Numerical Evaluation of Anisotropy in Frost Heave Based on Takashi's Equation

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1. INTRODUCTION

Nowadays freezing is not only a problem in cold regions, but also a big issue in the populated urban area. Many underground excavations such as those for subway tunnels, pump pits, and tunnels near or below lakes or rivers are difficult to construct because flow-sand and water seepage frequently occur and shields cannot be easily built. For these diverse geotechnical construction problems, ground freezing technology is effective as it can deal well with seepage problems as well as improve the strength of the soil. But on the other hand, the ground freezing may also cause some serious compressing pressures to the structures just as the LNG tanks. In a word, ground freezing, both in cold regions and urban areas, may be an effective technology to help us, on the contrary, may be a serious challenge to structures, due to the frost heave and thawing settlement. Therefore, predicting and controlling the frost heave is a critical challenge to the engineering construction in those fields.

However, until now the mechanism of frost heave in micro scope is still controversial. In the early days, Taber demonstrated that benzene or nitrobenzene in a porous medium could cause frost expansion by experiment, which demonstrates that frost heave is caused by water migration, not only the *in-situ* water. For theoretical analysis, in 1943, Edlefsen and Anderson advanced the Generalized Clausius-Clapeyron Equation (GCCE). And it is obvious that GCCE is the equation for state of the soil-particles-water-ice system when ice lens exist but no water migration happens. It is an attempt to explain the mechanism of frost heave. But typical analysis methods for frost heave were based on that frost heave accompanying water migration, thus these methods had inherently poor accuracy when they tried to predict frost heave. After that, in 1996, Yoshiaki Miyata improved this equation from static equilibrium to dynamic equilibrium, which means that water migration was considered. And also some experiments had been done to demonstrate this new theory. Unfortunately, it was not widely accepted in this field.

As to the practical methods aiming to estimate the volume of frost heave, extensive researches have been done. Nixon proposed a frost heave estimation model based on the segregation potential theory. Konrad and Morgenstern also applied this segregation potential theory for the prediction of frost heave. This method is the most widely accepted practical model, however, in this model, one critical aspect of frost heave is not included, the freezing rate. As in Japan, Takashi's model is widely accepted and has been successfully applied to the design of Liquid Natural Gas tank. This model was proposed by Tsutomu Takashi in 1974, which considered the constraining stress and freezing rate, also there are three constants in this model which can represent the soil property. But Takashi's equation was derived from one dimensional frost heave experiments, so it cannot be used in multi-dimensional problems directly. Accordingly we advance a new model based on three dimensional frost heave experiments aimed to be used in multi-dimensional situation.

2. TAKASHI'S MODEL

Takashi's equation relates the frost heave ratio with the constraining stress and freezing rate as Eq. (1).

$$\xi = \xi_0 + \frac{\sigma_0}{\sigma}\left(1 + \sqrt{\frac{U_0}{U}}\right) \tag{1}$$

ξ: frost heave ratio, σ: constraining stress in the freezing direction, U: freezing rate. ξ_0, σ_0 and U_0 are constants for the material obtained by experiment regulated by JGS. It is obvious that U and σ are two critical variables for obtaining frost heave ratio. For example, if the confining stress σ is small, there will be a large frost heave ratio. Concerning to this issue, if stress is very small, an infinite value could be expected due to the form of equation, so there should be a limit for the stress where it is suitable for this equation. And according to Takashi's paper, this equation is valid when stress is between 1kg/cm^2—15kg/ cm^2. Thus, in our calculation, this range of stress is obeyed.

In three-dimensional analysis, U is obtained by heat transfer analysis and σ is obtained by mechanical analysis at each time step. Then, the frost heave ratio is converted into equivalent strains in three directions with the anisotropic parameter corresponding to the volumetric change due to frost heave. If we assume that it is isotropic problem in three dimensions, in other words, that all strains in different directions equal to each other, the strains are related with the frost heave ratio as Eq. (2).

$$\xi = \xi_1 + \xi_2 + \xi_3 = 3\xi_1 \tag{2}$$

Where ξ_1 means frost heave ratio in freezing direction and ξ_2, ξ_3 are frost heave ratio perpendicular to the freezing direction. Here we assume the problem as two-dimensional anisotropic problem with plain strain condition. β is the anisotropic parameter that distribute frost heave ratio in different directions.

$$\xi_1 = \frac{1}{1+\beta}\xi, \quad \xi_2 = \frac{\beta}{1+\beta}\xi \tag{3}$$

Takashi's equation has a good performance when it is applied to estimate the frost heave in one dimensional problem, however, due to the inherent restriction, which was derived from one-dimensional experiments, it is better to improve it before applying it to multi-dimensional problems.

3. 3D EXPERIMENTS AND MODIFIED EQUATION

Fig.1 is 3D frost heave apparatus developed by our team. The soil specimen is frozen at a constant freezing rate from the top to bottom. Meanwhile, water is supplied from the bottom as an open system during the freezing phase, and the water absorbed could be recorded. Original Takashi's equation was relied on one-dimensional frost heave experiments and did not consider the frost heave occurred in lateral direction. This new frost heave apparatus is just aimed to solve such problems, to relate frost heave ratio with constraining stress.

The frost heave ratio in the heat flow direction is recorded using a dial gauge. It is quite accurate and reliable. However, the change in the radius direction is measured using a vernier calliper after the experiment. And the volumetric change of the specimen can be related with the expansion in three dimensions by this equation:

$$\xi_v = \xi_1 + \xi_2 + \xi_3 = \xi_1 + 2\xi_3 \tag{5}$$

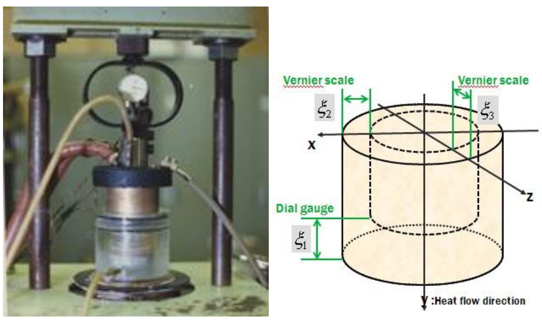

Figure.1 Three-dimensional frost heave apparatus developed by Ueda

Based on the data obtained by this new 3D experiment, a preliminary study of the results showed that the volumetric change due to frost heave could be expressed by the following equation:

$$\xi_v = \xi_0 + \frac{\sigma_0}{\sigma} + \frac{U_0}{U} \tag{6}$$

Where ξ_v is the volumetric change ratio; σ is the constraining stress in the heat flow direction and U is the freezing rate. σ_0 and U_0 are constants obtained experimentally, but they are different from the constants usually used in the original Takashi's equation. ξ_0 is the same as in Takashi's equation. The linear regression coefficient of this equation equals to 0.788 which proves that the total volumetric change due to frost heave can be estimated by this equation with sufficient accuracy (Fig.2).

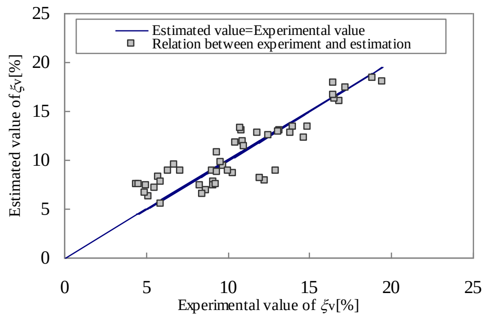

Figure.2 Comparison of volumetric change ratio ξ_v

From the form of this new equation, it seems similar to the old one-dimensional Takashi's equation; however, they have totally different essential meanings. In this equation, ξ_v is the total volumetric frost heave ratio which includes frost heave ratio in three different directions. As to Takashi's equation, ξ is just the frost heave ratio occurred in freezing direction. To be frank, this new equation may need more data to verify its reliability. However, now this new equation does improve the original one-dimensional equation into three-dimensional practical equation. It is not precise to call it as Takashi Equation again. Here we call it Hokkaido University Model (HUM).

For three-dimensional estimation of frost heave, the volumetric change given by Equation.(6) should be decomposed into the frost heave ratio in each direction: e.g., ξ_1, ξ_2, and ξ_3. The frost heave ratio in direction i was assumed to be given by Equation (7) with delivering coefficient α_i.

$$\xi_i = \alpha_i\left(\xi_0 + \frac{\sigma_0}{\sigma} + \frac{U_0}{U}\right) \tag{7}$$

Two types of coefficients were assumed, as shown in Equations.(8a) and (8b). In both of these two assumptions, frost heave ratio in each direction is related to the constraining stress in that direction. These kinds of assumptions distribute frost heave ratio based on confining stress.

$$\left.\begin{aligned}\alpha_1 &= \frac{\sigma_2+\sigma_3-\sigma_1-1/2|\sigma_2-\sigma_3|}{\sigma_1+\sigma_2+\sigma_3}\\ \alpha_3 &= \frac{\sigma_1+\sigma_2-\sigma_3-1/2|\sigma_1-\sigma_2|}{\sigma_1+\sigma_2+\sigma_3}\end{aligned}\right\} \tag{8a}$$

$$\alpha_i = \frac{1}{2}\left(1-\frac{\sigma_i}{\sigma_1+\sigma_2+\sigma_3}\right) \tag{8b}$$

By applying the above equations, the frost heave ratio ξ_1 in the heat flow direction can be calculated. The linear regression analysis shows that if Equation.(8a) was adopted as the delivering coefficient, the coefficient of determination becomes 0.796. When Equation.(8b) was applied, the estimation shows better agreement with the experimental results, and the coefficient of determination becomes 0.927. Thus, the frost heave ratio in the heat flow direction was evaluated appropriately.

In contrast, it is a little difficult to evaluate the frost heave ratio ξ_3 that is normal to the heat flow direction. The linear regression analysis of comparison between experimental and estimated values of ξ_3 showed that the coefficients of determination became worse: 0.635 for Equation.(8a) and 0.352 for Equation.(8b). This inaccuracy may be caused by the measuring method for the change in diameter of the specimen. Unlike the frost heave in the heat flow direction ξ_1, the change in diameter of the specimen was measured using a vernier calliper after the experiments, and the observed values themselves were less reliable. And the uneven frost heave in diameter direction may also affect the accuracy of measurement.

4. THE VALIDATION OF CALCULATION PROGRAM

In this part, we want to confirm the validation of our calculation program. The basic idea is that controlling the freezing rate, and then calculates the influence of each parameter and the total value of them. If the summation of the each calculation equals to the value of total influence, it is demonstrated that each part of the program works well. And furthermore, we also estimate the calculation result by hand. If they are really close to each other, the program does exactly what we expect. A table of these results is shown as follow:

Table.1 Influence of each parameter to frost heave

G	ξ_0	σ_0	U_0	Sum
-2.6E-05	2.5E-02	8.E-03	5.631E-04	3.353E-02
Total	3.353E-02			

Based on this table, it is obvious that the program considers each parameter just as our design. And also in the hand calculation part we set the freezing rate as a constant of 0.1m/2h, and ξ_0=0.025, σ_0=800Pa, and U_0=6.9×10^{-8}m/s, where all the parameters are just the same as used in program calculation.

$$\xi = \xi_0 + \frac{\sigma_0}{\sigma}\left(1+\sqrt{\frac{U_0}{U}}\right) = 0.025 + \frac{800}{100000}\left(1+\sqrt{\frac{6.9\times10^{-8}}{1.388\times10^{-5}}}\right) = 0.03356389$$

After considering the influence of gravity, frost heave =0.03356389×1-0.000026 =0.03353789m. If we consider the influence caused by rounded off, the accuracy of this calculation is amusing.

5. EXAMPLE AND COMPARISONS

After validation of program, here a very simple 1m×1m square model is given, two different model are used, Takashi’s model and Hokkaido University Model. Due to the limited experiment quantity, appropriate parameters used for Hokkaido University Model could not be obtained until now, so we adopt the same parameters for both of these two models. Boundary conditions are shown in Fig.3. And in both of these two models, the following assumptions are adopted:

- Soil is assumed fully saturated
- Water absorbed for heaving is provided from the boundary
- Water flow after thawing is not considered
- Nonlinearity due to temperature dependent elasticity and expansion in volume is not considered by step analysis
- Nonlinearity due to tension crack in soil is not considered
- Plain strain problem

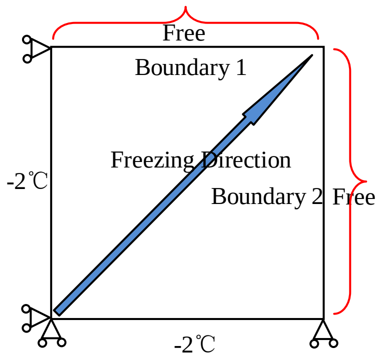

Figure.3 Simple 2D Model

Before this calculation, we can make some estimation concerning the results. First, due to the symmetrical characteristic, the results of temperature and frost heave at boundary 1 and boundary 2 should be the same if we omit the influence of gravity. Thus, in this calculation, gravity parameter is set as 0. Now we can check the results.

These are the results obtained by Takashi’s method where we set the anisotropic parameter β=0, 0.5, 1 and HUM. The results of temperature and frost heave distribution are just the same as we expected. Fig.4 shows the frost heave at boundary 1 and 2 equal to each other. However, frost heave obtained by HUM is a little larger than that of Takashi’s method. This may be caused by that we use the same parameters as Takashi’s equation for HUM. In fact, the constants used in HUM should be different from those of Takashi’s equation. Besides that, as the increasing of β from 0 to 1, which means from anisotropic to isotropic, the shape of deformation is changed, it seems the deformation shape turns for some angle.

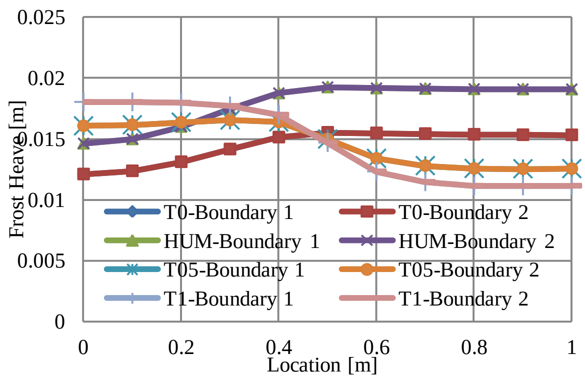

Figure.4 Comparison of frost heave at boundary 1and 2

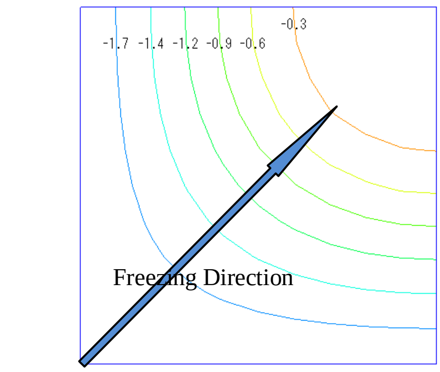

Figure.5 Temperature distribution after 20 hours

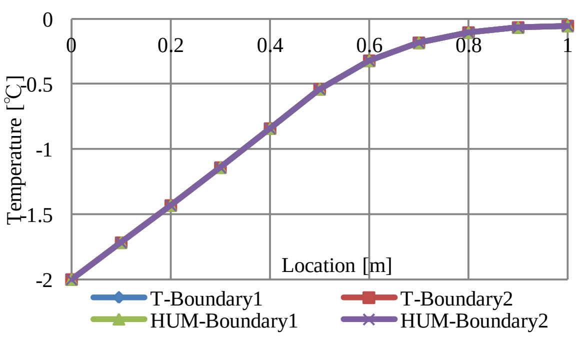

Figure.6 Comparison of temperature at boundary 1and 2

Fig.5 shows the temperature distribution of this model after 20 hours. And Fig.6 shows the temperature distribution at boundary 1 and boundary 2. All the results agree well with our expectations.

6. CONCLUSIONS

1) The program based on Takashi's equation and HUM works well. It inspires us to improve this program to consider more complicated situation, such as water pressure.

2) This study proposed a new three-dimensional experiment aimed to improve Takashi's equation, and the modified equation advanced in this paper was confirmed to be potential as a practical method for multi-dimensional estimation of frost heave.

3) In order to realize better estimation, additional experiments with sufficient accuracy in measuring ξ_3 and further discussion on the estimation equation may be necessary.

4) It is reasonable to evaluate the anisotropy in frost heave by the delivering factors based on the balance in the confining stress. However, the applicability of the delivering factors should be investigated further with a plenty of multi-dimensional experiments.

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