan deflection relationships for it developed analytical model and hand calculation and shows the almost same initial stiffness. The developed model shows an acceptable agreement with the ultimate load level by Zhang[5)]. For hand calculation, consider a short segment of a simply supported rectangular beam with width b, depth t, and span L that is subjected to an external load P at the middle. Flexural stress σ and midspan deflection δ according to simple elastic theory given by the following equations.

$$\sigma = \frac{1.5\,P\,L}{b\,t^2} \quad \& \quad \delta = \frac{P}{4\,E_c\,b}\left(\frac{l}{t}\right)^3 \qquad (5)$$

Hand calculation shows smallest strength value because it was not taken into the account the bridging stress concept for cracked concrete.

Fig. 9 shows the crack tip gradually extended up with increasing of static loading. This figure shows stress and strain distributions along beam section at midspan for different loading points (A: pre-peak loading, B: peak loading, and C: post-peak loading). The stress and strain distributions of the loading point (A) having linear relationship and showing satisfactory agreement with hand calculated. Increasing load to a point (B) leads to increasing tensile stress and generating the crack that expands from the lower-center of the beam to crack tip. However, the crack tip located when the tensile strain is just larger than ε_t to show a bilinear strain and bridging stress distributions along the crack length. According to this procedure, for loading point (C), the crack propagation leads to generate of new fracture zone that showing larger strain value and smaller bridging stress at its crack mouth.

3.4 Fatigue analysis results and discussions

Fatigue strength is generally defined as a friction of the static strength that can be supported repeatedly for a given number of cycles.

The beams were tested under different maximum loads. Every beam has a constant amplitude fatigue loading between maximum and minimum load levels, where the minimum load is equal to 20% of the maximum load.

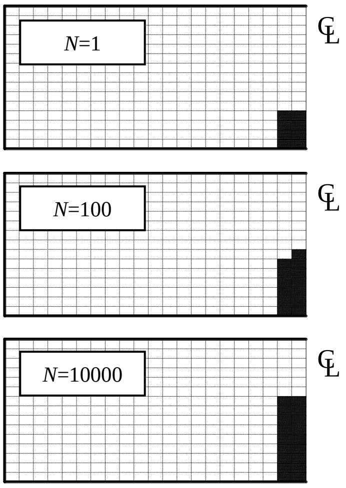

Fig. 10 Cracked elements propagation

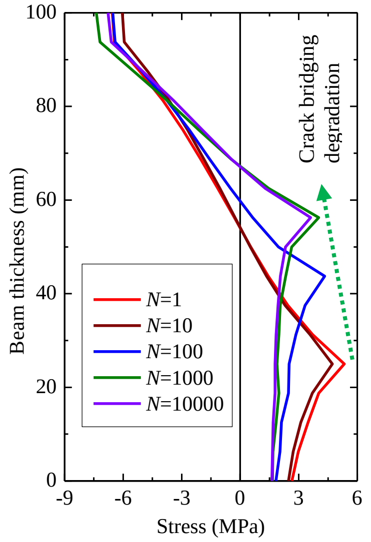

Fig. 11 Stress distribution curves

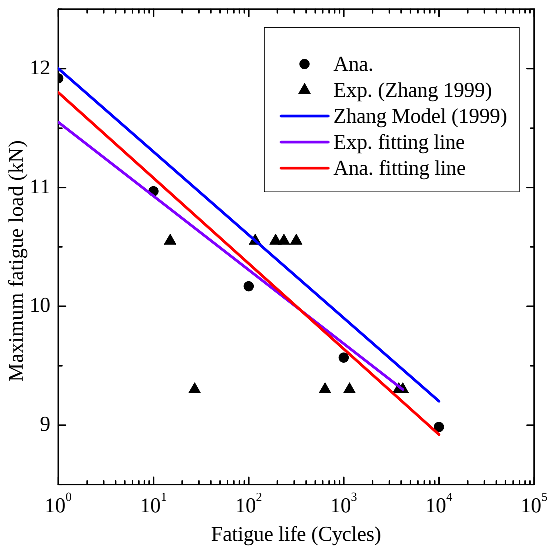

Fig. 12 Relationship of maximum load and fatigue life

As an example, a specific fatigue loading procedure with maximum load equal to 8.98 kN leads to generate a cracked zone at first cycle (**Fig. 10** *N*=1). The bridging formula without stress degradation was used in this area. With increasing number of cycles, the fatigue crack bridging degradation will occur in the fracture zone because it is subjected to closing and opening processes. The load capacity cannot reach the maximum load level with this already formed cracked area. Therefore, a new crack area is needed, in order to reach the maximum load level (**Fig. 10** *N*=100). The bridging formula with different cycle numbers will be used in the old fracture zones (N=2, 3~N_i) and the newly developed fracture zone (*N*=1). This progressive crack propagation proceeds until the final failure occurs (**Fig. 10** *N*=10000). The detailed expressions of bridging-stress degradation procedure along midspan section for different cycle numbers under constant maximum load equal to 8.98 kN are given in **Fig. 11**. This figure shows the crack bridging stress tip gradually extended up with increasing of cycle number.

By applying to this procedure for different maximum load levels, the fatigue life is predictable. The numerical result of the relationship between fatigue life and maximum load level compared to experimental results of Zhang et al.[5] is shown in **Fig. 12**. The numerical model shows satisfactory agreement compared with the experimental results by Zhang et al.[5]. One of the benefits of the developed FEM simulation is that it can capture much more crack in the material elements compared with the theoretical proposal of Zhang et al.[5] which considers only single crack at midspan.

4. CONCLUSIONS

The following conclusions can be derived from this analytical approach:

1) The fatigue failure mechanism and fatigue life prediction of the plain concrete beam can be simulated through this numerical model.
2) The fatigue results are obtained and good correlation between the experiments and the developed numerical model exists.
3) The developed model simulation takes into account multi cracks effect in the material.
4) By knowing bridging stress degradation relation, the fatigue properties such as the *S-N* relationship could be predictable.
5) Increasing of cycle number under constant amplitude fatigue loading leads to a decreasing of crack bridging stress and an increasing of the cracked zone area.

REFERENCES

1) Li, V. C., and Matsumoto, T.: Fatigue crack growth analysis of fiber reinforced concrete with effect of interfacial bond degradation, Cement and Concrete Composites, No. 20/5, pp. 339-351, 1998.
2) Matsumoto, T., and Li, V. C.: Fatigue life analysis of fiber reinforced concrete with a fracture mechanics based model, Cement and Concrete Composites, No. 21/4, pp. 249-261, 1999.
3) Zhang, J., and Stang, H.: Fatigue performance in flexure of fiber reinforced concrete, ACI Materials Journal No. 95/1, pp. 58-67, 1998.
4) Suthiwarapirak, P., Matsumoto, T., and Horii, H.: Fatigue life analysis of reinforced steel-fiber-concrete beams, Proc. Japan Concrete Institute, No. 23/3, pp. 127-132, 2001.
5) Zhang, J., Stang, H., and Li, V. C.: Fatigue life prediction of fiber reinforced concrete under flexural load, International Journal of Fatigue, No. 21/10, pp. 1033-1049, 1999.
6) Suthiwarapirak, P., and Matsumoto, T.: Fatigue analysis of RC slabs and repaired RC slabs based on crack bridging degradation concept, Journal of Structural Engineering, No. 132/6, pp. 939-948, 2006.
7) Maekawa, K., Toongoenthong, K., Gebreyouhannes, E., and Kishi, T.: Direct path-integral scheme for fatigue simulation of reinforced concrete in shear, Journal of Advanced Concrete Technology, No. 4/1, pp. 159-177, 2006.
8) Fukuura, N., and Maekawa, K.: Multi-dimensional crack model for in-plane reinforced concrete under reversed cyclic actions 4 way fixed crack formulation and verification, Computational modelling of concrete structure, de Borst, pp. 143-151, 1998.
9) Rots, J. G., and Blaauwendraad, J.: Crack models for concrete, discrete or smeared? Fixed, multi-directional or rotating?, Heron, No. 34/1, pp. 1–59, 1989.
10) Matsumoto, T., Suthiwarapirak, P., and Asamoto, S.: Model development of ECC fatigue analysis, Proc. Japan Concrete Institute in Japanese, No. 24/1, pp. 237-242, 2002.