CHAOS IN TRAFFIC FLOW DYNAMICS USING CAR FOLLOWING MODEL

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1. Introduction

Traffic flow is a complex dynamical system because of its nature to under go sudden changes especially in dense traffic conditions due to irregular stop-start situation and many other contributing factors such as drivers' behavior, vehicles' performance, traffic flow conditions, driving environment etc. Chaos theory has been proved very useful in exploring many complex dynamical systems e.g. weather forecasting, fluid dynamics etc. A system is said to be in chaotic state when long-term predictions is not possible due to omnipresent uncertainty in determining initial state that grows exponentially fast in time. The system is still deterministic in a sense that if the initial conditions can be determined exactly the future behavior can be predicted by integrating the time evolution equations of the system. As there is always some imprecision in specifying initial conditions of chaotic systems, the long-term behavior becomes unpredictable while short-term predictions can be made more reliable under certain circumstances. It was observed that a periodic perturbation to the Equilibrium State of car following model produces chaotic motion in some of the following vehicles for some particular initial conditions to some extent. The predictability of vehicular motion was measured based on the value for KS entropy.

2. Car Following Model

Car following models attempt to mimic the microscopic behavior of individual vehicles in a platoon. Collision Avoidance model developed by Gipps[1] in 1981 was used to simulate motion of vehicles. This model is attractive in a sense that it is very easy for calibration, as it needs only the maximal breaking rates drivers wish to apply which can be assumed using common sense assumptions about the drivers' behavior. This model was derived to calculate a safe speed with respect to the preceding vehicle by setting limits on the performance of driver and vehicle. This model assumes that the driver of following vehicle selects his speed to ensure that he can bring his vehicle to safe stop if the vehicle ahead comes to a sudden stop.

$$v_n(t+\tau)=\min\left\{\begin{array}{l} v_n(t)+2.5a_n\tau(1-v_n(t)/V_n)(0.025+v_n(t)/V_n)1/2, \\ b_n\tau+\sqrt{b_n{}^2\tau^2-b_n\left[2[x_{n-1}(t)-s_{n-1}-x_n(t)]v_n(t)\tau-v_{n-1}(t)^2/\hat{b}\right]} \end{array}\right\} \quad (1)$$

where a_n and b_n are the maximum acceleration and most severe braking the driver of vehicle n wishes to undertake, s_n is the effective size of vehicle n, V_n is the speed at which the driver of vehicle n wishes to travel, $x_n(t)$ and $v_n(t)$ are the location and speed of vehicle n at time t and τ is the apparent reaction time, a constant for all vehicles. This model has two terms, the first one limits a substantial proportion of the vehicles in free traffic flow conditions while the second one is the limiting condition for almost all vehicles. It was assumed that the transition between these two terms occurs smoothly. Gipps verified his model parameters and suggested that the some values that can be assigned to simulate vehicular motion.

A platoon consists of a lead vehicle and N following vehicles was simulated using linearized equation of motion. It was assumed that prior to time t=0, each vehicles were traveling at a constant speed v and distance headway d while at t=0, this equilibrium state was perturbed, introducing a small sinusoidal variation to the lead vehicle's speed, $v_0(t)=v + A \sin \Omega t$, where A, Ω, v>0. The effects of this disturbance in lead vehicle's speed on other following vehicles were analyzed systematically using simulation program. The above equations of motion were solved numerically to calculate position of each vehicle in platoon using Runga-Kutta Gill algorithm.

3. Numerical Results and Analysis

The computer simulation program was extended to produce time series diagrams, post-transient phase diagrams, Poincare sections, power spectrum, bifurcation diagram and spectrum of Lyapunov exponent from simulated data of motion of vehicles. These diagrams are popular tools to detect the occurrence of chaos in a system. The value of Lyapunov exponent and KS entropy were calculated to characterize vehicular motion and to measure the predictability of the system respectively.

3.1 Sensitivity Analysis

Sensitive dependence on initial conditions is an important characteristic of a chaotic system. It was conducted systematically changing the values of each of model parameters and observing effect on output results of simulation program. Figure 1 presents time series diagram for all following vehicles as an example. It is established that reaction time τ, braking rate b_n and speed V_n are sensitive parameters of this model as significant variations in output results were observed with small change in the values of these parameters. While no such variation were observed for all other parameters so those parameters were treated as insensitive parameters and assigned with some reliable values as suggested by Gipps.

The effect of V_n is dominant only when the vehicles are accelerating, as higher is the value of V_n the acceleration rate would be higher. The reaction time (τ) is sensitive in a sense that the response of vehicle will be quicker if its reaction time is smaller. The braking rate b^* influences the amlitude of disturbance that is if b^* is less than b_{n-1} the disturbance will damp, while if b^* is greater than b_{n-1}, disturbance will amplify.

Observing results of sensitivity analysis, some parameter values were selected for which chaos can be expected for further investigation using Lyapunov exponent and power spectrum.

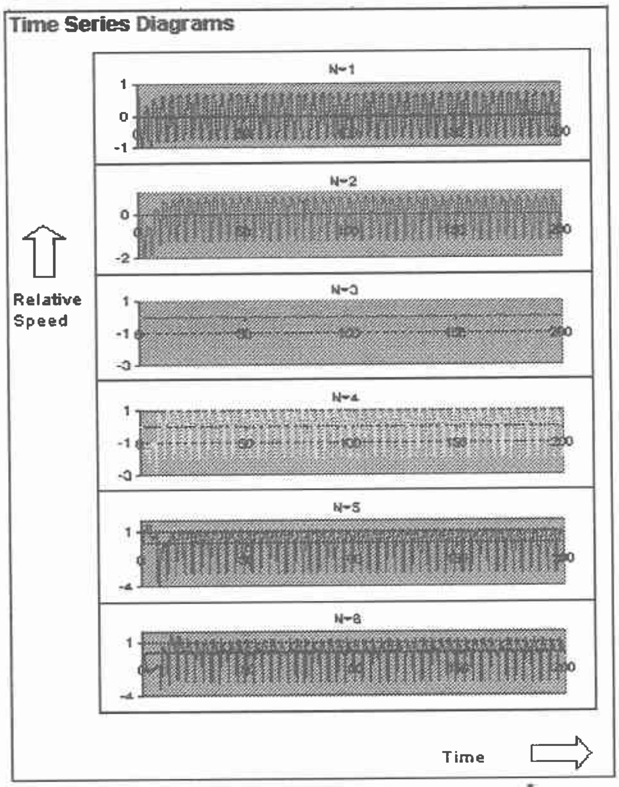

Figure 1: Time Series Diagram for τ = 0.4 sec, b^* = - 2.5 m/sec^2, V_n = N (25, 5^2) m/sec

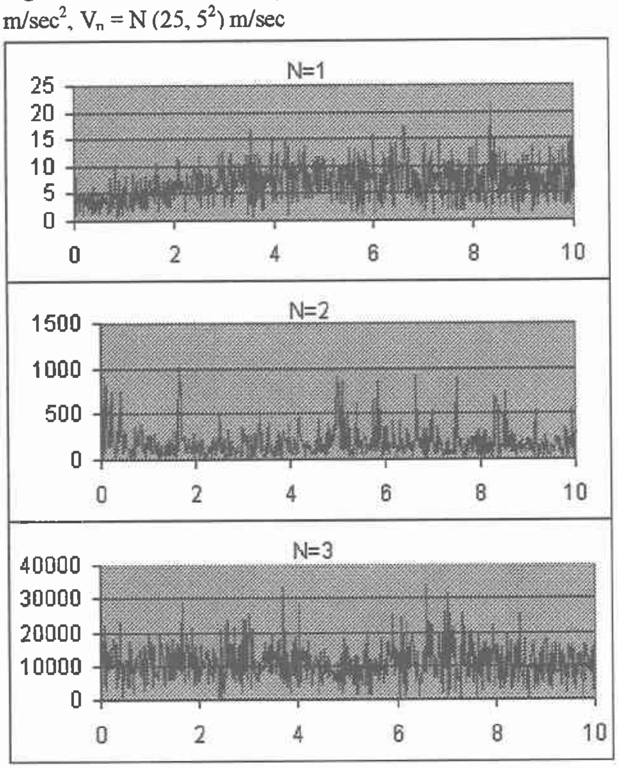

Figure 2: Power Spectrum for τ = 0.4 sec, b^* = -2.5 m/sec^2

3.2 Power Spectrum and Lyapunov Exponent

Although a broad-banded power spectrum does not guarantee the occurrence of chaotic motion but it definitely is a reliable indicator of chaos. Power spectrum for first three following vehicles are presented in Figure 2 for τ = 0.4 sec, b^* = -2.5 m/sec^2 as an example. It was noted that all vehicles except the lead vehicle have broad-banded power spectrum indicating possibility of occurrence of chaos.

Lyapunov exponent is one of the most effective and popular tools to characterize a chaotic system. Wolf et al.[2] method was used to calculate the value of Lyapunov exponent and it was calculated to be zero for leading vehicle and 0.011, 0.012, 0.014, 0.016, 0.017, 0.019 for 1st, 2nd, 3rd, 4th, 5th, 6th following vehicles for τ = 0.4 sec, b^* = -2.5 m/sec^2, N=7 and time=300 sec, while for τ = 0.4 sec, b^* = -3.5 m/sec^2, N=7 and time=300 sec, the values of Lyapunov exponent were calculated to be zero for all vehicles. This indicates that in former case, the vehicles exhibit chaotic behavior to some extent as the value of this exponent is just above zero. An increasing trend in the value of Lyapunov exponent was noticed along the platoon indicating that the following vehicle behaves more chaotic than the vehicle in the front.

3.3 KS Entropy and Predictability of Motion

The value of KS entropy can be determined approximately simply by adding the values of all positive Lyapunov exponents of the system. Car following model is a one-dimensional system so it has only one Lyapunov exponent. It means, the value of KS entropy shall be approximately equal to the value of Lyapunov exponent. Following relationship can be used to calculate the prediction time for vehicular motion.

$$T \sim (1/\lambda_+) \log_e(L / \varepsilon)$$

For the case of τ = 0.4 sec, b^* = -2.5 m/sec^2 and time=300, the 6th following vehicle has λ_+ = 0.019, if the precision of velocity data that is L / ε is 1 in 10^8 then the prediction time is calculated to be approximately 969.5 time units.

4. Discussion

It was observed that for particular initial conditions, a periodic perturbation to the equilibrium state of the car following model generates chaotic oscillations in some of following vehicles to some extent. The motion of vehicles is predictable even when chaotic behavior exists but such prediction is valid only for a short duration of time. This paper has presented theoretical approach to explore the complexity in traffic flow dynamics as the data used was produced by simulation program. The scope is limited further more using only Collision Avoidance car following model. As a matter of fact, no model is perfect to explain real-world traffic cases so there is always some noise coming from the model itself. Further study is recommended using time series data coming from real-world traffic flow.

5. References

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