Application of Equivalent Fracture Energy for Concrete Elements With Sand Cushion for Three Dimensional Elasto-Plastic Impact Response Analysis of Prototype RC Girder

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1. INTRODUCTION

The goal of this paper is the development of an objectivity algorithm for tensile failure of concrete elements based on the smeared cracking formulation for RC Girder with sand cushion. The algorithm has been implemented into LS-DYNA for hexahedron solid elements and correctly accounts for crack directionality effects. Thus enabling the control of energy dissipation will be associated with each failure mode regardless of mesh refinement.

The advantage of the proposed technique is that mesh size sensitivity on failure is removed leading to results, which converge to a unique solution for girder with sand cushion, as the mesh is refined. The proposed algorithm has been validated by a full-scale prototype test with sand cushion using different cases of mesh refinement.

Here, in order to establish a modification method for material properties of concrete so as to rationally analyze using coarse mesh, an equivalent fracture energy concept for concrete element with sand cushion is proposed and the applicability was conducted comparing numerical analysis results and experimental results.

2. EXPERIMENTAL OVERVIEW

2.1 Dimensions and static design of RC girder

RC girder, which is modeled considering roof of real RC rock-sheds, is taken for falling-weight impact test of prototype RC structures. The girder is of rectangular cross section and the dimensions are of 1.0 x 0.85 m and clear span is 8 m long. The dimensions of the sand cushion set on the center of girder are of 1.5 x 1.5 x 0.9 m. Figure 1 shows dimensions of the RC girder with sand cushion, arrangement of rebars, and measuring points for each response wave. 7#D29 rebars are arranged which is for 0.64% of main rebar ratio corresponding to designing of real RC rocksheds. Static flexural and shear load-carrying capacities P_{usc} and V_{usc} were calculated based on Japanese Concrete Standards. From these values, it is confirmed that the RC girder designed here will fail with flexural failure mode under static loading.

2.2 Experimental method

In the experiment, 5,000 kg heavy weight was lifted up to the prescribed height of 10 m by using the track crane, and then dropped freely to the mid-span of girder due to a desorption device. A heavy weight is made from steel in which outer shell is of 1 m in the diameter, 97 cm in height, and spherical bottom with 80 cm in radius and its mass is adjusted filling concrete and steel balls. In this experiment, impact force wave (*P*), reaction force wave (*R*), and displacement waves (*D*) were measured. The accelerometer is of strain gauge type and its capacity and frequency range

3. ANALYTICAL OVERVIEW

3.1 FE models

The purpose of this research is to propose the method for converting tensile strength of concrete element with arbitrary element size in span direction applying an equivalent fracture energy concept for full scale RC girder with sand cushion and the applicability is discussed by comparing with the experimental results. Therefore, the standard element division for precise numerical analysis result is needed. In this research, the suitable results of reference 1 were used and the standard analytical model such as MS35-*Gf* was decided for prototype RC girder. Similarly the standard mesh size of the span and the

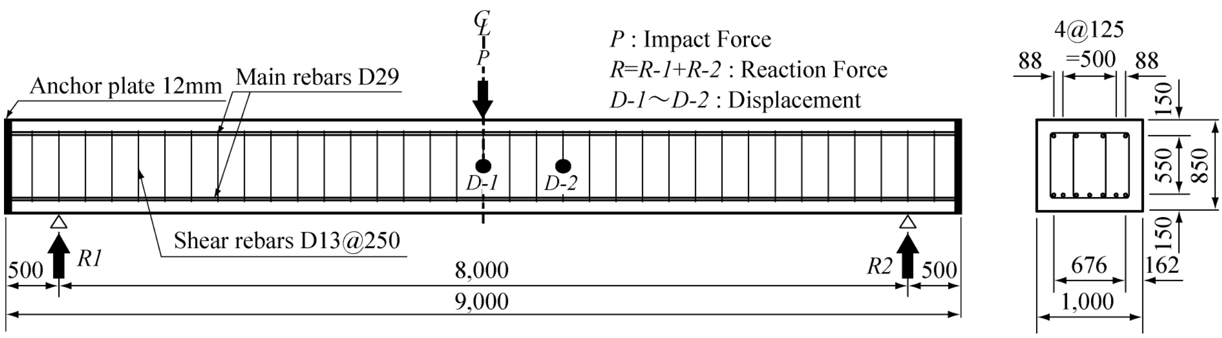


Figure - 1 Dimensions of RC girder and measuring items

Table - 1. Static design parameters of RC girder

Shear rebar ratio ρ_t	Static shear depth ratio a/d	Static shear capacity V_{usc} (kN)	Static bending capacity P_{usc} (kN)	Shear-bending capacity ratio α
0.0064	5.71	1794.0	619.8	2.894

Table - 2 Material properties of concrete and reinforcement

Type	Density ρ（ton/m^3）	Elastic coefficient E（GPa）	Poisson Ratio ν	Yielding *strength* $f'c$（MPa）
Concrete	2.343	25.4	0.177	31.2
Rebar D13	7.85	206	0.3	390
Rebar D29	7.85			400

cross section width and height of the RC girder were set as 35 mm, 41 mm and 31 mm, respectively based on the previous results[2)]

On the other hand, four cases were considered by assuming 1, 3, 5, and 7 division whose element sizes are 250 mm, 83 mm, 50 mm, and 35 mm, respectively and those cases are named as MS250-*Gf*, MS83-*Gf*, MS50-*Gf* and MS35-*Gf* respectively. For all cases, each mesh size along the girder near supporting area of 500 mm wide was set to be 35 mm long because of precisely analyzing an interaction between supporting gigues and girders[1)]. The mesh size distribution of each analytical case is shown in Fig. 2.

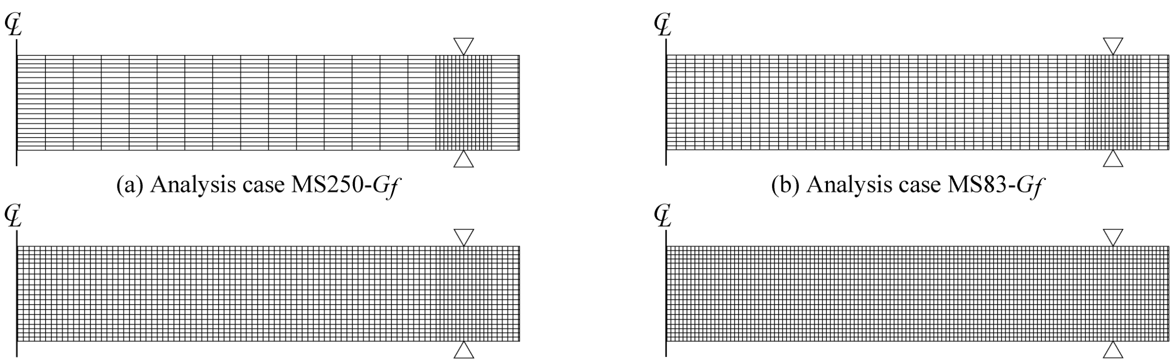
One quarter of RC girder with sand cushion was three-dimensionally modeled for numerical analysis with respect to the two symmetrical axis. Figure 2 shows a mesh geometry of the girder, which is finally used for numerical analysis with optimum design accuracy investigated here. Geometrical configurations of the heavy weight and supporting gigue were also precisely modeled corresponding to the real ones. In this model, axial rebar and stirrup were modeled using beam element having equivalent axial stiffness, cross sectional area and mass with those of real ones. The others were modeled using eight-node and/or six-node solid elements.

Total number of nodal points and elements for one-fourth model for MS35-*Gf* are shown in Fig. 3 are 49,547 and 42,487, respectively. Number of integration points for solid and beam elements are one and four, respectively. In order to take into account of contact interface between concrete, sand and head of heavy weight elements and between adjoining concrete and supporting gigue elements, contact surface elements for those were defined, in which contact force can be estimated by applying penalty methods for those elements but friction between two contact elements were neglected. Impact velocity was applied to all nodal points of the weight.

3.2 Modeling of materials

The stress and strain relations of concrete, rebar and sand are shown in Fig. 4. The outline of the material physical properties model such as concretes, rebar are shown in Table 2. For the compression region, assuming that concrete is yielded at 1,500 μ strain, perfect elasto-plastic bilinear model was used. For the tension region, tensile strength of concrete is assumed to be one-tenth of compressive strength for the standard case of MS35-*Gf* but for other cases equivalent fracture energy concept was applied. Fig. 4(c) shows the constitutive model for sand cushion. To rationally analyze in stress behavior of sand cushion when a heavy weight collides, second order parabolic stress-strain relation was applied for sand cushion.

Yielding of concrete has been judged based on the Drucker-Prager's yield criterion. Stress-strain relationship for main rebar and stirrup was defined using a bilinear isotropic hardening model. Plastic hardening coefficient *H'* was assumed to be 1% of Young's modulus *Es*. Yield of rebar and stirrup was judged



(a) Analysis case MS250-*Gf*　(b) Analysis case MS83-*Gf*

(c) Analysis case MS50-*Gf*　(d) Analysis case MS35-*Gf*

Figure - 2 FE mesh size distribution for each analytical case

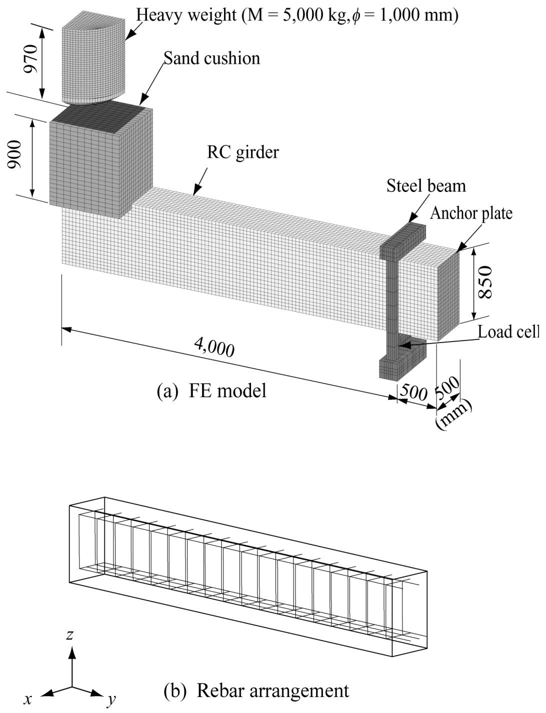


Figure – 3 FE Numerical analysis model

following von Mises yield criterion. Heavy weight, supporting gigues and anchor plates for axial rebars set at the both ends of RC girder were assumed to be elastic body because of no plastic deformation for those being found.

3.3 Equivalent fracture energy concept

In this paper, the element was assumed to be failed in the whole area of element because of applying smeared crack model, when negative pressure surcharged to the element reaches a tensile strength.

Assuming one flexural crack occurs in a element irrespective of magnitude of element size, the element must be set so as to be failed at the time when a strain energy stored in the element reaches fracture energy which is the same among all concrete element size irrespective of magnitude of element size.

Based on this equivalent fracture energy concept, each concrete element can retain the equivalent fracture energy due to setting a fictitious tensile strength corresponding to volume of element. In Fig. 5(a), assuming fracture energy of standard concrete element and volume of the element as G_f and V_o, respectively. G_f can be represented as Eq. (1) in which f_{to} and

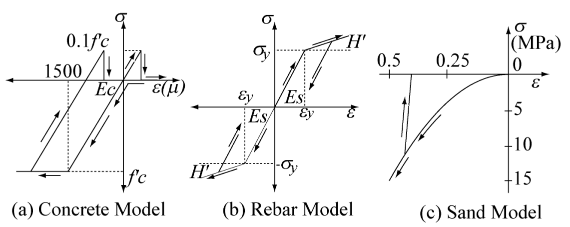


Figure - 4 Stress-strain relation of constitutive model

Table - 3. Tensile strength for different analytical cases

Element size in span direction (mm)	Fictitious tensile strength (MPa)
250	1.18
83	2.04
50	2.64
35	3.12

ε_{to} are tensile strength and strain at tensile failure of the standard concrete element.

$$G_f = \frac{f_{t0}\varepsilon_{t0}}{2} V_0 \tag{1}$$

Ultimate tensile strain ε_{to} can be determined as the following equation,

$$\varepsilon_{t0} = \frac{f_{t0}}{E_c} \tag{2}$$

Assuming each element size in x, y, and z direction of the standard element as x_o, y_o, and z_o, respectively, the volume of the standard element is as follows:

$$V_0 = x_0 y_0 z_0 \tag{3}$$

By putting the values of Eq. (2) and Eq. (3), the fracture energy G_f of Eq. (1) can be obtained as,

$$G_f = \frac{f_{t0}^2}{2E_c} x_0 y_0 z_0 \tag{4}$$

Here, setting the fictitious tensile strength and element size in y direction of i-element as f_{ti} and y_i and applying an equivalent fracture energy concept between standard element and i-element, following relationship can be obtained as;

$$\frac{f_{t0}^2}{2E_c} x_0 y_0 z_0 = \frac{f_{ti}^2}{2E_c} x_0 y_i z_0 \tag{5}$$

Fictitious tensile strength of i-element f_{ti} can be obtained as follows;

$$f_{ti} = f_{t0}\sqrt{\frac{y_0}{y_i}} \tag{6}$$

Therefore, taking the fictitious tensile strength f_{ti} obtained from Eq. (6) for i-element with y_i as the size in y-direction, the crack occurred in the i-element can be rationally estimated similarly to the standard element with fracture energy G_f. The fictitious tensile strength for each element size in y-direction used in this study is listed in Table 3.

4. COMPARISON OF RESUTLS

The applicability of the proposed method is examined for the set of each element length for different cases considering G_f in this section and comparing with experimental results. Analytical result concerning all cases with different element

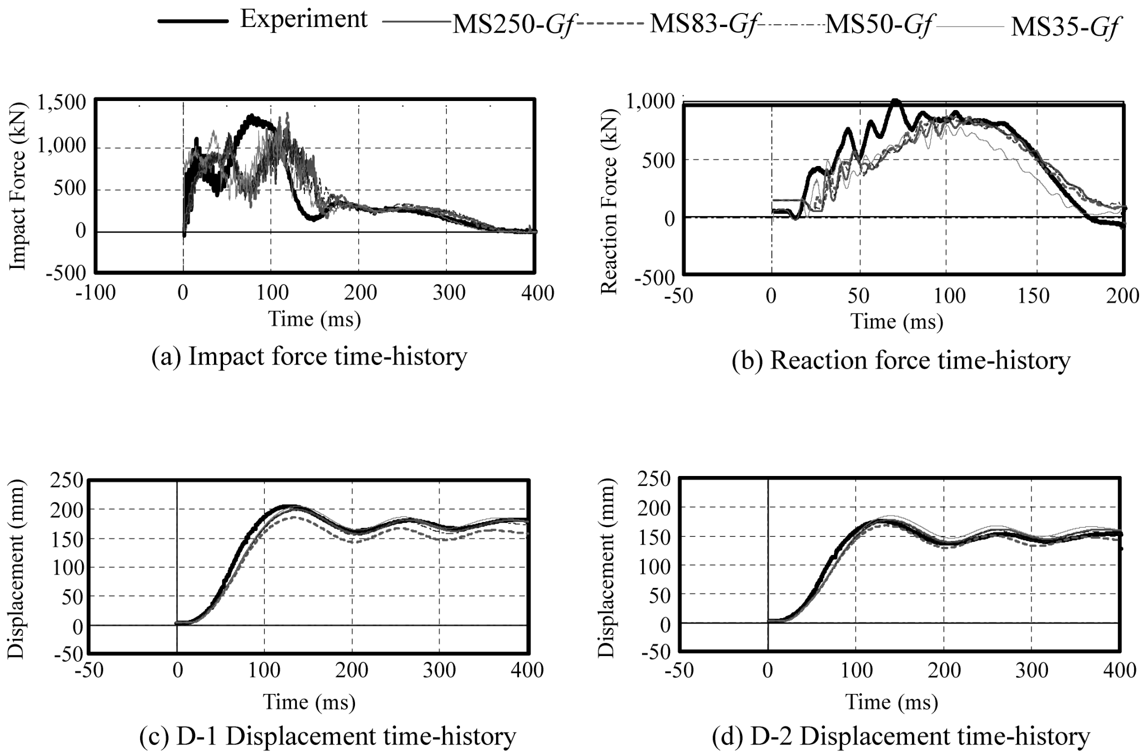


Figure - 5 Comparison between experimental and analysis results considering G_f

length considering G_f is compared with the experimental results as shown in Fig. 8. The comparison between results for coarse mesh with considering G_f are shown in Fig.5.

The maximum value of impact force wave is smaller than the experimental results regardless of the mesh size of the element length as previously observed. It is understood that the response characteristics are similar regardless of the size of the element length as well as the case of impact force and for the reaction force waveform. From Fig. 5(b), it is confirmed that the reaction force wave at the one supporting point was almost similar to experimental one. It is understood that the amplitude of D-1/2 as shown in Figs 5(c) and 5(d), the both cycle and the residual displacement are in the state of a free vibration after the maximum displacement regardless of the size of the element length by comparing the experimental results and the analytical one. By comparing the experimental results, among four cases the most underestimating case is MS83-G_f though the level of the error margin is not large when seeing in detail. By converting the tensile strength as the case of MS35-G_f as shown in Table 3, and it can be confirmed to an analytical result of MS250-G_f when the element length is the largest then the analytical accuracy is good enough even if the element division is coarse. Even if the span direction element length is different from a standard element, it means that the mesh size up to seven times the standard element length having the same accuracy as the case to use a standard element by using the fracture energy concept.

5. CONCLUSIONS

This paper presents a detailed formulation and numerical implementation of an objectivity algorithm for equivalent fracture energy concept of prototype RC girder with sand cushion. From this study, it is confirmed that even though coarse mesh was used for RC girder with sand cushion, similar results with those obtained using fine mesh can be assured and are in good agreement with the experimental ones. The results obtained from this study are as follows;

(1) The fictitious tensile strength for coarse mesh has been proposed for the prototype RC girder with sand cushion

(2) By using the fictitious tensile strength for coarse mesh with 250 mm mesh size, similar degree of accuracy can be obtained by using the fine mesh having mesh size of 35 mm.

(3) However, an incident of the beginning of impact and maximum amplitude of the impact force time history tend to be underestimated by comparing with those of experimental results.

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