Prediction Model of $M-\theta_r$ Relations for Top- and Seat-Angle Connections Taking Bolt Stiffness and Prying Action into Account

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1. INTRODUCTION

In the AISC-LRFD Specifications (1994), two types of steel frame constructions are categorized: *Type* FR (Fully Restrained) and *Type* PR (Partially Restrained). In these specifications, limited guidance is provided under the PR group on how this type of construction should be devised and practically no specific guidelines have been adopted for designing semi-rigid frame connections. The reason behind that is mainly lacking of ample understanding to capture the different aspects of connection flexibility on what model is appropriate for representing moment-rotation behavior of semi-rigid connections.

In the AISC-ASD specifications (1989), top- and seat-angle connection is treated as the top angle is used to provide lateral support of the compression flange of the beam, and the seat angle is to transfer only the shear force of beam to column and should not give significant restraining moment on the end of the beam. However, experimental evidences (Azizinamini 1985, Harper 1990) show that besides transferring beam-shear force, this type of connection is also capable of transferring fairly significant beam end moment to the column. In consequence, when transferring this moment through top angle to the column, an increase of tensile force is occurred in bolts due to local deformation of top angle's vertical leg. This additional tensile force is known as prying force. A few researchers enabled to consider this additional force in the formulations of design resistances, which are mainly based on T-stub model ("Eurocode 3" 1997). However, T-stub model shows different deformation configuration from actual deformation pattern of true top- and seat-angle connections at failure, and may show big deviation in estimation of design resistances from the actual values.

In the recent years, simple mathematical models are proposed to represent the $M-\theta_r$ curves of the connections linking connection parameters and adjusting with the experimental results (e.g., Rathbun 1936, Monforton and Wu 1963, Lightfoot and LeMessurier 1974, Frye and Morris 1975, Kishi and Chen 1990 etc.). Among them, three-parameter power model proposed by Kishi and Chen (1990) is the best to represent $M-\theta_r$ relations of connections with angles. However, in formulation of ultimate moment capacity for angle type of connections, power model only considered the bending and shear deformations of angle, and disregarded the bolt stiffness and prying effects. To establish a rational prediction model of $M-\theta_r$ curves for the connections, these affections on connection behavior need to be considered.

In this study, a method is proposed to efficiently determine the ultimate connection moment M_u based on three simple mechanisms. In this proposed prediction model, not only bending and shear deformations of tension angle but also the bolt stiffness and the effects of prying action are considered to represent nonlinear behavior of connection. Substituting ultimate connection moment obtained from the proposed prediction model in the three-parameter power model (Kishi and Chen 1990), $M-\theta_r$ relations of top- and seat-angle connection are predicted. Initial connection stiffness R_{ki} and shape parameter n are determined following exact the same procedure as power models (Kishi and Chen 1990). Performance of the proposed prediction model is assessed comparing $M-\theta_r$ curves predicted by the proposed prediction model and Kishi-Chen's power model (1990) with experimental ones.

2. PROPOSED PREDICTION MODEL

2.1. ASSUMPTIONS

The following assumptions are to be employed in determination of connection's ultimate moment capacity. Some of those were adopted by Kishi and Chen (1990).

1) Center of rotation of a deflected connection is located at the cross-point of the horizontal middle plane and the vertical cross-section at the toe of the fillet of angle leg adjacent to the compression beam flange (point C in Fig. 1).
2) Deformations of connection elements are small.
3) Materials of connecting elements are constituted with elasto-plastic behavior.
4) One plastic hinge is assumed always to be formed in the angle's leg adjacent to compression beam flange at the vertical cross-section through the center of rotation with other plastic hinges in tension angle and/or fasteners at the ultimate state of connection (H_4 in Fig. 1).

2.2. CONNECTION FAILURE MECHANISMS

Following the failure mechanisms of T-stub model ("Eurocode 3" 1997), three types of connection failure mechanisms are provided to evaluate ultimate connection moment, which are incorporated from experimental deformation configurations of connections reported by Azizinamini. (1985), Harper (1990) and Hechtmann et al. (1947). These mechanisms are also confirmed by the deformation configurations and plastic yielding areas of connections obtained from nonlinear FE analyses conducted by the authors (Ahmed et al. 2001). The assumed mechanisms of top angle at the failure of the connections are described in the following.

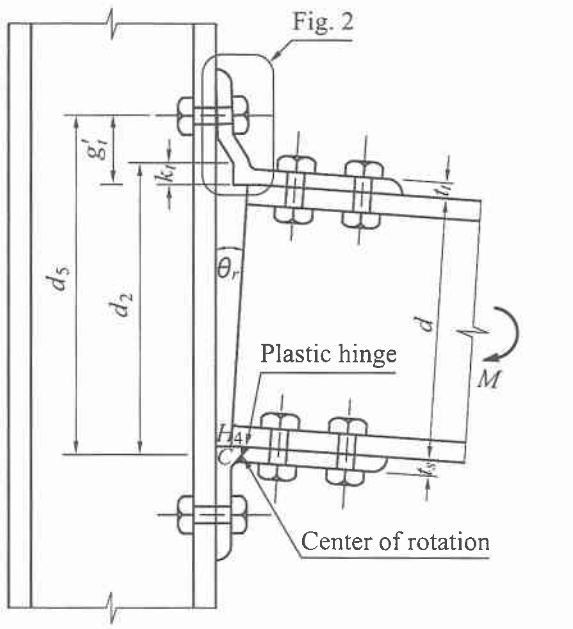


Fig. 1 Deflected configuration of top- and seat-angle connection

***Type I* mechanism:**

Two plastic hinges are assumed to be formed in the top angle; and bolts are considered as stiffer members than angles similar to Kishi-Chen's model. In this model, the location of formation of the upper plastic hinge is revised from Kishi-Chen's assumption based on the results obtained from 3D FE analyses (Ahmed et al. 2001). And it is assumed that prying force contributes to form the upper plastic hinge of the top angle (Fig. 2(a)).

***Type II* mechanism:**

One plastic hinge is assumed to be formed in the top angle and another is in the bolt shank due to combine action of bending and tensile forces. This type is prepared for the case in which both members top angle and tension bolt are equally stiff. Prying force is also considered for this mechanism (Fig. 2(b)).

***Type III* mechanism:**

No plastic hinge is assumed to be formed in the top angle, and connection failure occurs only by complete yielding of tension bolts. This type is prepared for the case in which the total strength of bolts is less than the bending and shear resistances of top angle (Fig. 2(c)).

These simple failure mechanisms in case of fastening bolts being arranged in one line are shown in Fig. 2.

In the cases of *type I* and *type II* mechanisms, it is assumed that prying force develop in between the centerline of bolt hole and the top edge of top angle; and the prying and bending-tension forces are determined considering the location of prying force and plastic moment capacity of angle at the plastic hinge. The smallest bending-tension force (hereinafter, indicated as shear force or shear resisting force) among those estimated from these three mechanisms is taken as the shear resistance V_t of a given connection and is used to evaluate the ultimate moment capacity of the connection.

2.3. MOMENT-SHEAR INTERACTION

Shear resisting forces acting on the plastic hinges of top angle corresponding to the *types I* and *II* mechanisms can be evaluated applying Drucker's moment-shear interaction (1956). According to Drucker's yield criterion (1956), yielding of top angle's vertical leg occurs under combine action of bending moment M_t and shear force V_t when the following condition is satisfied:

$$\frac{M_t}{M_{p,t}} + \left[\frac{V_t}{V_{p,t}}\right]^4 = 1 \qquad (1)$$

where $M_{p,t}$ is the pure plastic moment of top angle's vertical leg. According to Tresca's yield criterion, the pure plastic moment of top angle is given by:

$$M_{p,t} = \frac{l_t\, t_t^2}{4}\,\sigma_{y,t} \qquad (2)$$

where l_t is the length of top angle along the column; t_t is the thickness of top angle, and $\sigma_{y,t}$ is the yield stress of top angle's material; and $V_{p,t}$ is the pure plastic shear of top angle and can be found by:

$$V_{p,t} = \frac{l_t\, t_t}{2}\,\sigma_{y,t} \qquad (3)$$

2.4. DETERMINATION OF RESISTING FORCES AND ULTIMATE CONNECTION MOMENT

2.4.1. *Type I Mechanism*

Type I mechanism is characterized by the formation of three plastic hinges as shown mutually in Figs 1 and 2(a). Applying the work equation with considering moment-shear interaction to the *type I* failure mechanism, the shear force V_{t1} can be found by:

$$V_{t1} = \frac{2M_t}{g_4} \qquad (4)$$

where g_4 is the vertical distance between the two plastic hinges involved in the failure mechanism of top angle (Fig. 2(a)) and can be found by:

$$g_4 = g_t' + t_t - w_b - k_t \qquad (5)$$

in which w_b is the width of bolt head across the two opposite flat sides or the head diameter of rivet, and k_t is the distance from the top angle's heel to the toe of the fillet.

Combining Eqs 2 and 3, the relation between $M_{p,t}$ and $V_{p,t}$ can be obtained as:

$$M_{p,t} = \frac{t_t V_{p,t}}{2} \qquad (6)$$

Combining this relation with Eqs 4 and 1, a biquadratic equation regarding $(V_{t1}/V_{p,t})$ can be given by:

$$\left[\frac{V_{t1}}{V_{p,t}}\right]^4 + \frac{g_4}{t_t}\left[\frac{V_{t1}}{V_{p,t}}\right] - 1 = 0 \qquad (7)$$

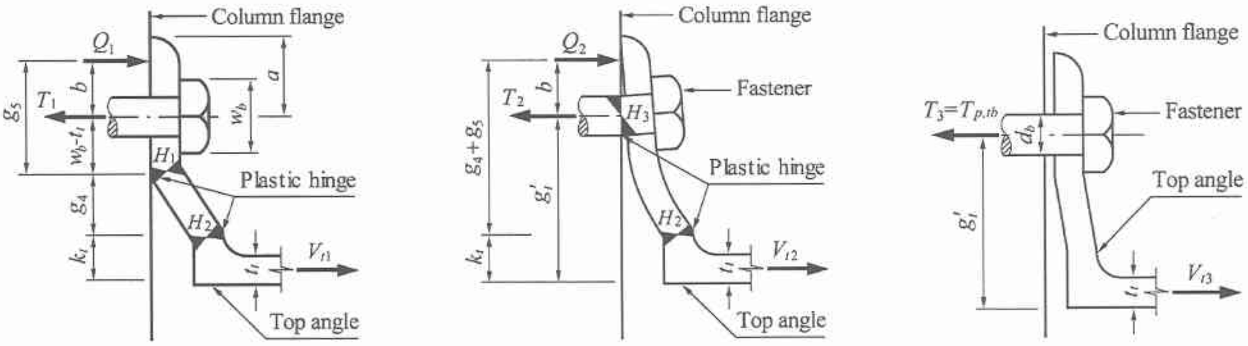
Following a simple iteration procedure, the value of V_{t1} can be easily determined from Eq. 7.

From the equilibrium condition of top angle, the tension resistance of the fasteners can be found by:

$$T_1 = V_{t1} + Q_1 \qquad (8)$$

in which Q_1 is the prying force for *type I* mechanism. From the plastic yielding of top angle's vertical leg at the plastic hinge H_1 (Fig. 2(a)), the prying force is given by:

$$Q_1 = \frac{1}{b}\left[V_{t1}\left(g_5 - b\right) + \frac{V_{t1}\, g_4}{2}\right] \qquad (9)$$



a) *Type I* mechanism b) *Type II* mechanism c) *Type III* mechanism

Fig. 2 Failure mechanisms of top angle

where g_5 is the distance from the plastic hinge H_1 to the location of prying force (Fig. 2(a)) and is defined by:

$$g_5 = w_b - t_t + b \tag{10}$$

and b is the distance from the centerline of fastener's hole to the location of prying force at the ultimate state of connection. Distance b is investigated previously by employing FE analyses method for top- and seat-angle connections (Ahmed et al. 2001) and approximated with some conservative provision for bolted connections by:

$$b = 2.4t_t \quad \text{if} \quad 2.4t_t < a \quad \text{or,} \quad b = a \tag{11}$$

and for riveted connections, it is assumed by:

$$b = 3t_t \quad \text{if} \quad 3t_t < a \quad \text{or,} \quad b = a \tag{12}$$

The ultimate moment capacity M_u of top- and seat-angle connection, taking moment about the center of rotation (Fig. 1), can be estimated from the following equation:

$$M_u = M_{p,s} + \frac{V_{t1} g_4}{2} + V_{t1} d_2 \tag{13}$$

in which $M_{p,s}$ is the pure plastic moment of seat angle's leg adjacent to the compression beam flange, which ignores the interaction with the axial forces, and can be determined by Eq. 2 substituting l_s, t_s and $\sigma_{y,s}$ in lieu of l_t, t_t and $\sigma_{y,t}$, and d_2 is the distance from the center of rotation to the plastic hinge H_2 and can be found by:

$$d_2 = d + 0.5t_s + k_t \tag{14}$$

where d is the depth of beam section and t_s is the thickness of seat angle.

2.4.2. *Type II* Mechanism

Type II mechanism is shown mutually in Figs 1 and 2(b). Considering moment-shear interaction effect for this mechanism, the plastic yielding of top angle's vertical leg at the plastic hinge H_2 (Fig. 2(b)) provides:

$$M_t = T_2(g_4 + g_5 - b) - Q_2(g_4 + g_5) \tag{15}$$

where T_2 is the tension resisting force of fastener for *type II* mechanism.

Combining Eqs 6 and 15 with Drucker's moment-shear interaction Eq. 1, the condition of plastic yielding of top angle (Fig. 2(b)) can be expressed by:

$$\left(\frac{V_{t2}}{V_{p,t}}\right)^4 + 2\frac{g_4+g_5}{t_t}\frac{V_{t2}}{V_{p,t}} - \left(1+\frac{T_{p,b}\,b}{M_{p,t}}\right) = 0 \tag{16}$$

where $T_{p,b}$ is the tensile resistance of fasteners' shank, but not the threaded area of the shank because yielding of tension fasteners occurs at the shank near the head due to bending (Ahmed et al. 2001), which is given by:

$$T_{p,b} = n_t' A_b \sigma_{y,b} \tag{17}$$

where n_t' is the number of fasteners in tension angle's leg adjacent to the column face, A_b is the cross-sectional area of fasteners' shank, and $\sigma_{y,b}$ is the yield stress of fastener's material.

Considering the nondimensional values $\mu = (g_4+g_5)/t_t$ and $\eta = 1 + T_{p,b}\,b/M_{p,t}$ into Eq. 16, the equation for estimating shear resisting force can be obtained as:

$$\left(\frac{V_{t2}}{V_{p,t}}\right)^4 + 2\mu\frac{V_{t2}}{V_{p,t}} - \eta = 0 \tag{18}$$

The tension resisting force of fastener is given by:

$$T_2 = V_{t2} + Q_2 \tag{19}$$

where Q_2 is the prying force for *type II* mechanism.

Applying this equation and substituting $M_{p,t}$ in lieu of M_t in Eq. 15, prying force developed in tension fasteners can be expressed with little conservative provision as:

$$Q_2 = \frac{1}{b}\left[V_{t2}(g_3 + g_4 - b) - M_{p,t}\right] \tag{20}$$

Taking moment of all forces developed in plastic hinges for *type II* mechanism about the center of rotation, the ultimate moment capacity of the connection can be obtained by:

$$M_u = M_{p,s} + M_{p,b} + V_{t2} d_2 \tag{21}$$

where $M_{p,b}$ is the pure plastic moment of fasteners, which ignores the interaction with the axial force, is given by:

$$M_{p,b} = \frac{n_t' \pi d_b^3}{16}\sigma_{y,b} \tag{22}$$

$\pi = 3.142$, and d_b is the fastener's diameter.

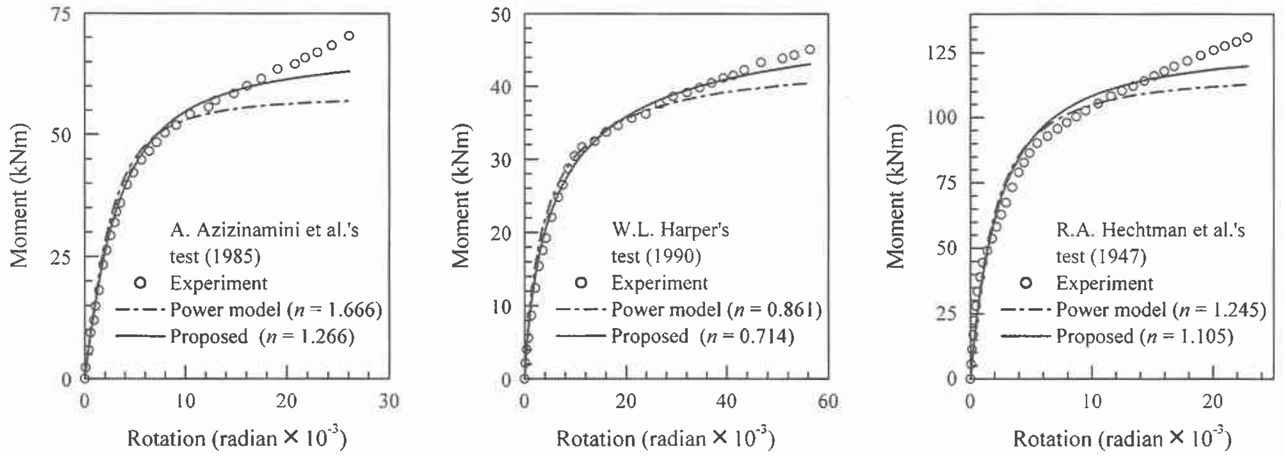
2.4.3. *Type III* Mechanism

This failure mechanism is depicted in Figs 1 and 2(c). From this mechanism, the value of shear force that is less than the shear resistance of top angle can be given by:

$$V_{t3} = T_3 = T_{p,tb} \tag{23}$$

in which T_3 is the axial resisting force of tension fasteners, and $T_{p,tb}$ is the axial tensile resistance of fasteners and can be determined by using Eq. 17 substituting A_{tb} in place of A_b; where A_{tb} is the net tensile area of fasteners.

Taking moment of fasteners' resisting force and seat angle's bending resisting force about the center of rotation, the ultimate moment capacity for mechanism *type III* can be found by:



a) A1 b) Test 3 c) NO 11

Fig. 3 Performance of proposed prediction model and three-parameter power model

$$M_u = M_{p,s} + V_{t3}\, d_5 \tag{24}$$

in which d_5 is the distance between the center of rotation and the centerline of tension fastener (Fig. 1) and given by:

$$d_5 = 0.5t_s + d + g_t' \tag{25}$$

3. ASSESSMENT OF THE PROPOSED PREDICTION MODEL

A prediction model is proposed to estimate the ultimate moment capacity M_u of top- and seat-angle connections based on three simple mechanisms. The initial connection stiffness R_{ki} of the connection is determined following the same procedure as for power model, and the shape parameter n is determined by employing a least-mean square fit technique devised in data base program SCDB (Chen and Kishi 1989). Several series of experiments of top- and seat-angle connection stored in this updated databank are used to conduct a comparison of $M-\theta_r$ curves predicted by proposed prediction model and power model with measured values. This comparison is depicted in Fig. 3. A summery of assessment of the proposed prediction model is shown in Table 1. It is obvious from Fig. 3 that $M-\theta_r$ curves predicted by the proposed prediction model and the power model represent close-fit with the experimental ones, and Table 1 reveals that the proposed prediction model estimates more accurately the values of ultimate moment of top- and seat-angle connections.

4. CONCLUSIONS

From the assessment of proposed prediction model, it can be concluded that:

1) Both the proposed prediction model and three-parameter power model show the same level of accuracy in predicting $M-\theta_r$ relations.
2) Proposed prediction model shows better performance and accuracy in estimating ultimate moment capacity of the connections.
3) Proposed prediction model is also able to estimate the resisting forces of tension angle and fasteners, and prying force at the ultimate state of top- and seat-angle connections.

Table 1 Assessment of the proposed prediction model

Model or Test ID	Test	Power model		Proposed prediction model		
	M_u (kNm)	M_u (kNm)	V_p (kN)	M_u (kNm)	V_t (kN)	Q (kN)
Bolted connections tested by Azizinamini (1985)						
A1	70.3	58.0	143.2	67.5	171.2	255.7
A2	95.8	111.7	274.9	102.4	248.7	229.9
Bolted connection tested by Harper W. L. (1990)						
Test 3	45.1	44.5	181.5	52.2	204.7	211.0
Riveted connections tested by Hechtmann et al. (1947)						
NO 2	64.7	88.7	243.8	61.7	169.5	82.0
NO 5	128.1	140.2	268.0	152.9	292.3	208.9
NO 9	147.3	227.7	440.8	172.3	329.3	171.6
NO 10	160.5	320.1	621.6	196.0	374.7	124.6
NO 11	130.9	117.9	223.4	129.4	244.9	175.4
NO 16	50.2	53.9	147.1	54.2	149.3	102.4
NO 17	51.2	53.9	147.1	54.2	149.3	102.4
NO 18	53.7	65.0	177.5	56.3	154.3	98.7
NO 20	112.5	180.4	442.2	134.7	329.3	171.6
NO 22	133.9	204.5	442.9	153.2	329.3	171.6
NO 23	138.5	171.1	333.9	162.2	315.6	180.7
NO 24	178.5	247.6	481.0	219.4	421.3	250.4

REFERENCES

1) Ahmed, A., Kishi, N., Matsuoka, K.G., and Komuro, M. (2001), "Nonlinear analysis on prying of top- and seat-angle Connections," *Journal of Applied Mechanics, JSCE*, Vol.4, 227-236.
2) Azizinamini, A. (1985), "Cyclic characteristics of bolted semi-rigid steel beam to column connections," *Ph.D. dissertation thesis*, University of South Carolina, Columbia, S.C.
3) Chen, W.F. and Kishi, N. (1989), "Semi-rigid steel beam-to-column connections: Data Base and Modeling," *Journal of Structural Engineering, ASCE*, Vol. 115(ST1), 105-119.
4) Harper, W.L. (1990), "Dynamic response of steel frames with semi-rigid connections," *Ph.D. dissertation thesis*, Department of Civil Engineering, University of South Carolina, Columbia, S.C.
5) Hechtman, R.A. and Johnston, B.G. (1947), "Riveted semi-rigid Beam-to-column building connections," *Progress Report No.1*, AISC research at Lehigh University, Bethlehem, PA.
6) Kishi, N. and Chen, W.F. (1990), "Moment-rotation relations of semi-rigid connections with angles," *Journal of Structural Division, ASCE*, Vol. 116, ST7, 1813-1834.