Applied Optimization for Transit Network Routing in Obihiro

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1. Background of the Study

This paper introduces a solution methodology for the transit (bus) route network problem for a sample city with available PT data. Several methodologies, in varying sophistications, have been developed to generate route sets for transit routing; however, these solutions appear viable only in virtual networks, where nodes and links are characterized to allow for shortest paths to be searched in time units. However, unlike that of the single vehicle problem, travel time is not the optimal criterion for transit lines. In practice, transit route design is not solely a result of random linkages between nodes. Otherwise, there can be an infinitely large number of candidate routes for only one OD pair. Without regard for existing OD patterns, the path search becomes more complex as the network size also increases. Searching for feasible routes must, therefore, incorporate existing characteristics in the sample city on which the network bed is based. In doing so, the search algorithm is simplified and the results are closer to the side of practicality.

2. Objectives of the Study

The novelty in this study is the reduction of the search space for feasible routes by using OD patterns derived from existing PT data so that links can be weighted according to preferential value (using a coefficient for link attractiveness). This ensures that the resulting *shortest paths* in a candidate route set reflect the actual preferred paths of non-users of transit services. The goal of the study is essentially to provide a practical procedural solution to the transit network problem by reducing the search space for feasible routes by incorporating existing path preferences to do avoid complex calculations for a large number of nuisance paths. Also, as transit routing is a systems problem, several optimizers are integrated in the methodology in order to ensure shortest, direct paths, with optimal fleet size.

3. Methodology

Figure 5 on the third page is a simplified schematic diagram for modeling the network, the steps in the path search algorithm with the search space reduction scheme, and fleet size optimization. Through the search space reduction scheme, link-attractiveness is calculated so that in applying Dijkstra's algorithm to find shortest paths, optimal criteria will include total travel time and total system cost (in time units) and link attractiveness. Recall that the main goal of the study is to provide a procedural solution for the transit routing problem, in order to generate the shortest routes that minimize the total system cost. PT data and traffic zones are both taken from the Obihiro Area General Urban Traffic System Investigation (Masterplan Development Investigation)[1] released in March 2006.

3.1. *Sample Network with OD Data*

3.1.1. *Obihiro Area*

The small-sized city of Obihiro was used to illustrate the usefulness of search space reduction schemes in generating candidate route sets for transit (bus) lines. OD tables are a rich source of mobility patterns. There are more inter-zonal trips in Obihiro than there are intra-zonal trips. However, due to insufficient digital data regarding bus stop locations, nodes were made to represent demand zones, or sub-centers in the city where trips are densely distributed.

As of 2006, the population of Obihiro was 171,153 for an area of 619 km^2. Obihiro is the sixth-largest city in Hokkaido; however, the city boundaries do not encompass all the conurbation and a large swathe of countryside is included. If the adjacent urban areas of Otofuke, Memuro, and Makubetsu were included, the population would be at least 200,000 to 220,000 and Obihiro would be the fourth-largest city in Hokkaido [1].

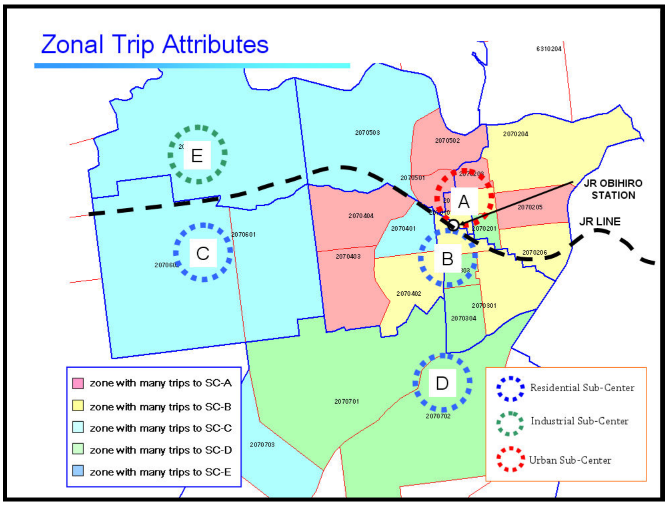

Figure 1. Zonal Attributes of the Obihiro Area

The proponent is deeply familiar with OD-based measures for restructuring the city's bus network. Car dependency is very high for a population with 60% of its drivers over 65 years of age[4]. Hence, triple efforts are exerted to promote public transportation. It is in this vein that transit studies in local cities are also important.

3.1.2. *Demand Matrix*

From figure 2 below we learn of zones with very active cross-trips and sixteen zones with denser trip distributions. First we deal with the inter-zonal movement patterns in order to test whether our search space reduction scheme is workable. If we were to schematically represent trip distribution for this OD matrix, we would see figure 3 (orthogonally arranged). Trip patterns connect nodes and redundant paths, vis-à-vis spatial proximity, are removed.

Figure 2. OD Matrix for the sample city on which the network bed is based

A ZONE	B ZONE	C ZONE	OBIHIRO CITY																												TOTAL
			01		02						03				04				05			06		07					08	09	
			01	02	01	02	03	04	05	06	01	02	03	04	01	02	03	04	01	02	03	01	02	01	02	03	04	05	01	01	
OBIHIRO	01	01	210	226	77	258	135	178	185	115	219	87	83	175	229	98	233	422	493	252	651	757	212	391	218	71	153	0	35	0	6163
		02	161	523	69	64	256	153	227	175	159	513	181	256	129	375	630	410	276	212	220	575	165	248	107	28	77	58	558	0	6805
	02	01	19	73	160	60	13	164	386	107	158	77	150	175	220	46	130	34	134	87	106	292	111	27	237	0	0	0	123	0	3089
		02	141	125	80	245	67	303	246	149	316	374	62	111	64	21	184	156	335	115	302	320	189	178	43	153	0	0	82	0	4361
		03	153	252	13	32	398	314	230	44	109	84	63	80	135	33	60	262	98	168	276	244	230	239	286	21	39	0	171	25	4059
		04	319	261	217	256	409	2758	1572	232	431	637	144	98	286	474	106	148	411	492	517	600	573	290	653	39	0	0	663	147	12733
		05	365	336	365	538	238	1819	3291	569	436	390	172	276	242	245	116	100	571	509	173	243	454	313	157	194	133	0	89	0	12334
		06	78	168	12	84	28	296	528	745	365	144	112	49	57	81	164	30	68	107	158	144	166	239	116	15	19	18	201	14	4206
	03	01	315	265	203	324	150	430	442	399	1493	191	497	430	364	292	178	186	134	183	462	190	161	612	601	29	279	44	135	72	9061
		02	158	344	52	333	48	458	389	174	165	824	49	164	278	353	375	512	385	217	426	437	258	318	535	224	15	34	191	31	7747
		03	73	60	150	107	32	217	180	236	445	49	265	799	129	267	121	0	118	29	381	439	148	443	426	88	17	13	32	0	5264
		04	93	376	127	163	14	199	260	64	534	149	614	2561	327	264	234	166	327	66	141	320	201	1494	962	207	231	38	174	38	10344
	04	01	127	91	441	67	78	297	199	70	315	201	142	226	2142	870	855	784	616	154	428	890	583	301	382	126	66	45	427	0	10923
		02	110	187	91	48	111	333	457	253	242	443	230	225	857	2547	1444	547	435	151	170	442	621	899	334	88	23	33	569	0	11890
		03	295	631	149	201	228	145	183	31	100	547	124	200	583	1652	3801	1213	581	326	543	2323	752	642	492	46	44	0	635	16	16483
		04	546	562	35	161	189	222	86	27	135	186	118	233	887	574	1075	3975	397	206	1196	1721	915	588	508	92	159	14	647	17	15471
	05	01	736	341	153	357	313	425	642	84	75	356	168	256	451	366	490	593	804	281	1000	788	696	694	415	252	85	96	215	20	11152
		02	251	292	90	138	157	482	430	90	410	293	33	93	141	189	335	193	377	983	915	443	385	290	403	14	160	14	468	23	8092
		03	697	273	73	225	396	727	255	121	424	485	341	126	471	265	497	1309	1063	1085	11961	1185	1734	513	588	265	150	44	2949	82	28304
	06	01	712	480	324	378	306	604	353	69	190	477	498	377	850	645	2201	1742	1122	508	1449	11764	4671	1278	1216	408	572	50	2900	125	36269
		02	271	314	106	271	167	742	422	151	344	332	199	101	561	453	841	859	946	291	1806	4636	19228	1438	1103	351	418	206	2615	29	39201
	07	01	345	280	93	332	351	375	339	152	756	279	407	1364	321	809	775	438	676	251	573	1218	1411	7267	5494	1231	576	184	525	23	26845
		02	328	435	222	42	283	541	187	115	550	383	476	1139	543	305	408	426	542	339	702	1327	880	4942	7743	370	425	406	889	106	25054
		03	103	34	0	109	23	43	169	32	212	191	126	212	121	97	34	272	262	14	155	392	439	1294	403	2037	211	15	87	0	7087
		04	85	105	0	0	27	0	295	0	262	44	15	207	85	63	57	104	100	29	266	454	509	499	442	248	1102	355	387	63	5803
		05	0	28	0	0	0	0	20	0	40	44	12	23	11	68	20	13	137	13	80	52	142	171	365	15	222	4028	42	127	5673
	08	01	194	306	223	38	228	618	138	139	151	216	14	215	403	321	609	805	253	323	2819	2551	2702	505	730	87	332	38	2600	43	17601
	09	01	0	0	0	0	0	122	23	13	45	41	0	38	0	0	14	15	20	37	71	116	26	30	58	0	0	133	40	724	1566
		TOTAL	6885	7368	3525	4831	4645	12965	12134	4356	9081	8037	5295	10209	10887	11773	15987	15714	11681	7428	27947	34863	38562	26143	25017	6699	5508	5866	18449	1725	

3.1.3. *Demand Zones and Known OD Patterns*

The proponents aim to incorporate existing characteristics (e.g., actual mobility patterns) of the sample city into the virtual network of nodes and links through which the best set of links for each route can be computationally determined. Coefficients for link attractiveness will be used to reflect such links. Figure 3 below puts on a view of how these preferential links are located in the network. Figure 4 illustrates the final network model.

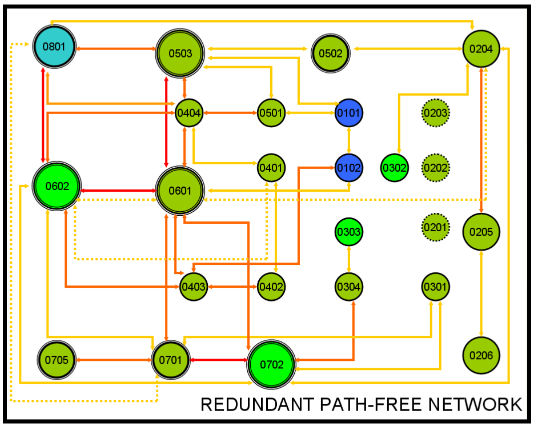

Figure 3. Dominant Inter-Zonal Trip Patterns

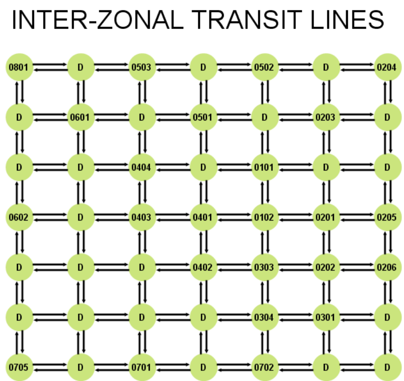

Figure 4. Network Model

3.1.4. *Network Model*

The proponents are using *Netlogo* to simulate the path search with reduced search space schemes in the virtual network (see figure 4) based on the sample city. G = (N, L) is the matrix representing all links connecting nodes *i* and *j*. N is the set of *n* nodes (representing demand zones) and L is the set of *m* direct links (i, j), connecting node *i* to node *j* in the network. Each distinct node represents a route station and is assigned a route ID.

Each path, which connects stations (nodes) of origin and destination, is an ordered sequence of nodes and links, $\{n_1=0, (n_1,n_2), n_2, (n_2,n_3), n_3, .., (n_{k-1},n_k), n_k = d)$. We define the set of paths for an OD to be $P_{i,j}$ and $C(P^{i,j})$ as the set of corresponding path times (equivalent costs) = $\{t(p_1), t(p_2), \ldots t(p_k)\}$. We assign dummy nodes to account for inter-zonal trips with large distances (*t* and *c* for links connected to dummy nodes are assigned zero values for transfer time and waiting time). This helps ease the work on assigning "t" values for links that represent long distances, vis-à-vis links representing short distances. For simplicity, links shall be treated as homogeneous road segments.

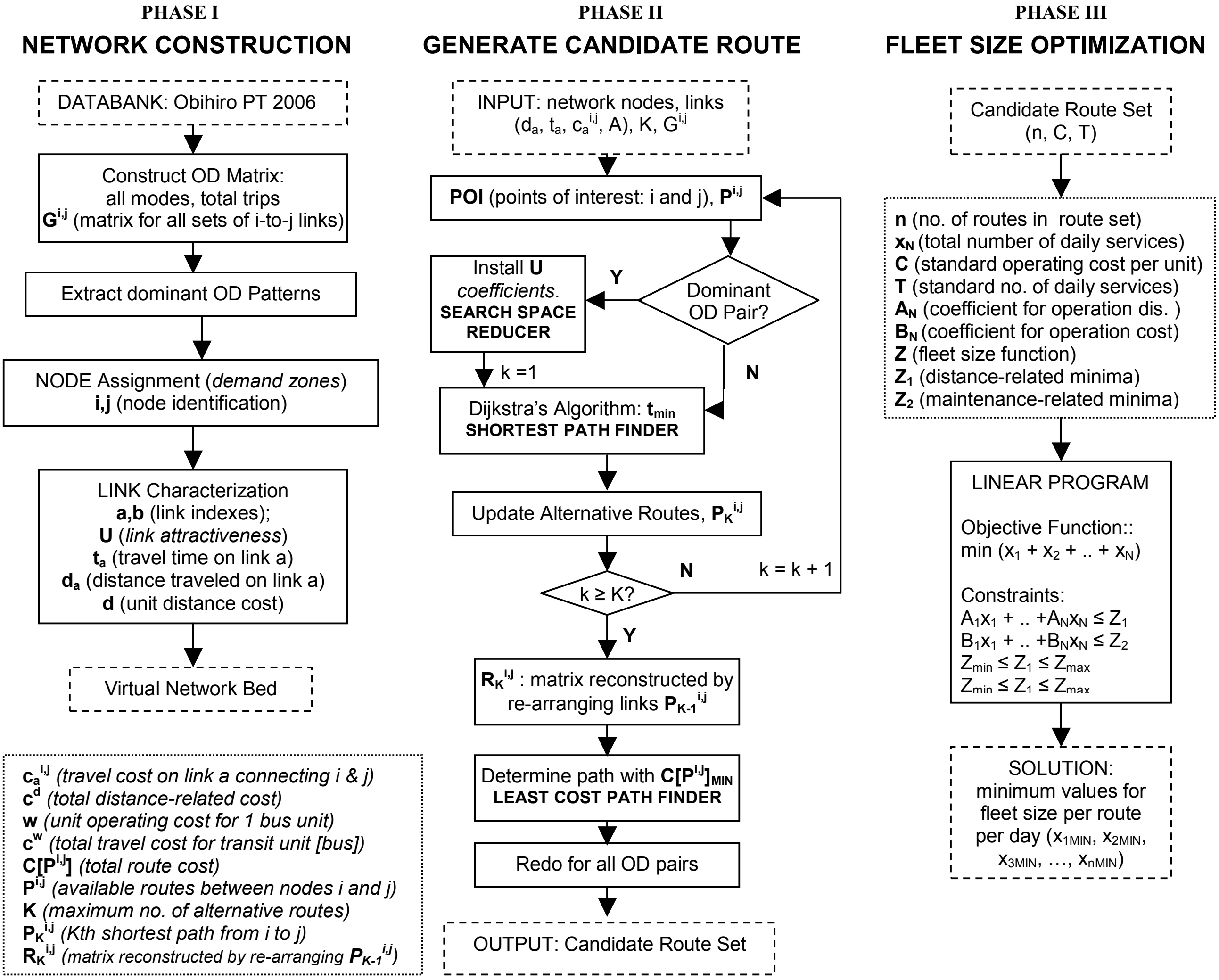

Figure 5. Schematic diagram for the overall flow of the solution methodology

3.2. *Route Set Generation*

3.2.1. *Shortest Path Algorithm*

Search Space Reduction is made possible by using *link attractiveness* (U), which is expressed as a coefficient in the equation for total route cost ($C[P^{i,j}]$). Such coefficients allow for the search space for feasible paths to be smaller in scale, instead of generating an infinitely large number of possible paths. Route Links correspond to segments of transit routes between consecutive prescheduled transit stations on the routes. Each link (i, j) has a value associated with it given by the time (cost) required to travel/transfer from node *i* to *j*. In particular, $\mathbf{c_{ij}}$ is the weight representing the sum of time (cost in time units) traveling from node i to node j if the link (i, j). Basically, we consider three attributes related to the total travel cost: (1) the riding time; (2) waiting time; and (3) operating cost converted into time units. .

Dijkstra's Algorithm, which is the basic multi-path search for most transit network problems, is used in the second phase of the methodology. It is also compatible with the procedure in this study, where *Netlogo* is used to simulate induced flow paths due to attraction to *preferential links* (see figure 6). First we let K=1 and find $P_K^{i,j}$ (the shortest path from node "i" to destination "j". All the paths that are found using this search statement are stored in a shortest path matrix, $R_K^{i,j}$, which will be used to find the K*th* shortest paths. When K (the maximum number of alternative *shortest* routes from which the *least costly* route is determined) is reached, the iteration stops and proceeds to the calculation of $C[P^{i,j}]$, with coefficients for link attractiveness (U) for a particular OD pair. Only the minimum cost path (in time units) in $K^{i,j}$ needs to be compared with the newly added paths.

Computational statements are also being studied in order to transform them to codable form in ***Netlogo***. For instance, the perceived travel time on the k*th* path between origin *i(r)* and destination *s(j)* is $T_K^{r,s} = t_K^{rs} + \xi_K^{rs}$, where t_K^{rs} is the measured travel time and ξ_K^{rs} is random term, the distribution of which is given by the Gumbel density function (Sheffi, [2]). The proponent is yet to decide how $T_K^{r,s}$ shall be determined for our sample city. Meanwhile, to test the performance of our solution methodology, assumed values are used.

Figure 6. R. Gaid's Shortest Path Algorithm via *Netlogo*

```
turtles-own [node-id L T prev-node]
links-own [weight]
globals [links-list nodes node-labels]

to setup
  clear-all
  set-default-shape turtles "circle"
  set nodes [matrix of nodes]
  set node-labels [node ID]
  set links-list [matrix for O1 D1 t1]
end
to layout [to set up network bed]
end
to set up node [node characterization]
end
to set up link [link characterization]
end
to shortest-path
  let source_id 0
  let destination_id 0
  foreach node-labels
  [
    if ((item 1 ?) = source) [ set source_id item 0 ? ]
    if ((item 1 ?) = destination) [ set destination_id item 0 ? ]
  ]

  if not (source_id = destination_id)
  [
    ask turtles
    [
      set L 1000
      set T 1
      set prev-node 1000
    ]
    ask get-node source_id [ set L 0 ]
    while [([T] of get-node destination_id) = 1]
    [
      let v [node-id] of min-one-of turtles with [ T = 1] [L]
      ask get-node v
      [
        set T 0
        ask link-neighbors
        [
          let w ( [L] of myself + [weight] of link [who] of myself [who] of
           self )
          if L > w
          [
            set L w
            set prev-node [node-id] of myself
          ]
        ]
      ]
    ]
    ask get-node destination_id [ set color red ]
    let prev [prev-node] of get-node destination_id
    while [not (prev = 1000)]
    [
      ask get-node prev [ set color red ]
      set prev [prev-node] of get-node prev
    ]
  ]
end
```

3.2.2. *Least Cost Paths*

The study aims to minimize total travel cost for one transit (bus) unit, given by C^w, which is a variation of C^d, the total distance-related cost. These two terms are calculated based on values derived for unit distance cost (d), unit travel cost of transit unit *w* (a^w), and the distance traveled on link a (d_a). To solve time and distance costs, we use the equations (presented by Jeong, Hong, and Kim, 2007)[3] in their study on flexible multi-path search algorithms.

$C^d = \Sigma_k d^k \Sigma_{a \in P^{k,i,j}} (d_a^k)$	total distance-related cost
$C^w = a^w \Sigma_k \Sigma_{a \in P^{k,i,j}} (t_a^k)$	transit unit operational cost
$C = C^w + C^d$	total path/route cost
$C[P^{i,j}] = C\ U$	total cost with link attractiveness coeff.

The shortest path search is done for all OD pairs, in such a way that those which reflect actual dominant OD patterns are calculated with coefficients for *link attractiveness* to induce preferences for such links, thereby decreasing the search space for feasible routes. The cycle is repeated until the K*th* shortest path is updated in the alternative route set, in which the path with the least cost (as aforementioned) is stored in the Candidate Route Set until the whole phase is performed for all OD pairs in the sample city.

3.3. ***Fleet Size Optimization***

Service frequency is a function of fleet size. In this part of the methodology, we deal with only the shortest, least costly routes stored in our Candidate Route Set. We use linear optimization to determine the best fleet size for this singular "best" route. Values for T and C will be taken from the study on the sample city's bus route restructuring (Gaid, 2008)[4]. The details for linear optimization are clearly presented in figure 5.

4. Conclusion

In this study, running the program with *Netlogo*, using assigned weights (expressed by coefficients for link attractiveness) for preferential links clearly reveals that the shortest, least costly paths selected also meet the preferred paths for each dominant OD pair reflected in the sample city's demand matrix. This reduces the search task as not all feasible paths need to be analyzed. With linear programming, the fleet size is also minimized without contradicting existing service frequency standards or existing demand size per route. For small networks, such as that of our sample city, this two-step program for determining the best set of routes in a transit (bus) network produces the shortest, least costly routes with optimal fleet size.

5. References:

[1] Obihiro Area General Urban Traffic System Investigation (Masterplan development Investigation) Hokkaido, 2006

[2] Sheffi, Y. (1985) Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods. *Prentice-Hall, Inc. Englewood Cliffs, N.J.* 07632

[3] Y.J. Heong, Y.C. Hong, T.J. Kim. (2007). Flexible Multipath Search Algorithm for Multipurpose Location-Based Activities. *Transportation Research Record: Journal of Transportation Research Board, No. 2039, Transportation Research Board of the National Academies, Washington D.C. 2007, pp. 50-57.* DOI: 10.3141/2039-06

[4] Gaid, R. (2008) OD-Based Measures for Bus Route Restructuring in the Obihiro Area. *The 38th Conference for Infrastructure Planning. Japan Society of Civil Engineers. Wakayama University. 2008.*