Comparison of the bed shear stresses using the original and the modified Chézy formulae in the vertically averaged and moment flow equations

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INTRODUCTION

Over two decades, most computational models of open-channel flows are based on the depth-averaged St. Venant equations. The depth averaging process used to derive these equations sacrifices flow detail over the vertical dimension. As a reason, these equations are not applicable to the complicated non-uniform flow. Steffler and Jin [1] proposed the new flow equations, the Vertically Averaged and Moment flow (VAM) equations, derived from the fundamental Reynolds equations by a moment weighted residual method. The VAM equations provide an additional component of the velocity, a vertically linear and zero mean distribution. Recently, a new local bed shear stress formula (a moment version of the Chézy formula) based on the moment of momentum approach has been developed, and it can describe the local bed shear stress using the mean velocity and the vertically linear and zero distribution of velocity [2]. In this study, we simulate two simple 1-D flow problems, flow over a gentle hump and flow over a step, using the VAM flow equations in order to investigate the advantage of using the modified Chézy formula.

FORMULATION

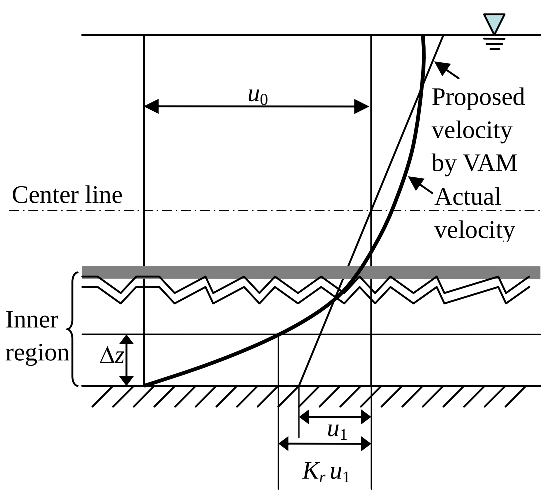

Figure 1 Scheme of the actual velocity, the proposed equivalent linear velocity and the near-bed velocity.

In Figure 1, the one-dimensional vertically averaged and moment (VAM) equations are express in the form

$$\frac{\partial h}{\partial t}+\frac{\partial u_0 h}{\partial x}=0 \tag{1}$$

$$\frac{\partial u_0}{\partial t}+u_0\frac{\partial u_0}{\partial x}+\frac{1}{3h}\frac{\partial h u_1^2}{\partial x}+g\frac{\partial h}{\partial x}+g\frac{\partial z}{\partial x}+\frac{\tau_b}{\rho h}=0 \tag{2}$$

$$\frac{\partial u_1}{\partial t}+u_0\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_0}{\partial x}-\frac{3\tau_b}{\rho h}+\frac{6\bar{\tau}}{\rho h}=0 \tag{3}$$

where t is time, x is the streamwise direction, g is the gravitational acceleration, ρ is the water density, h is the water depth, u_0 and u_1 are the depth-averaged velocity and the velocities at the water surface in excess of the means respectively, τ_b is the bed shear stress in the x-direction, and $\bar{\tau}$ is the vertically-averaged shear stress expressed as

$$\bar{\tau}=\rho\nu_z\left(\frac{2u_1}{h}\right) \tag{4}$$

where ν_z is the vertically averaged turbulent eddy viscosity in the vertical direction ($\nu_z=0.07u_* h$)

When the modified Chézy formula is used, it is given as

$$\frac{\tau_b}{\rho}=\frac{u_0\left(u_0-K_r u_1\right)}{C_2^2} \tag{5}$$

$$C_2=C_*\sqrt{1-K_r\alpha} \tag{6}$$

where the coefficient K_r is to give a more accurate estimation of the near-bed velocity and is assumed 1.50 in this study, C_2 and C_* are the modified and original Chézy coefficients respectively, α is the ratio between u_1 and u_0 in the case of uniform flow. The original Chézy coefficient is derived by

$$C_*=6+2.5\ln\left(\frac{h}{k_s}\right) \tag{7}$$

where k_s is roughness coefficient.

With the use of the CIP scheme, we divided the governing equations in the advection part and the non-advection part (Fractional Step Method). In the advection part, the equations are solved by the CIP scheme. In the non-advection part, the fully-implicit scheme is employed.

FLOW OVER A GENTLE HUMP

Let us consider the flow over a gentle hump. The simulation conditions are as follows. The discharge per unit width is 0.08 m^2/s. The height of the hump is 3.6 cm. The bed slope is 0.0001 and k_s is 1.508 mm.

As shown in Figure 2, the depth-averaged velocity u_0 gradually increases when the flow runs on the hump, and then it gradually decreases when the flow go down the hump. Meanwhile, the linear moment component of velocity u_1 is expressed the opposite trend. It can be explained as follows. When the flow is accelerating, the vertical distribution of the flow becomes more uniform and, correspondingly, the near-bed velocity increases. This results in the decrease in u_1. For the flow deceleration, the phenomenon will be reversed.

The comparison of the bed shear stresses over the water density using the original and the modified Chézy formulae is shown in Figure 3. It is found that far upstream and far downstream of the hump both formulae give the same value of the bed shear stress. However, in the vicinity of the hump where the flow is non-uniform the modified Chézy formula provides higher value.

FLOW OVER A STEP

In this section, we simulate the flow over a step which has steep slopes at the front and lee faces. The flow conditions are given as the preceding section.

In Figure 4, as expected the depth-averaged velocity u_0 shows the rapid increase at the front face of the step and the rapid decrease at the lee face of the step. Again, the linear moment component of velocity u_1 is expressed the opposite trend. However, some interesting patterns on u_1 are found. On the step, u_1 gradually increases. When u_1 reaches the lee face, the value rapidly increases and becomes higher than that far downstream. Then, u_1 decreases and will asymptotically achieves the value far downstream after some distance.

Figure 5 shows the bed shear stresses. The value by the modified Chézy formula shows the capability to simulate a lag between the bed shear stress and the bed profile downstream of the lee face in which it cannot be found in the original Chézy formula.

CONCLUSION

The one-dimensional flow model has been developed with the use of the new flow equations called the Vertically Averaged and Moment (VAM) equations. The VAM equations provide more details in velocity distribution, the depth-averaged velocity and its linear moment component. Two examples, flow over a gentle hump and flow over a step, are investigated. Because of the addition detail of the velocity distribution, the modified Chézy formula for computing bed shear stress is employed. From the result, it is found that the modified Chézy formula can express the lag between the bed shear stress and the bed profile qualitatively.

REFERENCES

[1] Steffler, P. M. & Jin, Y., 1993. Depth averaged and moment equations for moderately shallow free surface flow, J. Hydrau. Res., 31(1), 5–17.

[2] Elgamal, M. H. & Steffler, P. M., 2001. Stability analysis of dunes using 1-D depth averaged flow models, 2nd RCEM, Obihiro, 197-206.

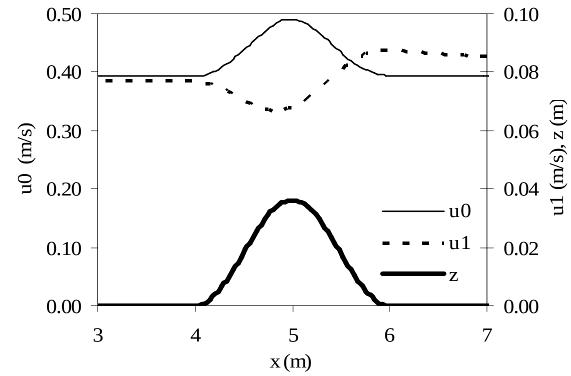

Figure 2 Bed and velocity components of flow over a hump.

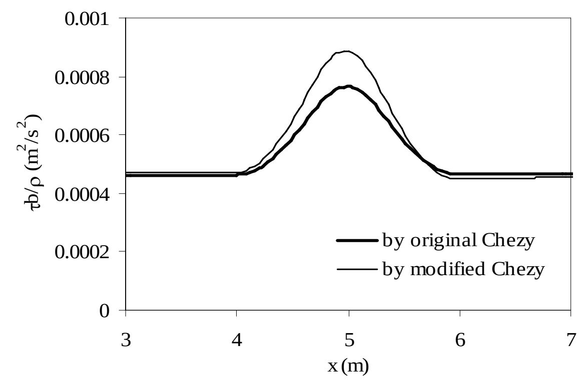

Figure 3 Bed shear stresses of flow over a hump.

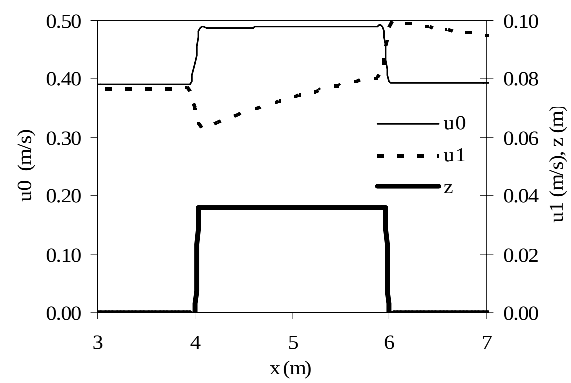

Figure 4 Bed and velocity components of flow over a step.

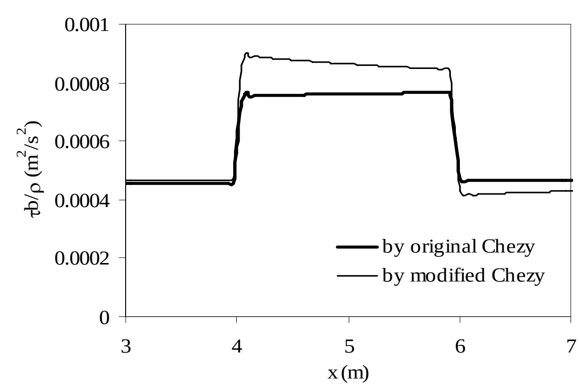

Figure 5 Bed shear stresses of flow over a step.