Macroscopic Traffic Flow Simulation Model Parameters Estimation

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1. Introduction

Macroscopic traffic flow model, which are able to handle large size of freeway systems with the fast simulation time, contains a set of parameters. Since model parameters have significant effects on the performance of the simulation, they have to be identified carefully. Various techniques can be used for this purpose. However, the choice primarily depends on the nature of the parameters, i.e. if they are sensitive to traffic condition, a dynamic method should be adopted. This study focuses on the parameters estimation of a particular model. First the Box technique was adopted to estimate the lump parameters. Then the parameters were estimated according to traffic condition. Finally, Kalman filtering technique was adopted as a dynamic one.

2. Macroscopic Traffic Flow Model

In this study, the model of Cremer [1] was adopted. It consists of three relationships. The first equation is the continuity equation that describes how density varies with time:

$$\rho_j(k+1)=\rho_j(k)+\frac{\Delta t}{\Delta L_j}\left(q_{j-1}-q_j+r_j-s_j\right)_{(k)} \tag{1}$$

where Δt is time increment, ΔL_j is length of j^{th} section, $\rho_j(k)$ is density, $v_j(k)$ is space mean speed, $r_j(k)$ is ramp entry flow rate, and $s_j(k)$ is ramp exit flow rate. k indicates the time step and j indicates the section. $q_j(k)$ and $w_j(k)$ are flow rate and time mean speed at a point of boundary between section j and $j+1$, respectively. The second equation, which is so called the momentum, defines the variation of space mean speed over time:

$$v_j(k+1)=v_j(k)+\frac{\Delta t}{\tau}\left\{v_e\left[\rho_j(k)\right]-v_j(k)\right\} + \frac{\Delta t}{\Delta L_j}v_j(k)\left[v_{j-1}(k)-v_j(k)\right]-\frac{\nu\cdot\Delta t}{\tau\Delta L_j}\frac{\rho_{j+1}(k)-\rho_j(k)}{\rho_j(k)+\kappa} \tag{2}$$

where v_e is the speed at equilibrium state, which can be obtained from density-speed curve. The third equation is the fundamental relationship among traffic volume, speed, and density:

$$q_j(k)=\alpha\left(v_j(k)*\rho_j(k)\right)+(1-\alpha)\left(v_{j+1}(k)*\rho_{j+1}(k)\right) \tag{3}$$

where α is weighting parameter ranging from 0 to 1. The parameters which have to identified in this study are τ (time parameter), ν (anticipation parameter), κ (density parameter), and α.

3. Parameter Estimation Technique

3.1 Box Complex Technique (BCT)

This method is a random search technique, which has proven effective in sol ving problem with nonlinear objective function subject to non-linear inequality constraints. The procedure tends to find the global optimum due to the fact that the initial set of points is randomly scattered throughout the feasible region [2].

3.2 Kalman Filter Technique (KFT)

The parameter identification problem can be integrated into state estimation problem by treating the parameters as another set of state variables [3]. The white noise errors were induced in both model formula and measurement process as follows:

$$\mathbf{x}(k+1)=\mathrm{f}[\mathbf{x}(k)]+\Gamma\varphi(k) \tag{4}$$

$$\mathbf{y}(k)=\mathrm{g}[\mathbf{x}(k)]+\psi(k) \tag{5}$$

where $\mathbf{x}(k)=(\rho_1, v_1,\ldots\rho_i, v_i,\ldots\rho_n, v_n,\tau,\nu,\kappa,\alpha)_{(k)}^{\mathrm{T}}$
$\mathbf{y}(k)=(q_1, w_1, q_n, w_n)_{(k)}^{\mathrm{T}}$

w is spot speed. $\varphi(k)$ and $\psi(k)$ are referred as to modeling errors and measurement errors, respectively. By linearizing both state and observation equation around the nominal solution, the model becomes:

$$\tilde{\mathrm{x}}(k+1)\cong \mathrm{f}[\hat{\mathrm{x}}(k)]+\frac{\partial \mathrm{f}}{\partial \mathrm{x}}(\mathrm{x}(k)-\hat{\mathrm{x}}(k))+\Gamma\varphi(k) \tag{6}$$

$$\tilde{\mathrm{y}}(k)\cong \mathrm{g}[\mathrm{x}(k)]+\frac{\partial \mathrm{g}}{\partial \mathrm{x}}(\mathrm{x}(k)-\tilde{\mathrm{x}}(k))+\psi(k) \tag{7}$$

$\tilde{\mathrm{x}}(k)$ and $\hat{\mathrm{x}}(k)$are the estimated state vector before and after obtaining actaul measurement data, $\mathrm{y}(k)$, respectively.

4. Numerical Experiments

Traffic data were collected from the Second-stage Expressway of Bangkok. Three cases of traffic data are examined:

1) Case 1: Off-peak period
2) Case 2: Peak period
3) Case 3: Smooth traffic situation with inflow volume between 4800 to 5200 vph.

The results by simulation runs of macroscopic model with a certain parameter sets estimated were compared with the real. As the statistics to evaluate each method quantitatively, the objective function (J) and root mean square of error (RMSE) of speed and volume were calculated. The objective function was set as the error between observed variables and model outputs:

$$J = \sum_{i=1}^{n} \left(\gamma_q \cdot (q_i - \hat{q}_i)^2 + \gamma_w \cdot (w_i - \hat{w}_i)^2\right) \qquad (8)$$

4.1 Parameters Estimation by BCT

Three sets of initial parameters were applied to BCT with different numbers of initial complex points. The estimated parameters for each case are presented in Table 1.

Table 1 Average Parameter Estimated by BCT

	Case 1	Case 2	Case 3
τ (sec)	8.12	242.51	419.93
ν (km^2/hr)	19.53	630.12	740.00
κ (vkp)	17.3	150.9	13.9
α	0.888	1.000	0.989
J	55.50	140.27	10.13
$RMSE_q$ (vph)	312.51	544.04	45.27
$RMSE_w$ (kph)	3.65	5.37	1.15

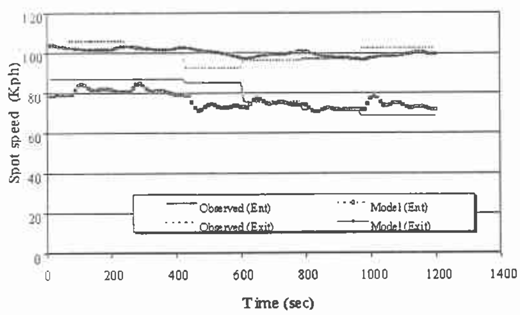

Fig.1 Spot Speed Estimated by Optimum Parameters Using BCT for Case 2

Table 2 Parameters before and after Speed Change

Period	first 600 sec	last 600 sec	Aggegate
τ (sec)	637.62	95	
ν (km^2/hr)	1901.79	340.93	
κ (vkp)	200.0	200.0	
α	1.000	1.000	
J	78.27	49.28	127.55
$RMSE_q$ (vph)	521.53	568.30	545.42
$RMSE_w$ (kph)	6.17	2.96	4.840

It was found from the analysis that the initial values had small effect on the final solutions because BCT has such a mechanism that generates a number of random points automatically with avoiding a local minimum. Consequently, the method successfully yielded the parameters that were substantially different from the initial values. Table 1 indicates that BCT was effective in estimating the parameters for the off-peak (Case 1) and smooth traffic state (Case 3). However, the estimation is not so successful in Case 2. As shown in Fig. 2, the difference in the short term variation is still large. There was a sudden speed drop around 600 seconds. To treat this phenomenon more precisely, the data set of Case 2 was divided into two parts; before and after the abrupt change of speed. And then the parameters were identified separately. Table 2 exhibits the new parameters for each time period. With being aggregated for both periods, the separation was effective in improving both objective function and RMSE of spot speed.

4.2 Parameters Estimation by KFT

The results in the previous section suggest that the parameter should be identified in accordance with traffic situations. That is the real time estimation of model parameters may work well in the ultimate sense. The KFT, which simultaneously estimates model parameters as well as traffic state variables in real time manner, was applied to this problem. Unfortunately, KFT was not effective in improving any indices as shown in Table 3. The deficiency may have arisen from some causes. First, it may be caused by the spatial variation of the traffic situation. In this study, the observation points are only at the entrance and the exit of the study section with locating far away each other. Next, generally in KFT, the statistics of noises have significant effect on the estimation precision. In this study, arbitrary values were assigned to them without any validation.

Table 3 Comparison of Performance Indices between BCT and KFT

Parameters	J	$RMSE_q$	$RMSE_w$
Initial Condition (Optimized by Box Complex)	140.23	544.13	5.37
Estimated by Kalman filtering	170.28	558.49	6.31

Conclusions

The BCT provides the superb outcomes for the low volume case, but its performance deteriorates in the high volume with abrupt change in traffic condition. That means the model parameters strongly depend on the traffic condition. The dynamic technique such as KFT should be applied to this problem. Anyhow, the program in this study has to be adjusted in its formulation, and the traffic condition in intermediate section should be considered. Furthermore, the relationship of the model parameters and traffic condition such as density should be investigated for further study.

References

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