Dynamic Response of Cylindrical Shells on Elastic Foundation under Radial Impact Loads

Hokkaido University	Student Member	○ Haryadi Gunawan
Hokkaido University	Member	Motohiro Sato
Docon Co., Ltd.	Member	Akio Koike
Hokkaido University	Member	Shunji Kanie
Hokkaido University	Fellow Member	Takashi Mikami

1. Introduction

Cylindrical shells are extensively used in engineering fields in the form of structural components for storage tanks, pressure vessels, processing equipment, water ducts, subsea/ground pipelines, and in other applications. The strength, lightness, and spatial properties of the shells have been recognized as major advantages over other types of structures. On the other hand, the shells exhibit complex behaviors related to their characteristics in the dynamic states. The problems are becoming more complex when considering the effects of foundations surrounding a shell and the interactions between fluid and shell.

Lakis *et al*.[1-2] have developed the hybrid finite element formulation for circular cylindrical shells based on the analytical shape functions which are derived from governing equations of shell. The analysis of whole buried pipeline subjected to sinusoidal seismic wave, differential settlement, and dislocation of ground has been investigated by Yang *et al*.[3] using the shell finite element. Paliwal *et al*.[4-5] have studied the free vibrations of the whole buried cylindrical shells in Winkler and Pasternak foundations by direct solution to the governing equations of motion. In the paper, the elastic foundations are distributed uniformly both in the circumferential and in the longitudinal directions. However, cylindrical shells are generally laid on elastic foundation, so that the foundation only covers certain parts of the shell in the circumferential direction. This leads to more complex problem. Amabili *et al*.[6] have investigated the free vibrations of cylindrical shell simply supported at both ends with a non-uniform elastic foundation in the circumferential direction based on the Rayleigh-Ritz method. The elastic foundation has to be assumed distributed uniformly over the whole cylinder length in the longitudinal direction. Gunawan *et al*.[7-8] have studied the static and free vibration of cylindrical shells partially buried in the elastic foundation based on the semi-analytical finite element method where the simple polynomials were used as shape functions. Free vibrations of fluid-filled cylindrical shells on elastic foundations using the analytical shape functions have been investigated by Gunawan *et al.* [9].

This paper presents the dynamic response of cylindrical shells partially buried in elastic foundations under radial impact loads by means on the semi-analytical finite element method. The shell is discretized into cylindrical finite elements where the analytical shape function based on the governing equations of the empty is used. The foundation is represented by elastic radial springs. Empty and fluid-filled shells are considered here. The internal fluid is assumed to be stationary. The effects of the spring stiffness and enclosed angle on dynamic response of the system are investigated. In addition, the dynamic load factors of empty and fluid-filled shells for different impulse duration.

2. Model and formulation

The structure is an isotropic thin elastic circular cylindrical shell with Young's modulus E, Poisson's ratio υ, radius of the middle surface R, thickness h, and length L. The radial spring coefficient is denoted by K_w. In the analysis, the spring coefficient is assumed to be constant along the enclosed arc. The angle that define the enclosed arc is denoted by φ. The shell is subjected to a localized radial impact load $Q(t) = q\,f(t)$ in the radial direction, where q is the magnitude of surface load and $f(t)$ is the time-varying function. The load is located symmetrically about the midspan. The geometry of the structure, loading distribution, and the reference directions are shown in Fig. 1. The foundation and load are distributed symmetrically about the $\theta = 0$ axis.

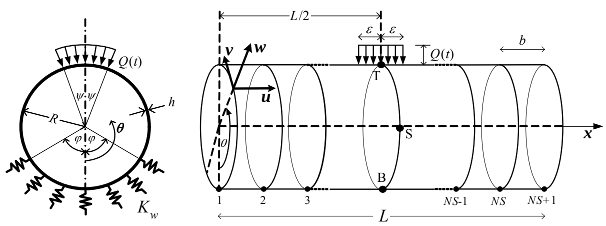

Fig.1. A shell subjected to a localized radial impact load.

The displacement of a point on the middle surface in the axial, circumferential, and radial directions is indicated by u, v, and w, respectively. Due to the symmetry of the problem, the displacements in the spatial coordinate can be defined as follows:

$$u(x,\theta) = \sum_{m=0}^{M} U_m^{\mathrm{S}}(x)\cos(m\theta);\ v(x,\theta) = \sum_{m=0}^{M} V_m^{\mathrm{S}}(x)\sin(m\theta);$$

$$w(x,\theta) = \sum_{m=0}^{M} W_m^{\mathrm{S}}(x)\cos(m\theta) \qquad (1)$$

where m is a typical circumferential wave number. The derivation of the stiffness and mass matrices of the shell and stiffness matrix of the foundation is explained in details in the paper by Gunawan et al. [9], and therefore is not given here.

The fluid is assumed to be incompressible, invicid, and the fluid motion is irrotational so that the flow can be described by a velocity potential, Φ, which satisfies the following Laplace equation:

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial x^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}=0 \tag{2}$$

The deformation potential ϕ is related to the velocity potential Φ by

$$\Phi(x,\theta,r,t)=i\omega\phi e^{i\omega t} \tag{3}$$

The contact condition at the shell-fluid interface is defined by

$$\left(\frac{\partial\Phi}{\partial r}\right)\bigg|_{r=R_i}=\frac{\partial w^*}{\partial t} \tag{4}$$

where $w^*=w(x,\theta)\,e^{i\omega t}$. Solution of the Laplace equation satisfying additional fluid boundary conditions at the ends of the shell (ϕ = 0 at x = 0, L) and regularity condition (finite value at the axis of the shell) can be written as follows:

$$\phi(x,\theta,r)=\sum_{m=0}^{\infty}\sum_{k=1}^{\infty}D_{mk}\sin\left(\frac{k\pi x}{L}\right)\mathrm{I}_m\left(\frac{k\pi r}{L}\right)\cos(m\theta) \tag{5}$$

where k is the half-wave number and I_m is the m-th order modified Bessel function of the first kind. D_{mk} are unknown coefficients which can be determined by using (4) and employing the orthogonality properties of the function with respect to x. Once the potential is obtained, the hydrodynamic pressure, p, acting on the wall of the shell can be determined from the linearized Bernoulli equation and is given by

$$p=-\rho_L\frac{\partial\Phi}{\partial t}\bigg|_{r=R_i} \tag{6}$$

where $R_i = R - h/2$ is inner radius of the shell and ρ_L is density of the fluid. By using the finite element method, the mass matrix of the fluid can be obtained readily.

By neglecting any physical damping effects, finally, the resulting governing equation of the system under consideration is given by

$$\mathbf{M}\,\ddot{\boldsymbol{\delta}}^{\mathrm{e}}(t)+\mathbf{K}\,\boldsymbol{\delta}^{\mathrm{e}}(t)=\mathbf{F}(t) \tag{7}$$

where $\mathbf{M}$ is composed of mass matrices of the shell, $\mathbf{M}_\mathbf{S}$ and of the fluid, $\mathbf{M}_\mathbf{L}$. $\mathbf{K}$ is composed of stiffness matrices of the shell, $\mathbf{K}_\mathbf{S}$ and of the foundation, $\mathbf{K}_\mathbf{F}$, and $\mathbf{F}(t)$ is the load vector. $\mathbf{M}_\mathrm{L}$ reduces to a zero matrix for cases corresponding to empty shells. The spatial distribution of the load is handled by the Fourier series and an element mesh strategy. (7) is solved by using the well-known Newmark integration scheme.

The natural frequency ω is non-dimensionalized by $\Omega = \omega/\omega_0$ where $\omega_0^2 = E/(\rho_S L^2(1-\upsilon^2))$. For all numerical results presented here, the computations used the following parameters: υ = 0.30, $\varphi_1 = \varphi_2 = \varphi$, ε/L = 0.05, and $\psi = \pi/36$. Based on the convergence studies which are not shown here, the total number of finite elements, NS = 40 and the total number of circumferential waves, M = 40 are used through out. For convenience, the time coordinate is non-dimensionalized by $\tau = \omega_0 t$. For the time integration scheme, the following Newmark's parameters are used: β = 0.25 and γ = 0.50. The time step is taken to be $\Delta\tau$ = 0.005. The initial conditions for displacement and velocity are assumed to be zero. Unless otherwise stated, a unit step function load with magnitude q is used. The displacements at coordinates (L/2,0), (L/2,π/2), and (L/2,π) are denoted by subscript B, S, and T (Fig. 1), respectively. The total stress in the l ($l = x$ or θ) direction is denoted by σ_l.

3. Numerical results

This section mainly describes the response of empty shells. In the present study, the shell is assumed to be simply supported at both ends. The effects of foundation parameters such as spring stiffness and enclosed angle are also investigated. The latter part discusses the response of fluid-filled shells, while results for empty shells are simultaneously presented for comparison.

First, one investigates how the response of the shell is affected by the spring stiffness. Fig. 2 shows the radial displacement response of the shell for different values of K_wL/E. The figure reveals that the presence of the foundation reduces the magnitude of w_B drastically. However, the largest absolute magnitude for the radial displacement is found at T. From Fig. 2b, one can find that w_T is less influenced by the changes in radial spring stiffness, K_wL/E.

Secondly, one investigates the ratio of the maximum response for displacements and total stresses with variations in φ. The results are presented in Fig. 3, where d stands for v or w, while d' and σ' are the displacement and stress of shell in absence of the foundation. At early increment of φ, w_B is directly influenced by φ. Fig. 3a shows that w_B decreases about 90%. However, for φ > 30 degree, further increase in φ has no significant influence on w_B. Similar to w_B, w_S decreases gradually as φ increases. It can be seen that the rate of decrement for w_S is slower than that for w_B. The same figure also suggests that for φ < 90 degree, φ does not significantly affect w_T. It is interesting to note that v_S decreases as φ increases. However, at the end of the curve, v_S tends to increase. This is due to the existence of the foundation on most part of the shell which gives an extra confinement so that the shell tends to move vertically. Apart from the discussion of the displacements, the stresses at T for

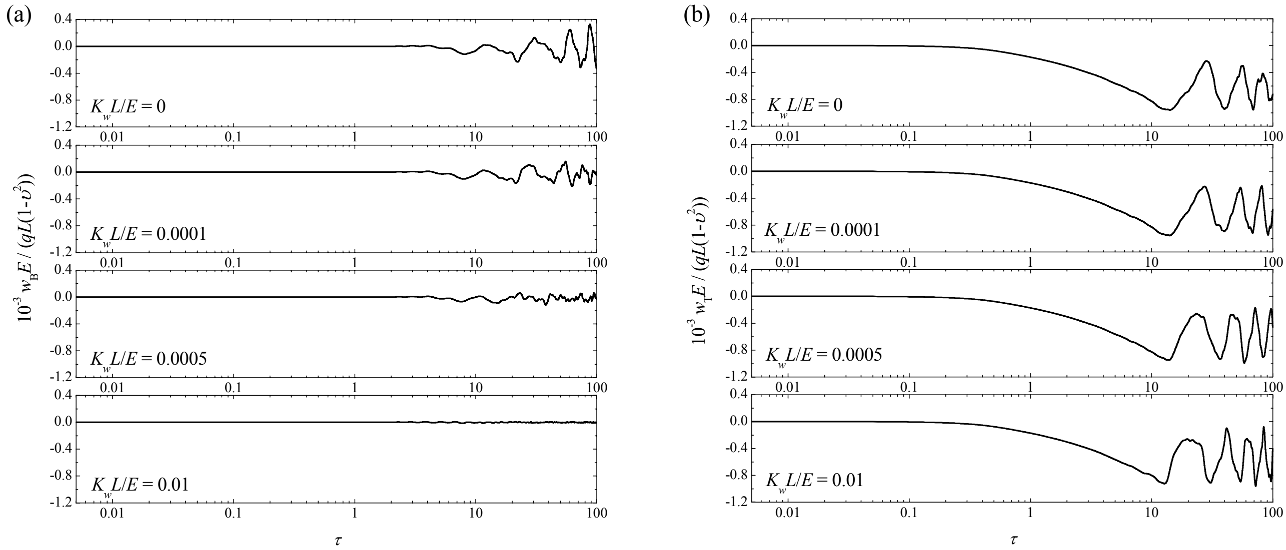

Fig. 2. Radial displacement response for different values of K_wL/E
(Empty, υ = 0.30, R/L = 0.20, R/h = 100, K_wL/E = 0.003, and φ = π/3): (a) at B; (b) at T.

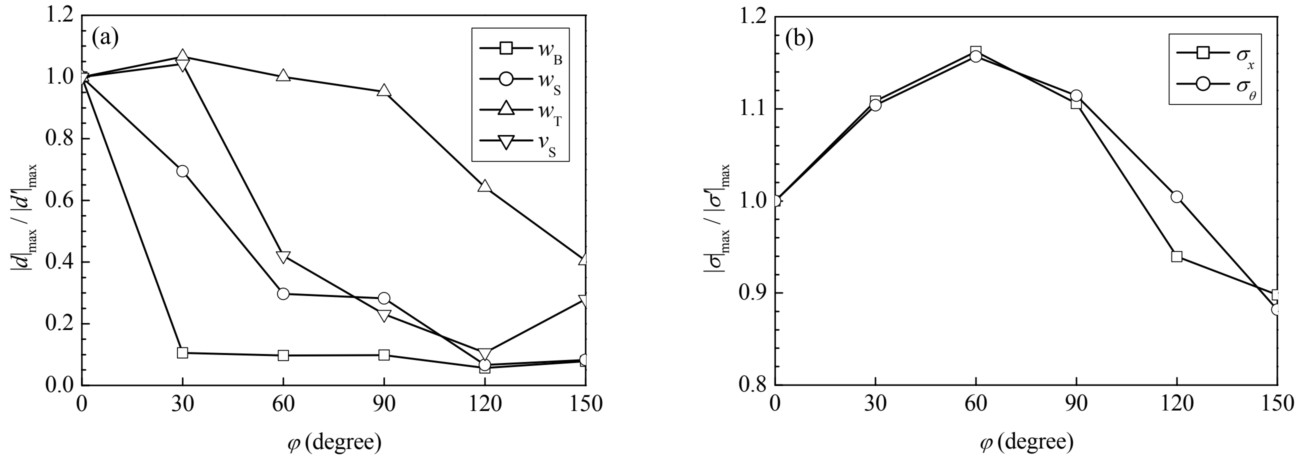

Fig. 3. Effect of enclosed angle on the maximum response of displacements and total stresses
(Empty, υ = 0.30, R/L = 0.10, R/h = 100, and K_wL/E = 0.003): (a) displacements; (b) total stresses at T.

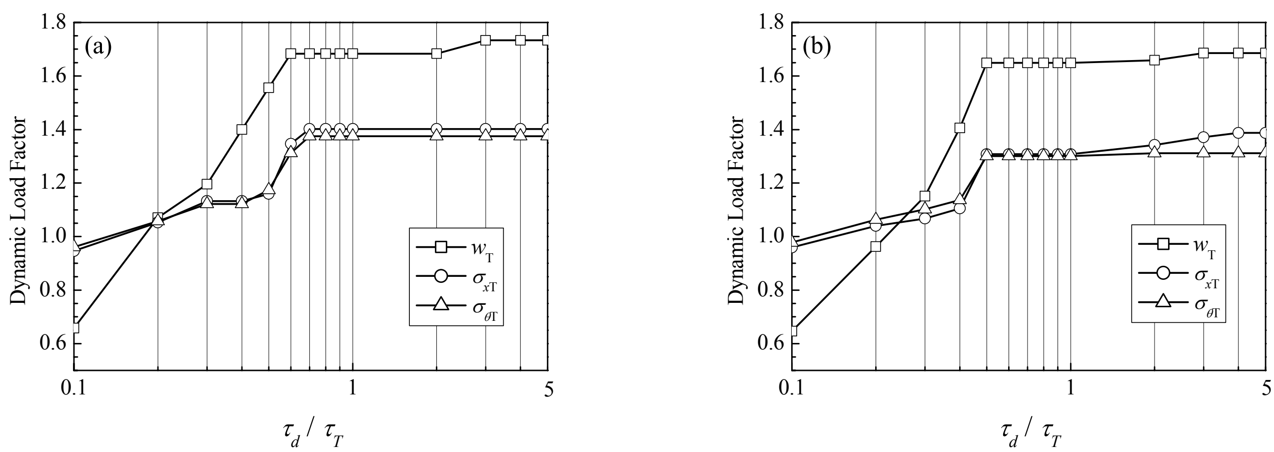

Fig. 4. Dynamic load factor for displacement and stresses (υ = 0.30, R/L = 0.10, R/h = 100, K_wL/E = 0.001, and φ = π/3):
(a) empty, τ_T = 22.841; (b) full, τ_T = 53.163 (ρ_L/ρ_S = 0.128).

different φ are given in Fig. 3b. As φ increases, both stresses increase and then decrease gradually.

Finally, one investigates how the impulse duration affected the response of the shell. The rectangular impulse load distribution in the time coordinate is considered. The duration of impulse is denoted by τ_d. The results are presented in terms of dynamic load factor which is defined as the ratio of maximum absolute response divided by the absolute static solution. Fig. 4 shows the dynamic load factors of the displacement and stresses at T for different values of τ_d / τ_T, where τ_T is the non-dimensionalized fundamental period of the system. Generally for $\tau_d / \tau_T \geq 0.3$, the dynamic load factor for displacement is larger than the dynamic load factors of the stresses. The dynamic load factor for σ_x does not show significant difference from that for σ_θ. Comparing the results for empty and fluid-filled shells, the dynamic load factor for displacement at relatively large value of τ_d / τ_T is found to be around 1.7 for both shells, while the dynamic load factors for the stresses range from 1.3 to 1.4. The dynamic load factors for empty shells are similar to those of fluid-filled shells.

4. Conclusions

This paper demonstrates the applicability of the semi-analytical finite element method to the dynamic analysis of cylindrical shell partially buried in elastic foundations subjected to localized impact loads. First, the response of empty shells is presented. The effect of the foundation parameters is discussed. Later on, the response of fluid-filled shells in terms of the dynamic load factor is examined. The results for both empty and fluid-filled shells are given for comparison.

The main conclusions of the present study may be summarized as follows:

- The foundation parameters such as the radial spring stiffness and enclosed angle influence the response of displacement and stresses, especially at parts of the shell which are enclosed by the foundation.
- For thin shells, it is found that increase in the radial spring stiffness has no significant effect on the maximum response of the displacement on the top edge. In this case, the enclosed angle is more pronounced especially for $\varphi \geq \pi/2$.
- The fluid does not influence the dynamic load factor of the displacement. However, the dynamic load factors of the stresses are slightly decreased by the existence of the fluid. Therefore, one may use the dynamic load factor of the empty shells for designing shells with internal fluid.

References

1) Lakis, A.A. and Paidoussis, M.P.: Dynamic Analysis of Axially Non-uniform Thin Cylindrical Shells, J. Mech. Sci., Vol 14(1), pp.49-71, 1972.
2) Lakis, A.A. and Sinno, M.: Free Vibration of Axisymmetric and Beam-like Cylindrical Shells, Partially Filled with Liquid, Int. J. Numerical Methods in Eng., Vol 33, pp.235-268, 1992.
3) Yang, R. *et al.*: Shell model FEM Analysis of Buried Pipelines under Seismic Loading, Bul. Disas. Prev. Res. Int. Kyoto University., Vol 38(3), pp.115-146, 1988.
4) Paliwal, D.N. *et al.*: Free Vibration of Circular Cylindrical Shell on Winkler and Pasternak Foundations, Int. J. Pres. Ves & Piping, Vol 69, pp.79-89, 1996.
5) Paliwal, D.N. *et al.*: The Large Deflection of an Orthotropic Cylindrical Shell on a Pasternak Foundation, Comput Struct., Vol 31(1), pp.31-37, 1995.
6) Amabili, M. and Dalpiaz, G.: Free Vibration of Cylindrical Shells with Non-axisymmetric Mass Distribution on Elastic Bed, Meccanica, Vol 32, pp. 71-84, 1997.
7) Gunawan, H. *et al.*: Static and Free Vibration of Cylindrical Shells on Elastic Foundation, J. Struc. Eng. JSCE, Vol 50A, pp.25-34, 2004.
8) Gunawan, H. *et al.*: Finite Element Analysis of Cylindrical Shells Partially Buried in Elastic Foundation, Comput Struct, Vol 83(21/22), pp. 1730-1741, 2005.
9) Gunawan, H. *et al.*: Free Vibrations of Fluid-Filled Cylindrical Shells on Elastic Foundations, Thin-Walled Struct, Vol 43(11), pp. 1746-1762, 2005.