# Transport network vulnerability: application analysis of vulnerability theory in some transport network degradation cases

Hokkaido University Hokkaido University Zhejiang University

Associate Professor Associate Professor

 $\bigcirc$  Research student

Du Qianqian Kunihiro Kishi Wang Fujian

## 1. Introduction

In the past decade, transport network vulnerability has received more and more attention. There have been many literatures defining transport vulnerability from different aspects. Berdica [1] proposed that vulnerability analysis of transport networks should be regarded as an overall framework through which different transport studies could be conducted to determine how well a transport system would perform when exposed to different kinds and intensities of disturbances.

Transport network vulnerability analysis provides valuable insights into certain aspects of network performance. Most articles focus on strategic level network especially in large-scale, sparse regional or national networks. They try to find out the most important nodes or links and a replaced routine of the weak nodes and links in case these elements are cut by accidence or disaster [2]. However, efficiency now is more and more important for traveler as the development of social economy. In addition, the transport network manager must handle with more and more accidences or disasters which degrade or even cut part of elements in the transport network. These problems stem from a variety of causes, including natural disasters (e.g. floods, fire or earthquake), human malevolence such as acts of sabotage or terrorism, maintenance of road network and municipal pipe network. Planners and policy makers want to make rational assessments of the consequences of network degradation and failure at various locations and under different circumstances. Are there particular locations in that network where loss or degradation of certain road links will have most significant impacts? If that is the case, what we should do about it. This study will try to find the vulnerable sections in the transport network.

## 2. Methodology

How should such impacts be assessed? Thus there are needs for the development and application of a methodology to assess vulnerability of transport networks. Transport network reliability has the subject of considerable international research interest in recent years [3]. The concepts of reliability and vulnerability are much similar, but they are essentially different. This study is based on the concept that vulnerability is more strongly related to the consequences of link failure, irrespective of the probability of failure [4]. In fact, in some cases, link failure may be statistically unlikely but the resulting adverse social and economic impacts on the community may be sufficiently large to indicate a major problem warranting remedial action [5].

Social and economic benefits flow from the ability to plan for and manage the impacts of transport network degradation. This study proposes a method and decision support tools to assess the consequence of urban transport network degradation or failure using time as the performance measure. The performance of transport network is inherently dependent on the congestion effects caused by the interaction between driver behavior and built environments. In the static transportation planning context, the congestion effects can be capture using the user equilibrium model [6]. The methodology used in this study assesses the criticality by computing the congest effects based on user equilibrium with and without the transport network link.

In transportation network analysis, Wardrop's first principle states that every user seeks to minimize his transportation cost which under this perspective is the individual travel time. The flow that satisfies this condition, where no traveler can improve his/her travel time by unilaterally changing route, is referred as the user equilibrium (UE). The problem involves the assignment of origin and destination (O-D) flows to the network links such that the travel time on all used paths for any O-D pair equals the minimum travel time between the O-D.

A generic measure of criticality in a network can be defined by the change in the performance of the network after the removal or damage of one its components. Therefore, the criticality of any of its components can be expressed by:

$$R(i) = \frac{f_{G}'(i) - f_{G}(i)}{f_{G}(i)}$$
(1)

Where R(i) is the vulnerability index,  $f_G(i)$  is the performance measure of the network without disruption and  $f_G'(i)$  is the performance measure of the network after the disruption of the component i. The key in using this expression is finding an appropriate performance measure.

One potential measure is the length of the shortest path. Some researchers have simplified the problem through the construction of equivalence classes on the set of all possible input graphs [7]. If a set of origins and destinations is set as subsets of N. If there exists a path connecting any O-D pair, the distance dij between these two nodes is positive and if there exist no path then dij =  $\infty$ . The shortest path length lij between nodes i and j can be defined as the smallest sum of the physical distances throughout all possible paths [8]. However, this measure is not suitable to address effect of congestion in transportation networks.

This study chooses the equilibrium travel time which satisfied user equilibrium condition as an appropriate measure. The measure is given as the summation of all link travel time (t) represented by:

$$f_{G}(a) = \sum_{\forall a} t_{a}(x_{a}) \tag{2}$$

Where  $x_a$  is the flow at link a. If there is at least one path connecting any O-D pair this value is a positive number but if there is not a path the travel time will became infinite assuming that there is typically path choice between any two given O-D pairs.

Under Wardrop equilibrium each vehicle seeks to minimize its journey time [9]. This principle is used to assess the criticality of the links by determining the change in total travel time due to the deletion or degradation of a link. The algorithm is based on the convex combinations algorithm, also called the Frank-Wolfe. The proposed algorithm is an iterative process of choosing one link and eliminating or reducing its capacity. In each iteration of the algorithm, the UE solution is computed for the disrupted network. This process is repeated until all links have been evaluated. Finally, the algorithm compares the results with UE solution without disruptions. The links are subsequently ranked using the measure defined in Eq. (2).

## 3. Examples

This section will introduce some cases using the theory described above to analysis the vulnerability character of transportation network and try to find out the weak links in the network.

## 3.1 Network description

This study applies the methodology on a theoretical transport network (see Fig.1). The network consists in a total of 29 links and 21 nodes. Node17, 18, 19, 20 and 21 are the centroids of the traffic areas. Links 25, 26, 27, 28 and 29 is the free links connecting the centroids to the road network. The convergence rate for the UE algorithm has been set at 0.001. The algorithm is run to evaluate the performance of the network in variant cases.



#### 3.2 Case 1

The situation that one link is cut in the transport network is supposed. It means that the capability of one link in the network will become 0 when it is selected. There is only one link is selected in one time calculation circulation. The algorithm is run until evaluating the importance of every link in the entire network is finished. Then the vulnerability index of all links will be ranked. The calculation result is shown in Table 1.

Link	R	Link	R	Link	R
3	27.48%	21	2.36%	17	1.00%
4	16.31%	23	2.36%	24	1.00%
1	9.10%	19	1.84%	9	0%
2	9.09%	5	1.70%	10	0%
14	6.80%	6	1.64%	11	0%
15	6.70%	7	1.62%	20	0%
16	4.48%	12	1.00%	22	0%
8	3.24%	13	1.00%	18	-0.07%

Table 1 Vulnerability index of links

#### Analysis results

The vulnerability index is calculated when one link completely loses its capability in one circulation. The table shows us the result of all links ranked by the vulnerability index. There re links with much higher vulnerability index such as link 3, link 4 than other links. It means that loss of these links greatly increase the travel time in the entire transport network. In other words, these links are the vulnerable points of the network from the perspective of transport network operating efficiency. There is another interesting phenomenon shown in the table. There are some links whose vulnerability index is 0 or even negative. It is similar to the situation that the network performance will not be better or even worse when extra routes are added into the transport network.

## 3.3 Case 2

The situation that the capacity of each link changes from 0% to 100% is supposed. The capacity of every link increases at the rate of 10% in one calculating circulation. The algorithm is run until the evaluation of all links in the entire network is finished. The calculation result is shown in Fig. 2.

#### Analysis of results

The lateral axis is capacity increasing from 0% to 100%. The vertical axis is the vulnerability index R varying as the capability degradation of links. The picture shows us that the vulnerability index is sensitive to the capability and the relationship between vulnerability index and capability is not simply linear. It means that the links with largest capability may be not the most important ones whose degradation or loss will give most impacts on the whole transport network. The picture also shows us that for one single link its vulnerability index keeps almost the same level when the capability degrades in a range. This result is useful when the policy maker need to degrade some links in the transport network as the road network is in maintenance. They can choose the point which satisfies the maintenance requirement at the same time minimize the impact to the whole network caused by the degradation.



Fig.2. Sensitivity of vulnerability to capability

# 3.4 Case 3

Supposing that up to 5 links of the network will degrade its capacity in half, how to evaluate the performance of the network when more than one links degrade their capability at the same time? There are many combination options in that case. This study select 5 links (link1, link3, link9, link15, limk17) form the theoretical network. The algorithm is run to calculate any combination from the 5 links. The calculation result of different combination including numbers of links varying from 1 to 5 is shown in Table 2. The results shown in the table is the vulnerability index of combination options with highest vulnerability index R.

Links combination	R	
3	1.04%	
3&9, 3&15	1.04%	
3&9&15	1.04%	
1&3&9&15	0.99%	
1&3&9&15&17	0.81%	

Table 2 Vulnerability index of combination options

Analysis results

Many combination options have been calculated in case 3. Supposing that there are 5 links need maintenance at the same time, and the manager have sufficient resource (such as money, human and material), however, degradation or loss of the key sections in the urban road network will greatly influence residents' travel time as we feel in daily life. Table 2 shows us that there are some options which most impact the whole network compared with others including the same numbers of links. There is an interesting intimation in the calculation results. The more degraded links do not mean more impact to the whole transport network measured by travel time. If there are parts of links suffering loss or degradation caused by flood or earthquake in the urban transport network, repairmen of some important links will much more upgrade the efficiency of the entire transport network.

#### 4. Conclusions

The concept of network vulnerability is new, and it is important to define the measurement of vulnerability. There has been study of vulnerability focusing on the entire transport network travel time from the view of network manager. In fact, the impacts of transport network degradation related to many consequences on economic and social activities in cities or regions. More detail research is needed to study the characteristics of network vulnerability based on different social and economic benefits under different circumstances. More detail research is needed to apply the vulnerability theory into more large and complicated urban transport network. If the question where are these locations of potential network vulnerability is solved and there is another question about network vulnerability that what is the best response.

#### 5. References

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