Noise sensitivity study on inverse analysis of rebar force in RC

Hokkaido University	Student member	Pengru Deng
Hokkaido University	Member	Takashi MATSUMOTO
Hokkaido University	Fellow member	Toshiro HAYASHIKAWA
Hokkaido University	Member	Xingwen He

1. INTRODUCTION

Deteriorate aging infrastructure systems exhibits easily measuring information which reflects the present state of structures and can be used in structural health monitoring (SHM). Evaluation of physical and mechanical states of buried rebar, such as rebar force, location and diameter, is a non-destructive approach belonging to SHM and maintenance of RC structures. In this method, inverse analysis theories are employed and crack opening displacement (COD) is treated as input. Unlike the straightforward direct problem which maps crack bridging stresses into COD, the inverse problem is ill-posed due to the noisy COD. In this paper, Tikhonov regularization method[1] is adopted in dealing with this problem, whose practicability and efficiency have been verified[2].

Even though the applicability of inverse method has been verified, the accuracy of it depends largely on the noise level which is caused by various reasons, such as complicated COD profile, test conditions and artificial reasons. Therefore, analyzing the effect of noise level on the accuracy of inverse method is of great significance. However, since the noise level of experimental COD is uncontrollable, the synthesis noisy COD which consists of COD obtained from forward analysis and random numbers was regarded as study object. The random numbers are created with a normal (Gaussian) distribution having zero mean and a certain standard of derivation, which means the noise level can be easily changed by modifying the derivation.

This paper is structured in the following way: Firstly, introducing theoretical background of both forward and inverse analysis; then synthesizing CODs of different noise levels; lastly, conducting inverse analysis and determining the influence of noise level on accuracy of the method presented in this paper.

2. Theoretical background

2.1 Forward analysis

A two-dimensional bridged crack model is assumed for a through-the-thickness cracked RC beam, after the crack has passed all rebar layers. Linear elastic behavior of rebar and concrete is assumed at this stage, after the initial slip of rebar has occurred at crack initiation. These assumptions restrict the application of the current model within the loading interval between crack initiation and rebar yielding. Progressive debonding between steel and concrete is accounted for as [3,4].

The net COD profile composed of two effects which are crack opening due to external load and crack closing owing to bridging force. Since the bridging force passing though aggregates is much smaller than that of rebar, only bridging force contributed by rebar was taken into consideration in COD calculation. Following the procedures described by [5, 6], and simulating rebar force as shown in Fig.1, [7] derived COD profiles as

$$u(x) = u_a(x) - u_b(x) \tag{1}$$

where

$$u_a(x) = \frac{4(1-\nu^2)}{E_c}\int_x^a \int_x^{a'} G(x',a',b)\sigma(x')dx'G(x,a',b)da' \tag{2}$$

and

$$u_b(x) = \frac{4(1-\nu^2)}{E_c}\int_x^a \int_x^{a'} G(x',a',b)f(x')dx'G(x,a',b)da' \tag{3}$$

where E_c and ν are the Young's modulus and the Poisson's ratio of concrete respectively, and a is the crack length. The weight function for a particular crack geometry is denoted as $G(x,a,b)$, standard forms of which are available for a variety of geometry in stress intensity handbooks[8]. b is the beam total depth and x',a' are the dummy variables for x, a. The term $f(x')$ is the rebar force, considered as the force per unit length along the crack **Fig**.1, which must be integrated over the cracked domain to obtain the total rebar force F for an acting bending moment M as

$$\int_0^a f(x)dx = F = \frac{M}{jd} \tag{4}$$

jd is the internal lever arm between the total tension force in the rebar, and the total compression force in concrete, as simulated in the flexural of cracked RC beam cross-section, where d is the effective beam depth. The transformation of a point rebar force into a distributed force per unit length $f(x)$ is mathematically simulated by Unit Step Function for any (m) number of rebar layers as

$$f(x) = \sum_{i=1}^{m} f_i[H(x-h_i) - H(x-h_i-d_{b_i})] \tag{5}$$

Where, by $f_i=(F_i/d_{bi})$ the point loads of rebar are converted into line loads Fig.1 along the crack, F_i is the total force at the ith layer where the rebar diameter is d_{bi} , and h_i is the clear distance of a layer from the bottom face.

The CODs obtained from **Eq**.(1) are noiseless which should be made noisy by adding random numbers. The random numbers are created with a normal (Gaussian) distribution having zero mean and a certain standard of deviation.

2.2 Inverse analysis

Unlike the straightforward direct analysis, the inverse analysis of noise CODs is ill-posed, which should be treated by Tikhonov regularization method.

In **Eq**.(1), the first part of left term $u_a(x)$ can be divided by a grid of p points as

$$u_a(x) := \{u_{a_k}, k = 1,\cdots,p\} \tag{6}$$

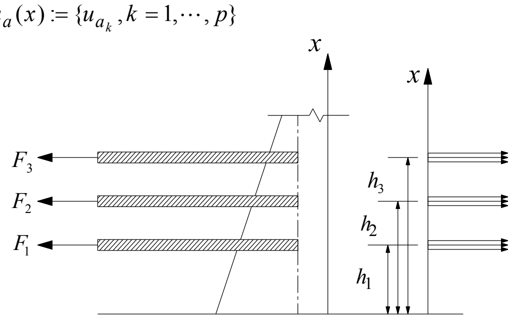


Fig. 1 Rebar forces are assumed as stepped functions

Similarly, u is denoted as

$$u(x) := \{u_k, k = 1,\cdots,p\} \tag{7}$$

As such, data points relevant to the second part are determined by subtracting u from $u_a(x)$ as

$$u_b(x) := \{u_{b_k}, k = 1, \cdots, p\} = u_a - u \tag{8}$$

But, the synthesis CODs contain errors, where an incorrect u^δ is obtained rather than a correct u. Consequently u_b is perturbed as u_b^δ up to noise level δ. We consider **Eq.**(3) as a linear operator equation T:Z→U between Hilbert spaces, which maps the rebar force f∈Z into crack closings u_b∈U, and we adopt the Tikhonov regularization method[1], where the extremals of the following function are sought

$$M^\alpha[f] = \left\| T_h f - u_b^\delta \right\|_U^2 + \alpha \left\| f \right\|_Z^2 \tag{9}$$

T_h is the numerical approximation of the transformation T, and α>0 is the regularization parameter. Then we reach a normal equation

$$Bf + \alpha C = v \tag{10}$$

The matrix C is a p×p identify matrix for linear space, while the matrix B and the vector ν have entries given by

$$B_{ij} = h_x \sum_{k=1}^{p} \left(\sum_{i=1}^{p} g_{li} g_{ki} \right) \cdot \left(\sum_{i=1}^{p} g_{li} g_{ki} \right) \in B \tag{11}$$

$$v_l = \sum_{k=1}^{p} \left(\sum_{i=1}^{p} g_{li} g_{ki} \right) \cdot u_{b_k}^\delta \in v \tag{12}$$

The weight function G of different situation can be found in [6]. For numerical computations, G is approximated by its finite difference equivalent tensor, entries of which are found within the current grid as

$$\left. \begin{aligned} g_{ij} &= G(x_i, a_i) & i &= 2, \cdots p-1 \\ g_{ij} &= \frac{G(x_i, a_i)}{2} & i &= 1, p \end{aligned} \right\} j = 1, \cdots, p \tag{13}$$

After the rebar force curve is obtained, the location of the centroid of rebar can be identified as the positon corresponding to the peak of curve, and the total rebar force is computed by the area under the bridging stress profiles.

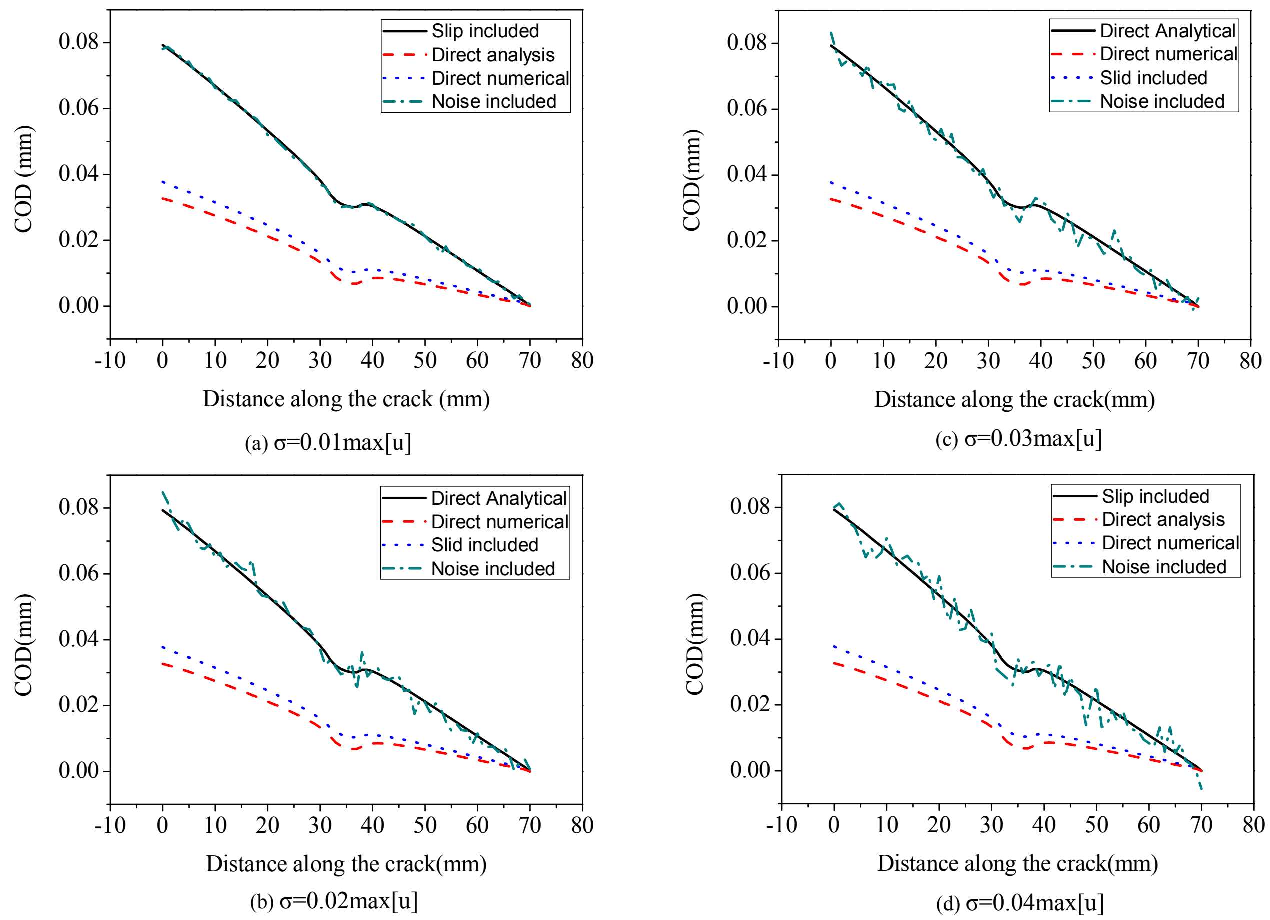
Accuracy of the inverse analysis method is checked by two ways (1) by comparing the estimated rebar force of inverse analysis with that obtained from other theoretical methods of analysis, and (2) by comparing the estimated rebar strain by inverse analysis with that obtained by the attached by the attached strain gauges.

The rebar diameter can be determined through methods, such as Half Point Method, Equivalent Area Method and Transition Point Method.

3. Noise effect study

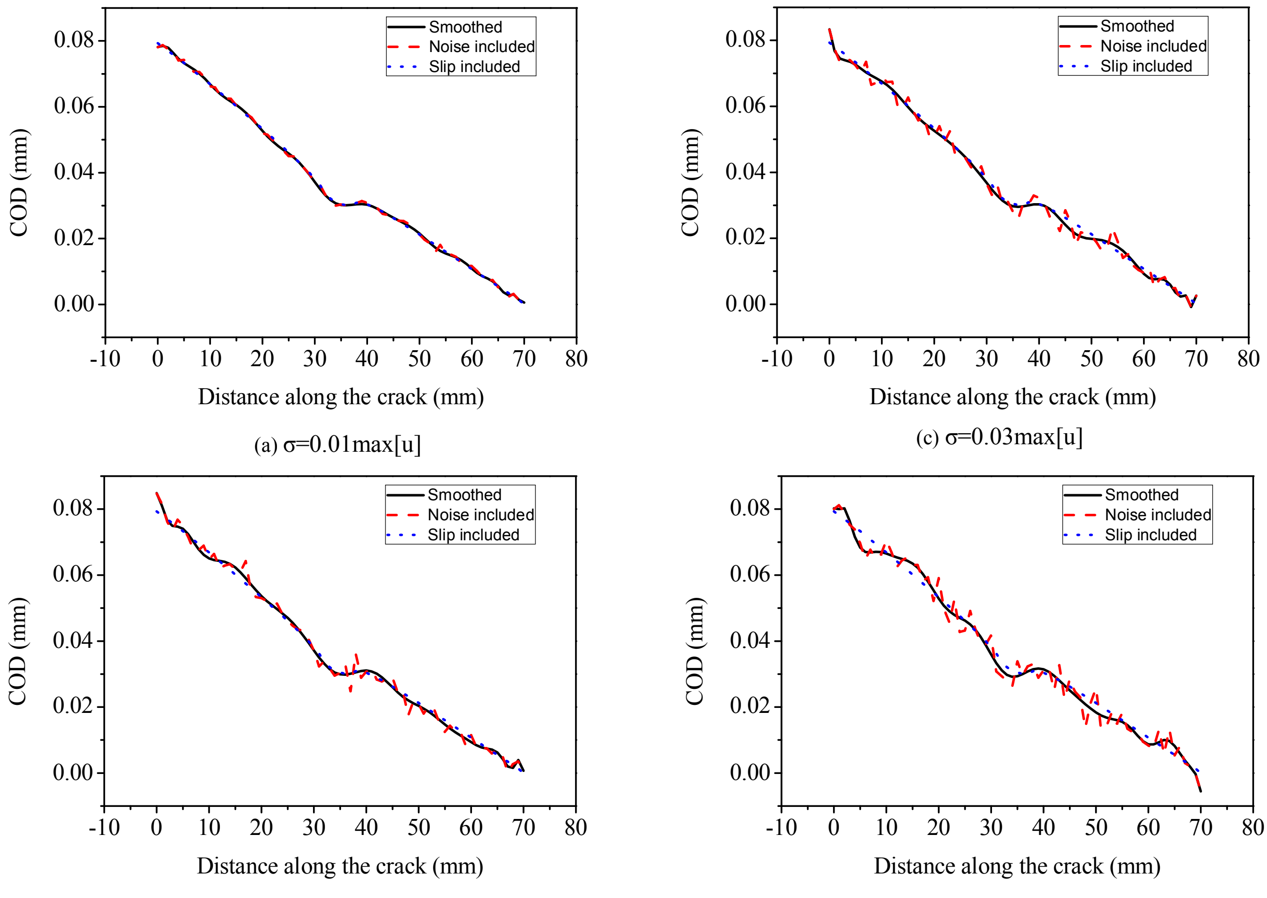
Due to fracture surface roughness which depends primarily on the size and type of aggregates, existence of impurities or voids, heterogeneity in compaction, etc, and errors in image collection and analysis, the experimental CODs contain certain levels of noise which determine the accuracy in prediction of material properties. Therefore, exploiting the influence of noise level on the accuracy of inverse analysis is of great significance and the target of this paper.

Another important source of derivation of analytical COD profiles from the actual one is from neglecting bond-slip behavior of rebar, which has been taken into consideration in this study according to [3].



(a) σ=0.01max[u]
(c) σ=0.03max[u]
(b) σ=0.02max[u]
(d) σ=0.04max[u]

Fig. 2 COD profile



(a) σ=0.01max[u]

(c) σ=0.03max[u]

(b) σ=0.02max[u]

(d) σ=0.04max[u]

Fig. 3 Smoothed COD

3.1 COD profile

In this paper, the noisy CODs are simulated by synthesizing analytical CODs and random number. The analytical CODs can be determined by the direct solution of **Eq**.(1) in which rebar forces were calculated by a cracked RC beam section analysis (which uses the cross-section geometry) using **Eq**.(4). In order to correspond to the experimental CODs which are measured at grid-points for further study, only analytical CODs points with positions, from crack mouth to crack tip with space 1mm, were calculated and added into corresponding random noise number.

The four graphs in **Fig**. 2 show COD profile from different approach and different noise level. Since the noise is simulated as a normal (Gaussian) distribution having zero mean (μ) and a certain standard of deviation (σ), the only standard which can be used in measuring noise level is σ. And the percentage of error invoked in the COD is

$$\delta = \frac{\left\| u - u^{\delta} \right\|}{\left\| u \right\|} \times 100 \tag{14}$$

where u^{δ} is the noisy CODs which is the input of inverse analysis.

Therefore, this paper alters noise level by adjusting σ as is showed in the graphs, where u is the analytical COD profile including slip.

As is showed in **Fig**.2, local of rebar can be primarily identified at the depression of a COD profile. And it can be seen from (d), whose noise level is σ=0.04$Max[u]$, that the noise fluctuate amplitude can be even larger than the depression, which means the rebar force prediction based on it is suspectable. The rebar force prediction will be presented later.

3.2 Inverse analysis

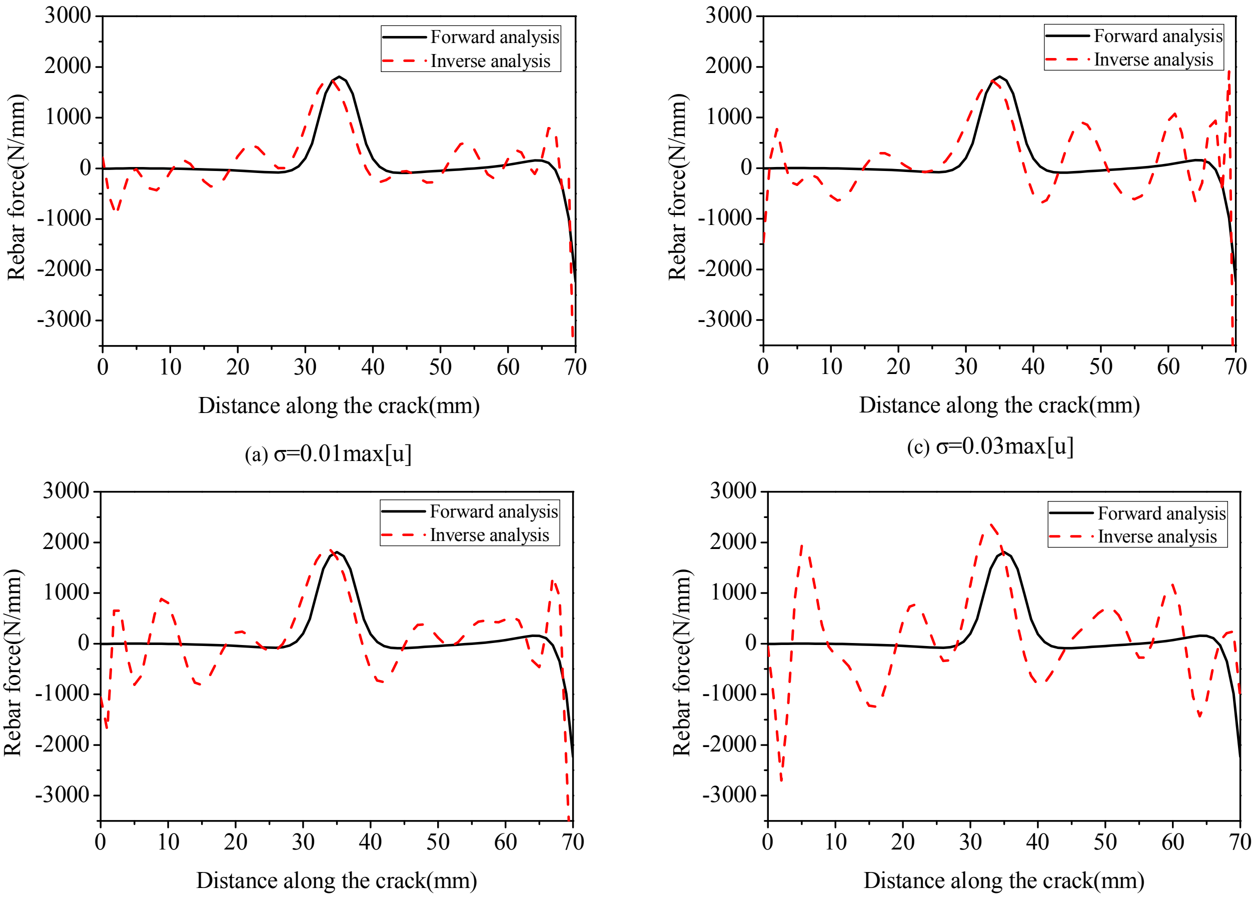
Before being inputted for inverse analysis, the noisy CODs profile should be smoothed by utilizing least square fit method. The smoothed CODs of different noise level are showed in **Fig**.3. It can be seen from (d) that the difference of depression amplitude between Smoothed and Slip included COD profile is largely aggregated due to the noise effect whose fluctuate amplitude is similar to that of depression. As a result, the smoothed at such a high noise level can't reflect the truth.

Then, the smoothed CODs data were processed by the inverse analysis theory introduced in Section 2.2. The obtained rebar force profile is showed in **Fig**.4.

In **Fig**.4, Location of the centroid of rebar is identified by the peak, and the total rebar force is computed by the area under the bridging stress profiles. The dash line which indicates the inverse analytical results contains undulations due to noise in the data, which at some locations indicates existence of absurd negative crack bridging stresses. It can be seen from the **Fig**.4, the undulations aggregate with the increasing of noise level. As to the rebar diameter, methods, such as Equivalent area method, Half point method and Transition point method, can be employed.

Accuracy of the inverse analysis method is checked by comparing the estimated rebar elements, such as rebar force, rebar diameter, obtained through inverse analysis with that from other theoretical methods. In noise level σ=0.04$Max[u]$, the amplitude of undulations lead by noise are equal to or only a little less than that of rebar force, as a result, the accuracy of inverse analysis in such situation is poor. Therefore, improving the accuracy of CODs data collection is of great importance.

Table.1 presents the error of every noise level by comparing rebar diameter and rebar force of forward analysis and inverse analysis. Since the error invoked in the COD (δ) are changeable even for Normal distribution noises of same deviation (σ), both of them are presented in the table. From the table we can see



(a) σ=0.01max[u]

(c) σ=0.03max[u]

(b) σ=0.02max[u]

(d) σ=0.04max[u]

Fig. 4 Rebar force

Table 1 Error of inverse analysis

σ ($\times 10^{-3}$)	*δ (%)*	Error of diameter (%)	*Error of rebar force* (%)
0.793	1.99	9.178	5.489
1.587	3.24	13.227	16.895
2.380	5.55	13.874	10.051
2.174	7.98	-5.226	21.909

that, basically, the error increases as the rise of noise level. Therefore, the influence of noise level on the accuracy of inverse analysis should be determined by conjunctively analyzing results in **Table**.1 and **Fig**.4. From this approach, it can be concluded that the accuracy of inverse analysis decreases along with the rising noise level, and the inverse analysis result is wrong when noise reaches certain level.

4. CONCLUSIONS

Through exploiting the accuracy of inverse analysis inputted into deferent level of noise, the following conclusions can be reached:

1) The applicability of inverse analysis method on predicting the rebar force in RC beams by using the easily measuring CODs has been verified.

2) Through conjunctively analyzing rebar curves and other rebar elements, it can be concluded that the accuracy of inverse analysis method decreases along with the increase of noise. Therefore, the accuracy of COD data collection is of great significance.

3) The inverse analysis method is regarded as incorrect when the noise exceed a certain level because the amplitude of noise is equal to or just a little smaller than the depression owing to rebar bridging effect.

REFERENCES

1) Tikhonov, A.N., Goncharsky, A.V., Stepanov, V.V., Yagola, A.G.: Numerical methods for the solutions of ill-posed problem, Kluwer Academic Publishers Group, Dordrecht, The Netherlands, 1990.
2) Nazmul, I.M., Matsumoto, T.: High resolution COD image analysis for health monitoring of reinforced concrete structures through inverse analysis, Solids and Structures, 45(1): 159-174, 2007.
3) Nazmul, I.M., Matsumoto, T.: Regularization of inverse problem in concrete fracture and determination of rebar force from crack opening displacement, PhD thesis, the University of Tokyo, 2004.
4) Carpinteri, A., Massabo, R.: Continuous vs. discontinuous bridged crack model for fiber reinforced materials in flexure, International Journal of Solids and Structures, 34(18), 2321-2338, 1997.
5) Cox, B.N., Marshall, D.B.: Stable and unstable solutions for bridged crack in various specimen, Acta Metallurgica Materialia, 39(4): 579-589, 1991a.
6) Buchanan, D.J., Join, R., Johnson, D.A.: Determination of crack bridging stress from crack opening displacement profiles, International Journal of Fracture, 87(2): 101-117, 1997.
7) Nazmul, I.M., Matsumoto, T.: Determination of steel stress in reinforced concrete structures from crack opening profile, In: Proceedings of the First International Conference on Structural Health Monitoring and Intelligent Infrastructure, Tokyo, Japan, 2003.
8) Tada, H., Paris, P.C., Irwin, G.R.: The stress analysis of cracks handbook, Paris Productions Inc., St. Louis, Missouri, 1985.