# Numerical Simulation of 1- and 2-D Overland Flow by Coupled Model

# 1・２次元複合型の流れの数値シミュレーション

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**ABSTRACT**

A hydrodynamic model of coupled 1- and 2-D overland flow with sediment transport is developed in this paper. Nodal domain integration (NDI) method finite difference scheme used for solving water flow with combination of upwind scheme for 1-D sediment transport are developed to simulate overland flow and sediment transport in channel and floodplain. Some Numerical results and comparisons with other schemes are presented to demonstrate applicability of the models.

## 1. INTRODUCTION

Numerous 1- and 2-D numerical models have been used to simulate flows in overland flow. For application in civil engineering, the interaction between channel flow and floodplain are required to simulate the drying and wetting processes in the floodplain. Ideally, the numerical model should possess the following features:

(1) to handle complex topography;
(2) to handle wetting and drying floodplain;
(3) to handle tributary and slough inflows;
(4) to handle rainfall, infiltration and evaporation;
(5) simulation of steady flow and unsteady flow.

Lately, several investigators contributed to the development of 1- and 2-D river basins by many numerical computation schemes. Katopodes and Strelkoff [1] used a method of characteristics for moving grid to the simulation of shallow water flow, but cannot handle wetting and drying floodplain. Akanbi and Katopodes [2] developed a model for simulation river basins by dissipative finite element technique. In this model, some assumptions made are that the flood wave spread radially on impervious bed; and the speed of propagation of wave front is equal to the fluid velocity just behind the front, these assumptions may not be satisfactory in most real phenomena.

## 2. GOVERNING EQUATIONS

### 1- and 2-D Continuity and Momentum Equations

The governing equations, based on the conservation of mass and momentum for one-dimensional unsteady open-channel flow can be expressed as

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial X}=q_L \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial\left(Q^2/A\right)}{\partial X}+gA\left(\frac{\partial H}{\partial X}+S_f\right)=0 \tag{2}$$

where $A$ = cross-sectional area of flow; $Q$ = flowrate; $q_L$ = lateral flowrate; $g$ = acceleration due to gravity; $H$ = water surface elevation ($H=\eta+h$) ; $\eta$ = bed elevation; $h$ = water depth; $S_f$ = friction slope; and $t$, $X$ = time and flow direction coordinate, respectively. The friction slope $S_f$ is computed by the Manning formulation for steady flow by

$$Q=\frac{1}{n}AR^{2/3}S_f^{1/2} \tag{3}$$

where $R$ = hydraulic radius; and $n$ = Manning's coefficient.

and two-dimensional unsteady open-channel flow can be expressed as

$$\frac{\partial h}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=RF-ET-IN \tag{4}$$

$$\frac{\partial q_x}{\partial t}+\frac{\partial\left(q_x^2/h\right)}{\partial x}+\frac{\partial\left(q_x q_y/h\right)}{\partial y}+gh\left(\frac{\partial H}{\partial x}+S_{fx}\right)=0 \tag{5}$$

$$\frac{\partial q_y}{\partial t}+\frac{\partial\left(q_x q_y/h\right)}{\partial x}+\frac{\partial\left(q_y^2/h\right)}{\partial y}+gh\left(\frac{\partial H}{\partial y}+S_{fy}\right)=0 \quad (6)$$

where $q_x$, $q_y$ = unit width flowrate; and $S_{fx}$, $S_{fy}$ = friction slope in $x$- and $y$-direction, respectively; $RF$ = rainfall intensity; $ET$ = evaporation rate; and $IN$ = infiltration rate.

### 1-D Sediment Transport

There are many empirical and semiempirical bed load formulas based on laboratory data and probabilistic approach. In this paper, Ashida and Michiue`s bed load formula is adopt, $q_{Bi}$, of sediment transport rate per unit width can obtained from (7) as

$$\frac{q_{Bi}}{\sqrt{sgd_i^3}}=p_i 17\tau_{*i}^{\prime 3/2}\left(1-\frac{\tau_{*ci}}{\tau_{*i}}\right)\left(1-\frac{u_{*ci}}{u_*}\right) \quad (7)$$

where $d$, $s$ = diameter and specific gravity of bed material; $p$ = percentage of bed material; $\tau_*$ = non-dimensional bed shear stress; $\tau_{*c}$ = non-dimensional critical bed shear stress (which calculated by Iwakaki`s formular); $u_*$ = shear velocity; $u_{*c}$ = critical shear velocity of sediment; and subscript $_i$ = grain size $i$.

A pick-up rate, $q_{sui}$, of the grains from unit width of riverbed can be expressed as (Itakura and Kishi [3]),

$$q_{sui}=p_i K\left(\alpha_*\frac{sgd_i}{u_*^{\prime}}\Omega_i-w_{fi}\right) \quad (8)$$

$$\Omega_i=\frac{\tau_{*i}^{\prime}}{B_{*i}}\frac{\int_{a'}^{\infty}\xi\frac{1}{\sqrt{\pi}}\exp\left(-\xi^2\right)d\xi}{\int_{a'}^{\infty}\frac{1}{\sqrt{\pi}}\exp\left(-\xi^2\right)d\xi}+\frac{\tau_{*i}^{\prime}}{B_{*i}\eta_0}-1 \quad (9)$$

where $u_*^{\prime}$ = effective shear velocity; and $w_{fi}$ = fall velocity; $a'=B_{*i}/\tau_{*i}^{\prime}-1/\eta_0$ ; $\eta_0$ = 0.5; $K$ = 0.008; $\alpha_*$ = 0.14; and $B_{*i}$ = 0.143.

The continuity equation of depth average suspended sediment is written as

$$\frac{\partial}{\partial t}\left(<c_i>h\right)+\frac{1}{B}\frac{\partial\left(Q<c_i>\right)}{\partial X}=q_{sui}-w_{f_i}c_{bi} \quad (10)$$

where $<c_i>$ = depth average suspended sediment concentration, and $B$ = channel width.

The percentage of bed material grain size $i$ can obtained from (11) as

$$\delta\frac{\partial p_i}{\partial t}+p_i^*\frac{\partial\eta}{\partial X}+\frac{1}{1-\lambda}\left[\frac{1}{B}\frac{\partial\left(q_{Bi}B\right)}{\partial X}+q_{sui}-w_{f_i}c_{bi}\right]=0 \quad (11)$$

when $p_i^*=p_i \quad ;\partial\eta/\partial t\geq 0$

$p_i^*=p_{i0} \quad ;\partial\eta/\partial t<0$

where $\delta$ = exchange layer thickness; $\lambda$ = void ratio.

The time dependent change of bed elevation calculated by the following continuity of bed material transport.

$$\frac{\partial\eta}{\partial t}+\frac{1}{1-\lambda}\left[\frac{1}{B}\frac{\partial\sum_i\left(q_{Bi}B\right)}{\partial X}+\sum_i\left(q_{sui}-w_{f_i}c_{bi}\right)\right]=0 \quad (12)$$

where $\sum_i$ = summation of bed material transport load.

## 3. METHOD OF SOLUTION

Upon integrating over the uniform grid elements, the finite difference of the nodal domain integration (NDI) method is used. Channel and flood plain geometry are showed in Fig.1 and 2, respectively.

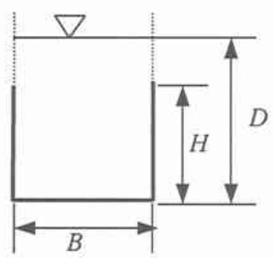


Fig. 1 Channel geometry

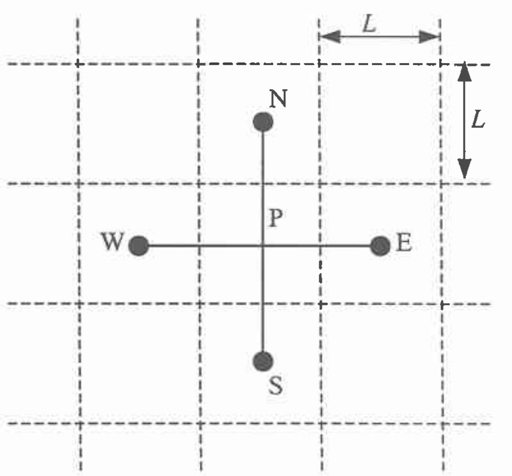


Fig.2 Floodplain geometry

## 4. NUMERICAL EXAMPLE AND RESULTS

Performance of the foregoing numerical scheme was applied to three flow problems, namely, junction flow in channel, rainfall runoff from floodplain and bed deformation in channel.

### Junction Flow in Channel

Junction flow in channel often leads to the formation of complicate river basin. A problem with junction flow channel is illustrated in Fig.3. At the upstream are constant discharge $Q_2$=1000 and $Q_3$=600 $m^3/s$, and downstream is set as free outflow condition. The initial condition in channel is normal depth throughout its length. The proposed model is then applied to simulate this problem, and the result is compared with well-known MIKE11 model (Fig.4). The proposed model produces results that give almost identical with MIKE11 model.

### Rainfall Runoff from Floodplain

The hydraulic event following rainfall runoff is importance for designing hydraulic structures. The knowledge on the runoff rate and volume are essential for designing channel and structure with banks cannot be overtopped. Fig.5 shows the problem. The initial condition has a depth equal to zero throughout the watershed $B$=10 m, $L$=100 m, $n$=0.003, and $\theta=15^{\circ}$ of floodplain. A storm with uniform rainfall intensity $i$=36 mm/hour, lasting 10 hours over a watershed. The critical outflow is set at the downstream end. The simulated results at time 24 hours from this proposed model and Ichikawa et al. [4] are given in Fig.6. The proposed model predict runoff rate from floodplain very accurately. The computation time is about 20 seconds.

### Bed Deformation in Channel.

Deformation of channel bed resulting from the flood in a channel is a one of the most important hydraulics and morphological situation for engineering works. Fig.7 is illustrated a channel with hump in the middle $B$=100 m, $L$=6000 m, $Q$=500 $m^3/s$, $n$=0.025, bed material grain sizes are $d_{10}$=0.2 mm, $d_{50}$=0.5 mm, and $d_{90}$=1.0 mm. The initial condition in channel is normal depth throughout its length. Fig.8 show the water surface and bed deformation profiles at the different times. The bed traveled from upstream to downstream. The proposed model results compared with upwind scheme, the traveling speed is slightly slow.

## 5. CONCLUDING REMARKS

A hydrodynamic model of coupled 1- and 2-D is presented to simulate the overland flow and sediment transport in channel and floodplain. Three test problems, namely, junction flow, rainfall runoff, and bed deformation are used to verify the numerical model. The computed results indicate that this proposed model satisfactorily predicted all the flow. However, it can extended to simulate floodplain with complicate topography.

## REFERENCES

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2)Akanbi A. A., and Katopodes, N. D., Model for flood propagation on initially dry land, J. Hydr. Engrg., ASCE, Vol.114, No.7, pp.689-706, 1988.

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4)Ichikawa Y., Ogura T., Tachikawa Y., Shiba M., and Takara K., Lumping general kinematic wave equation of slope runoff system, Annual J. of Hydraulic Engineering, JSCE, Vol.44, pp.145-150, 2000.

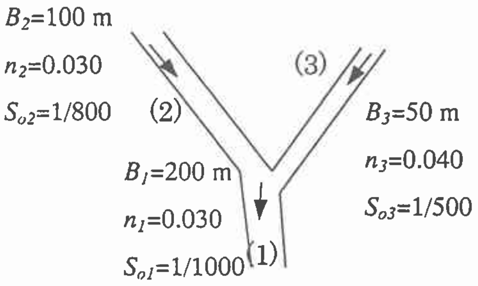


Fig.3 Definition of junction flow in channel

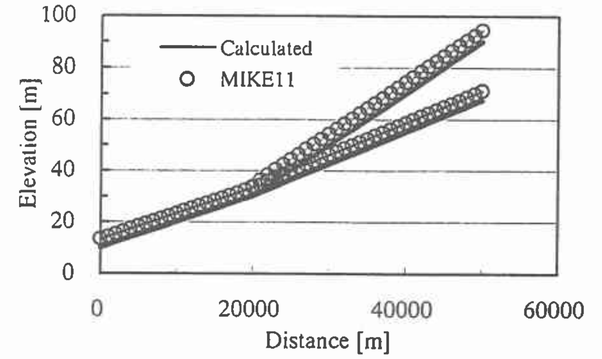


Fig.4 Water surface profile of junction flow in channel

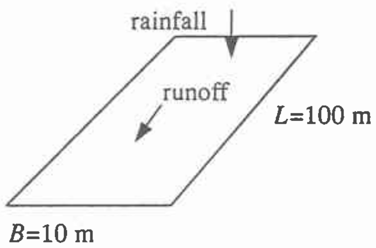


Fig.5 Definition of rainfall runoff from floodplain

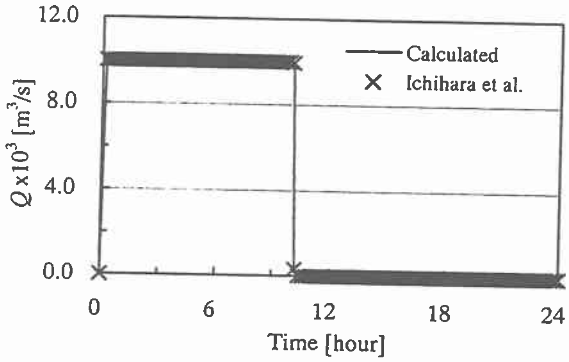


Fig.6 Rainfall runoff from floodplain

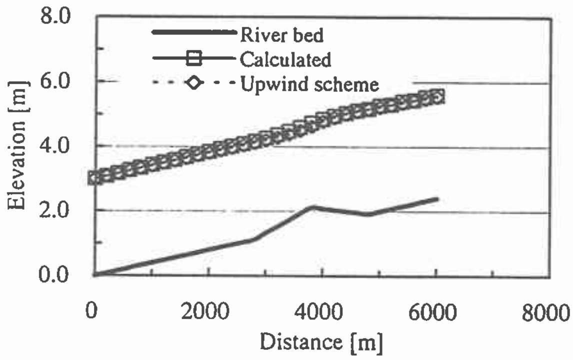


Fig.7 Definition of bed deformation in channel and initial water surface profile

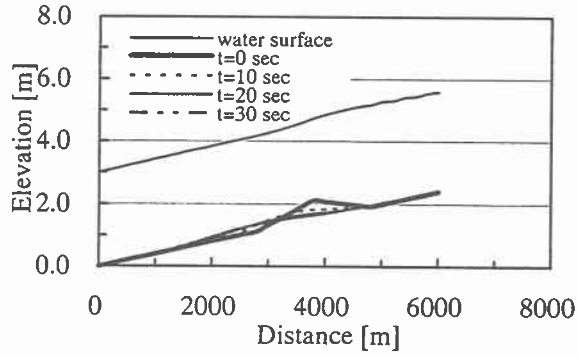


Fig.8 Water surface and bed deformation profiles in channel at the different times (t=10, 20, and 30 hours)