Free Vibration of Partially Suspended Cylindrical Shells on Elastic Foundation

Hokkaido University　Student Member　○ Haryadi Gunawan
Hokkaido University　Fellow Member　Takashi Mikami
Hokkaido University　Member　Shunji Kanie
Hokkaido University　Member　Motohiro Sato

1. Introduction

Cylindrical shells as structural components are extensively being used in many engineering fields for pressure vessels, process equipments, storage ducts, pipelines, etc. Free vibration of cylindrical shells in air has been investigated by many authors, while free vibration of whole buried cylindrical shells has been studied thoroughly by Paliwal et al. [1]. Amabili [2] has studied the Rayleigh-Ritz based free vibration analysis of cylindrical shells with non-uniform distributed elastic bed in circumferential direction, but in his paper, the distribution of foundation in longitudinal direction has to be assumed uniform. In practical applications, cylindrical shell structures might be exposed to the condition shown in Fig. 1(a). Gunawan [3] presented an application of semi analytical finite strip method to analyze cylindrical shells partially buried in elastic foundation by using ring shape element. Adopting the same method, the non-uniformity of distribution of foundation in longitudinal direction can be taken into account.

The purpose of this paper is to introduce a method to analyze partially suspended cylindrical shells on elastic foundation. Effect of the gap's length (αL in Fig. 1(a)) on the natural frequencies of the vibrating system is presented and compared with the cases which the whole shell is in the air or on the foundation.

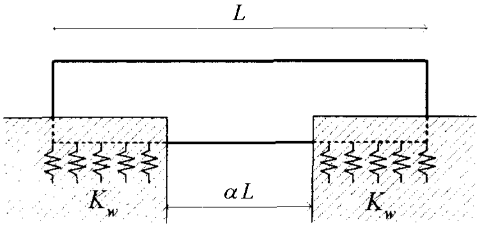
2. Model and formulation

The cylindrical shells are modeled by using isotropic thin elastic cylindrical shell element. The considered problem, geometry, reference direction and discretization of the model are shown in Fig. 1. The soil as foundation is modeled by elastic spring which is connected to shell in radial direction. For the sake of simplicity only radial spring is considered ($K_u = 0$, $K_v = 0$, $K_w \neq 0$, $K_\beta = 0$) and distributed on a limited arc along the circumferential direction.

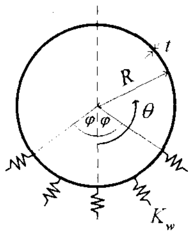
For an axi-symmetric problem, the axial, circumferential and radial displacements can be sought as follows,

$$\begin{aligned} u(x,\theta,t) &= \sum_{m=0}^{\infty} U_m(x)\cos(m\theta)\, e^{i\omega t} \\ v(x,\theta,t) &= \sum_{m=0}^{\infty} V_m(x)\sin(m\theta)\, e^{i\omega t} \qquad (1) \\ w(x,\theta,t) &= \sum_{m=0}^{\infty} W_m(x)\cos(m\theta)\, e^{i\omega t} \end{aligned}$$

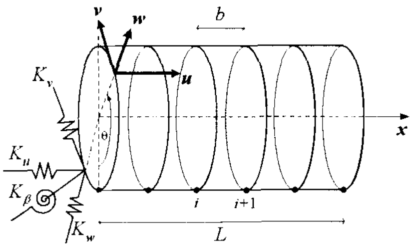
where t is a temporal coordinate, x and θ are spatial coordinates in longitudinal and circumferential directions, respectively.



(a) Partially suspended cylindrical shell



(b) Geometry of the shell



(c) Reference direction and discretization

Fig. 1. Generalized model

Strain displacement relationship in the form of three strain components of middle surface and three curvature changes can be expressed as follows [4],

$$\varepsilon = \{\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}, \chi_x, \chi_\theta, \chi_{x\theta}\}^{\mathrm{T}} = \sum_{m=0}^{\infty} \mathbf{B}_m\, \boldsymbol{\delta}_m^e \qquad (2)$$

where $\boldsymbol{\delta}_m^e$ is a nodal displacement vector corresponding to m-th circumferential wave number. Formulation of the shell stiffness matrix ($\mathbf{K_S}$) and mass matrix ($\mathbf{M_S}$) is given in [4]. Fourier series is used as a distribution function in θ. The foundation stiffness matrix ($\mathbf{K_F}$) can be obtained by imposing the well-known orthogonality property to the integral developed in each element of $\mathbf{K_F}$. For brevity, the detail of formulation of $\mathbf{K_F}$ is given in [3].

By arranging $\mathbf{K_S}$, $\mathbf{K_F}$ and $\mathbf{M_S}$, the governing eigensystem can be expressed as

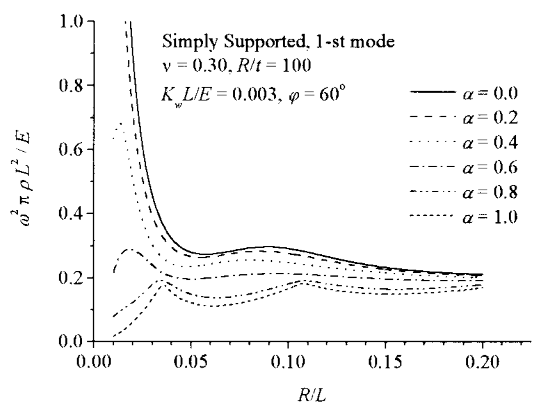
$$\sum_{i=1}^{NS}\sum_{n=0}^{\infty}\left[\mathbf{K}_{\mathrm{S}m,n}^{\,i}+\mathbf{K}_{\mathrm{F}m,n}^{\,i}\right]\boldsymbol{\delta}_n^{\mathbf{e}}=\omega^2\sum_{i=1}^{NS}\sum_{n=0}^{\infty}\mathbf{M}_{\mathrm{S}m,n}^{\,i}\,\boldsymbol{\delta}_n^{\mathbf{e}} \qquad (3)$$

$$, \text{for } m = 0, 1, 2, \ldots \infty$$

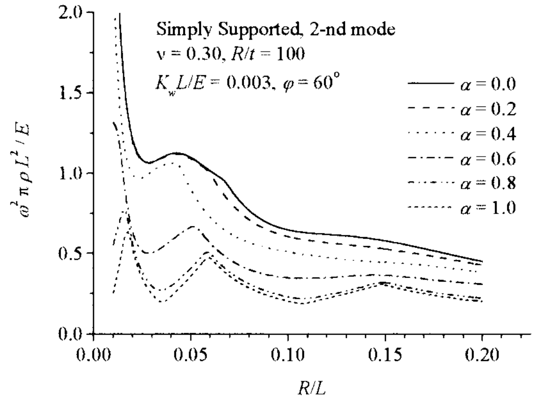
where ω and NS are natural frequency and total number of strip, respectively. Basically n has the same interpretation as for m. Only a finite number of term m say $M(=N)$, in the truncated series are taken into account. By increasing M any degree of accuracies can be obtained.

3. Numerical results

Simply supported partially suspended cylindrical shells on elastic foundation were analyzed. The system considered in this section has the following properties: $\nu = 0.30$, $R/t = 100$, $K_wL/E = 0.003$, $\varphi = 60°$, where E is Young's modulus of the shell. Six values of α (=0.0, 0.2, 0.4, 0.6, 0.8, 1.0) are considered. $\alpha = 0.0$ and $\alpha = 1.0$ correspond to the case of cylindrical shell on elastic foundation along its length and cylindrical shell in the air, respectively.



(a) 1-st mode

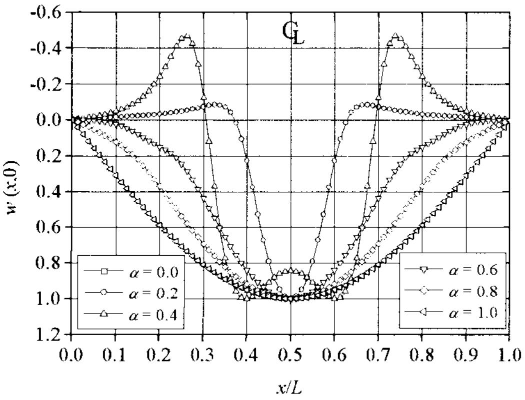


(b) 2-nd mode

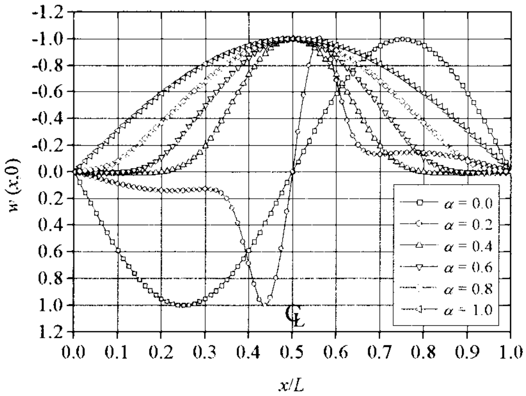
Fig. 2. Variation of natural frequency with R/L

Fig. 2 shows the variation of first and second natural frequencies with dimensionless value R/L for different α. Shells with small α show rather smooth curve. The changes of gradient in the curve are not sudden as in the case of shells on the air.

Fig. 3 gives the w on bottom edge corresponding to first and second mode shapes of shell having $R/L = 0.02$.



(a) 1-st mode



(b) 2-nd mode

Fig. 3. First two modes ($R/L = 0.02$)

4. Conclusions

Present method can be applied easily for cylindrical shells with non-uniformity of foundation distribution both in circumferential and longitudinal directions. The natural frequency of cylindrical shell on elastic foundation is rather smooth compare to one without foundation. For the associated modes with $0.0 < \alpha < 1.0$, simple sine curve can not be expected.

References

1) Paliwal, D.N. et al.: Free Vibration of Circular Cylindrical Shell on Winkler and Pasternak Foundations, Int. J. Pres. Ves & Piping, Vol 69, pp.79-89, 1996.
2) Amabili, Marco. and Dalpiaz, Giorgio.: Free Vibration of Cylindrical Shells with Non-Axisymetric Mass Distribution on Elastic Bed, Meccanica, Vol 32, pp.71-84, 1997.
3) Gunawan, Haryadi.: A Method to Analyze Cylindrical Shells Partially Buried in Elastic Foundation, Hokkaido University, Master of Engineering Thesis, EG-M13, September 2003.
4) Cheung, Y.K.: Finite Strip Method in Structural Analysis, Pergamon Press, 1st Edition, 1976.