A Formulation of Moment-Rotation Curve of Top- and Seat-Angle Connections Taking Prying Action into Account

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1. INTRODUCTION

Recently top- and seat-angle connections are used to transfer partially beam end moment in the steel frame structure. In designing of this type of connections, seat angle is frequently used to carry out gravity type load and top angle is provided for resisting lateral forces. When top angle transfers the beam end moment to the column, it develops an additional tensile force on fasteners due to local deformation at the end portion of vertical leg. The additional force is well known as prying force. Many researchers are enable to consider this additional force in their mathematical model of designing T-stubs. Later those are modified for extended end plate connections and for top- and seat-angle connections (CEN, 1997). The T-stub formulation for connections with angles gives so conservative results of resisting forces for thin angle that they differ widely from those of experiments show.

In this study, a three-parameter power model proposed by Kishi and Chen (1990) disregarding the strain-hardening effect, is selected, and the power function is used to predict flexural behavior of connections subjected to prying action, for its versatile acceptance in second-order frame analysis with semi-rigid connections. The power function containing three parameters: initial connection stiffness R_{ki}, ultimate moment capacity M_u, and shape parameter n, has the following form:

$$M = \frac{R_{ki}\theta_r}{\left[1+\left(\dfrac{\theta_r}{\theta_0}\right)^n\right]^{\frac{1}{n}}} \tag{1}$$

where M and θ_r are moment and relative rotation in connection, respectively; and θ_0 is a reference plastic rotation , $\theta_0 = M_u/R_{ki}$. Figure 1 shows the general shapes of M–θ_r curves of Eq. (1) with different values of shape parameter n. In the proposed model, initial connection stiffness, R_{ki} is estimated as recommended by Kishi and Chen (1990), which is formulated

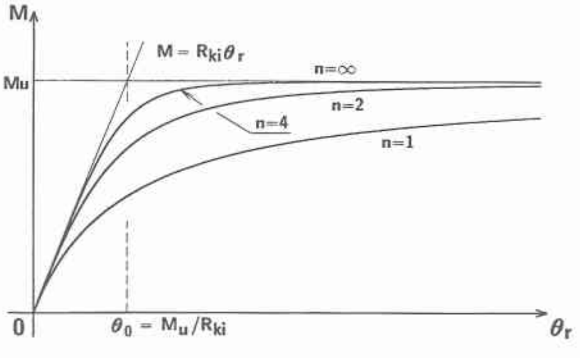


Fig. 1 Curves for three-parameter power model

as follows:

$$R_{ki} = \frac{3EI_t}{1+\dfrac{0.78t_t^2}{g_1^2}}\frac{d_1^2}{g_1^3} \tag{2}$$

in which EI_t is the bending stiffness of top angle's vertical leg; $g_1 = g - D/2 - t_t/2$; g is the gage distance from the heel to the bolt centerline in the top angle leg adjacent to the column flange; $D = w$ for bolted connections; or, $D = d_b$ for riveted connections; w is the nut's width across the flats; d_b is the fastener's diameter, $d_1 = h_b + t_t/2 + t_s/2$; h_b is the beam height; t_t and t_s are the thicknesses of top and seat angles; respectively. The shape parameter, n is determined by using least-square fit technique. The empirical equation of shape parameter, n will also be derived for design purpose to devise M–θ_r curve of top- and seat-angle connections in our following paper. In this paper, a mathematical formulation is developed to determinate rationally the ultimate moment capacity, M_u of present connections.

In the power model formulation predicting ultimate moment capacity, the distance between two plastic hinges in the top angle failure mechanism defined by Kishi and Chen (1990), is too small compared to the top angle thickness. Even in connections with very thick angles, this distance can be zero, which causes the increase of the shear force in the plastic hinges equal value of pure plastic shear of top angle. To overcome the situation, three mechanisms for each type of connection failure cases are developed, in which different definition of plastic hinges is used. From the FE analysis, it is observed that the minimal vertical distance between the sections, where the plastic hinges form in a top angle, increases with increasing angle thickness and decreases with increasing bolt diameter. This observation is considered when defined the plastic hinges in the connection failure mechanisms. Moreover, Kishi-Chen power model is totally ignored the contribution of prying action on top angle failure mechanism and this can also cause of higher estimation of ultimate moment capacity of top- and seat-angle connections. Therefore, a mathematical formulation is developed for predicting both the moment-rotation characteristics of top- and seat-angle connections and resisting forces of connection assemblages considering prying action.

2. DETERMINATION OF ULTIMATE MOMENT CAPACITY

A typical top- and seat-angle connection is shown in Fig. 2. The proposed formulation governs three mechanisms for this type of connection failure cases. The mechanisms are provided with plastic hinges in angles and bolts at the places so that those can suit with the real connections deformations reported in Azizinamini et al. (1985) and Hechtman et al.'s test data (1947). The mechanisms of plastic hinge formation are also confirmed with the deformation configurations of connections predicted by FE analyses conducted by authors (2000). The following assumptions are to be employed in determination of connection ultimate moment capacity those were adopted by Kishi and Chen (1990).

1. The center of rotation for the connection is located at the middle of angle leg adjacent to the compression beam flange at the top of angle heel (Fig. 3).
2. The restraining moment at the center of rotation is so small that it can be neglected.
3. The deformation of connection elements is small.
4. The material is constituted with linearly elastic and perfectly plastic behavior.

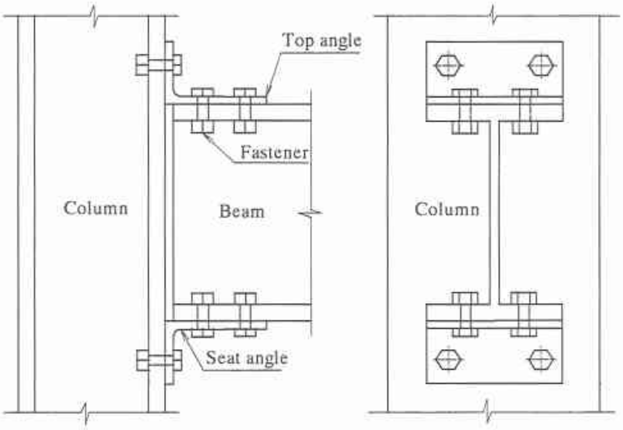


Fig. 2 Typical top- and seat-angle connection

2.1 Collapse mechanisms of connection

i) *Mechanism type I*

The first mechanism is characterized by the formation of three plastic hinges. Two plastic hinges are developed on the top angle leg adjacent to the column flange, and the other one is formed at the seat angle leg adjacent to the compression beam flange as shown in fig. 3(a).

From the work equation to the collapse mechanism *type I*, ignoring moment-shear interaction effect, the shear force developed in top angle plastic hinges can be found by:

$$V_{t1} = \frac{2M_{p,t}}{m_t} \qquad (3)$$

where the value of the pure plastic moment is,

$$M_{p,t} = \frac{w_t t_t^2}{4} f_{y,t} \qquad (4)$$

and the minimal vertical distance between two plastic hinges of top angle are given by (Fig. 3(a)):

$$m_t = g - w_b - r \qquad (5)$$

in which g is the gage distance from the heel to the bolt centerline in the top angle leg adjacent to the column flange, w_b is the bolt head width across the flats or the head diameter of rivet, and r is the radius of top angle leg-to-leg joint curvature.

ii) *Mechanism type II*

The mechanism *type II* is shown in Fig. 3(b). Ignoring moment-shear interaction effect, the yield condition of the top angle vertical leg in the 2-2 section (Fig. 3(b)) provides:

$$M_{2-2} = T_2(m_t + n_t - b) - Q_2(m_t + n_t) = M_{p,t} \qquad (6)$$

where b is the distance from prying force acting point to the tension fastener centerline at the ultimate state of connection. Distance b is defined by FE analyses for top- and seat angle connections (Kishi et al., 2000) and can be approximated for bolted connections with conservative provision by:

$$b = 2.4t_t \text{ if } 2.4t_t < a \text{ or, } b = a \qquad (7)$$

where a is the distance from the tension bolt centerline to the top edge of tension angle leg adjacent to column flange, and for riveted connections is assumed as:

$$b = 3t_t \text{ if } 3t_t < a \text{ or, } b = a \qquad (8)$$

$$n_t = w_b - t_t + b \qquad (9)$$

T_2 is the fastener tensile resisting force that can be found by:

$$T_2 = T_{p,b} = n_b A_b f_{y,b} \qquad (10)$$

where $T_{p,b}$ is the pure plastic tensile resistance of fastener, n_b is the number of fasteners those sustain tensile forces, A_b is the nominal area of fastener's shank cross section, and $f_{y,b}$ is the yielding stress of fastener's material.

Taking into account from the equilibrium condition prying restraining force for mechanism *type II*, $Q_2 = T_2 - V_{t2}$ in the Eq. 6, the top angle shear restraining force yields:

$$V_{t2} = \frac{T_{p,b}\, b + M_{p,t}}{m_t + n_t} \qquad (11)$$

iii) *Mechanism type III*

The collapse mechanism *type III* is depicted in Fig. 3(c). The resisting forces for mechanism *type III* is assumed not be affected by the moment-shear interaction and prying force does not exit at the ultimate state of connection, as the angles are much stronger than fasteners. That is $Q_3 = 0$. Therefore, the shear resistance of the top angle can be given by:

$$V_{t3} = T_3 = T_{p,b} \qquad (12)$$

where T_3 is the fasteners tension resistance for mechanism *type III*.

Thus, the ultimate moment capacity for mechanism *type III* can be found by:

$$M_{c3} = M_{p,s} + V_{t3}\, d \qquad (13)$$

in which $M_{p,s}$ is the pure plastic moment of seat angle leg adjacent to the beam flange, which ignores the interaction with the axial force, is given by:

$$M_{p,s} = \frac{w_s t_s^2}{4} f_{y,s} \qquad (14)$$

in which w_s and t_s are the width and thickness of seat angle, respectively, and $f_{y,s}$ is the yielding stress of seat angle material, and $d = 0.5t_s + h_b + g$, h_b is the beam height.

2.2 Boundary definitions for collapse mechanisms

The parameter β prevailing the collapse mechanisms relates the axial strength of tension fasteners and the flexural strength of top angle which is the ratio between the shear resisting forces corresponding to mechanism *type III* (Eq. 12) and that corresponding to mechanism *type I*, (Eq. 3) and has the following form:

$$\beta = \frac{m_t \; T_{p,b}}{2 \; M_{p,t}} \tag{15}$$

Accounting for Eq. 15 and non-dimensional geometrical parameters $\lambda = m_t / b$ and $\gamma = n_t / b$, and solving the equations, set for the boundaries between mechanisms *type I* and *type II*, and *type II* and *type III*, where the top angle shear restraining forces on the plastic hinges are equal for two relative cases (i.e., V_{t1}=V_{t2}, and V_{t2}=V_{t3}, respectively), the following boundary definitions are achieved:

- Mechanism *type I* (i.e., flange angles yielding) occurs when $\beta \geq \frac{\lambda}{2} + \gamma$
- Mechanism *type II* (i.e., bolt failure with flange angles yielding) occurs when $\frac{\lambda}{2} + \gamma > \beta \geq \frac{\lambda}{2(\lambda + \gamma - 1)}$
- Mechanism *type III* (i.e., bolt failure) occurs when $\beta < \frac{\lambda}{2(\lambda + \gamma - 1)}$

i) ***Mechanism type I***

From the work equation, it can have the following relation between shear resisting force and bending moment at the plastic hinges of top angle:

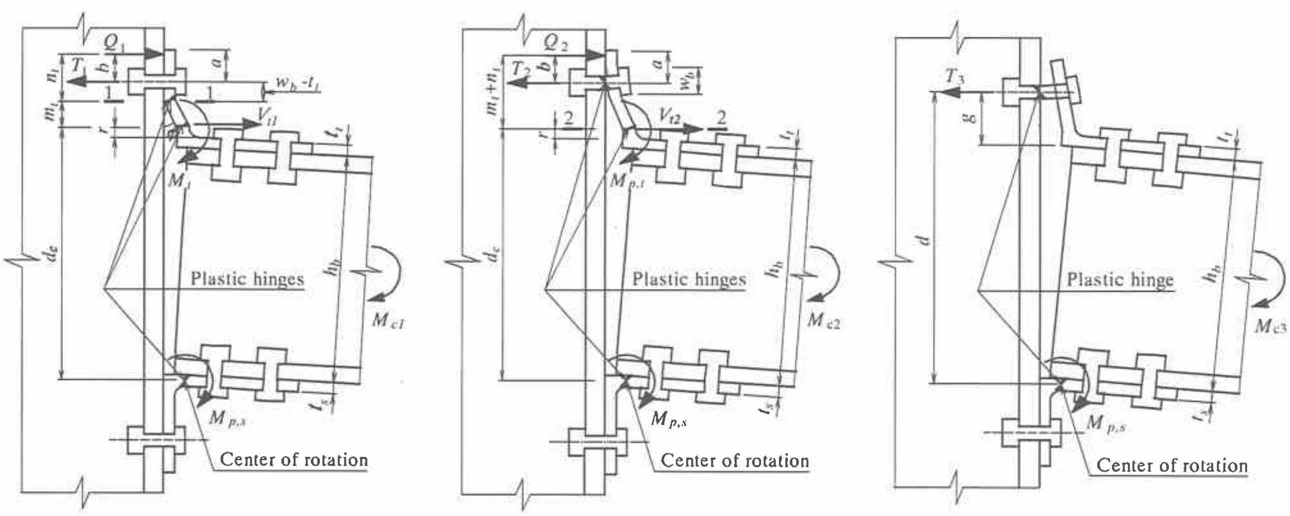
$$M_t = \frac{V_{t1} \; m_t}{2} \tag{18}$$

Dividing and multiplying to the Eq. 4 with the value of $V_{p,t}$ employed Eq. 17, the relation between the pure plastic moment and the pure plastic shear at the full yielding of top angle section yields:

$$M_{p,t} = \frac{\sqrt{3}}{4} t_t \, V_{p,t} \tag{19}$$

Substituting Eq.18 and Eq. 19 into Drucker's moment-shear interaction domain Eq. 16, the following fourth order equation can be obtained:

$$\left[\frac{V_{t1}}{V_{p,t}}\right]^4 + \frac{2}{\sqrt{3}} \frac{m_t}{t_t} \left[\frac{V_{t1}}{V_{p,t}}\right] - 1 = 0 \tag{20}$$



a) Mechanism *type I* b) Mechanism *type II* c) Mechanism *type III*

Fig. 3 Collapse mechanisms of top- and seat-angle connections

2.3 Consideration of moment-shear interaction

The formulations for computing the ultimate moment corresponding to the mechanisms *type I* and *type II* accounts for Drucker's moment-shear interaction. The Drucker's moment-shear domain has the following form:

$$\frac{M_t}{M_{p,t}} + \left[\frac{V_t}{V_{p,t}}\right]^4 = 1 \tag{16}$$

where M_t and V_t are the bending moment and shear force acting on the plastic hinges of top angle vertical leg, respectively. The values of the pure plastic moment, $M_{p,t}$ is described in Eq. 4 and pure plastic shear, $V_{p,t}$ of the top angle according to von Mises yield criterion are given by:

$$V_{p,t} = \frac{w_t \; t_t}{\sqrt{3}} f_{y,t} \tag{17}$$

where $f_{y,t}$ is the yielding stress of top angle material, and w_t is the width of top angle.

Following a simple iteration procedure the value of V_{t1} can be easily determined from Eq. 20.

From the equilibrium condition of the mechanism *type I*, the tension resistance of the fasteners can be found by:

$$T_1 = V_{t1} + Q_1 \tag{21}$$

in which Q_1 is the prying force for mechanism *type I*.

From the yield condition of the top angle vertical leg at the 1-1 section (Fig. 3(a)), it can have the following relationship for prying resistance:

$$Q_1 = \frac{1}{b}\left[V_{t1}\,(\,n_t - b\,) + M_{p,t}\,\right] \tag{22}$$

where n_t is the distance from the plastic hinge at the 1-1 section to the acting point of prying force (Eq. 9).

The ultimate moment capacity, M_{c1} of top- and seat-angle connection for mechanism *type I*, taking the moment about the center of rotation (Fig. 3(a)), can be obtained by the following equilibrium equation:

$$M_{c1} = M_{p,s} + M_t + V_{t1}\, d_c \tag{23}$$

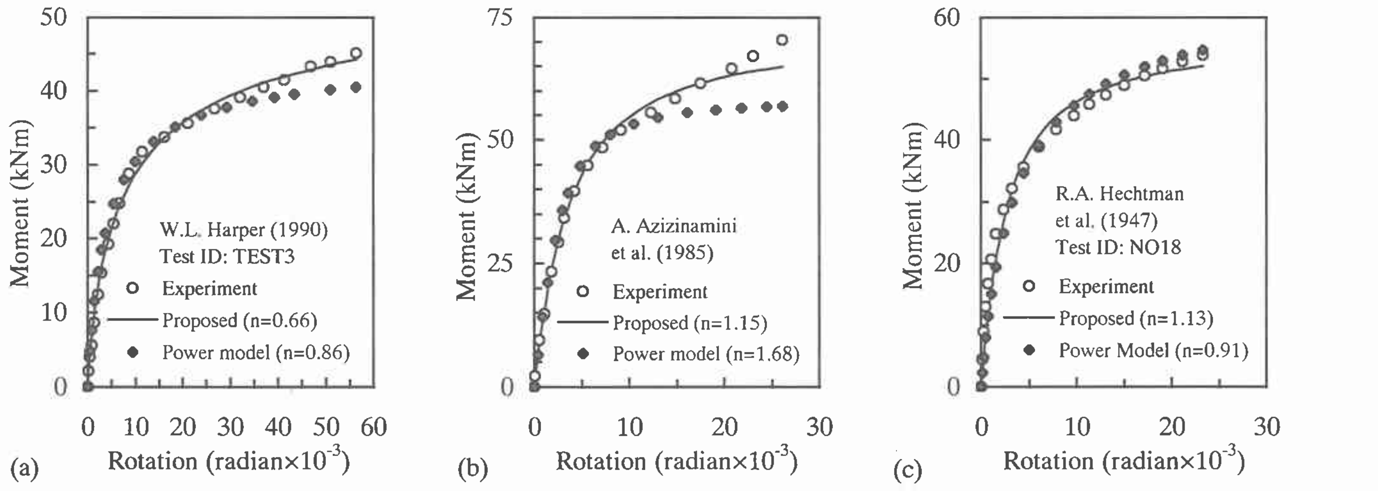


Fig. 5 Comparison among moment-rotation characteristics

ii) ***Mechanism type II***

Consideration of moment-shear interaction for mechanism *type II*, the pure plastic moment, $M_{p,t}$ in Eq. 6 need to be replaced by the plastic moment, M_t of the top angle that can be reduced due to moment-shear effect, that yields:

$$M_t = T_{p,b}\left(m_t + n_t - b\right) - Q_2\left(m_t + n_t\right) \tag{24}$$

Combining Eqs. 19 and 24 with Drucker's moment-shear interaction domain Eq. 16, and the yielding condition can be written in the following form:

$$\left(\frac{V_{t2}}{V_{p,t}}\right)^4 + \frac{4}{\sqrt{3}}\frac{m_t+n_t}{t_t}\frac{V_{t2}}{V_{p,t}} - \left(1+\frac{T_{p,b}\,b}{M_{p,t}}\right) = 0 \tag{25}$$

Considering the non-dimensional values $\mu = \left(m_t + n_t\right)/t_t$ and $\eta = 1 + T_{p,b}\,b/M_{p,t}$ into Eq. 25, can have the following equilibrium equation:

$$\left(\frac{V_{t2}}{V_{p;t}}\right)^4 + \frac{4\mu}{\sqrt{3}}\frac{V_{t2}}{V_{p,t}} - \eta = 0 \tag{26}$$

V_{t2} can be determined by iterating the Eq. 26.

Therefore, investigating from Eq. 6, prying restraining force of tension bolts can be expressed as:

$$Q_2 = \frac{1}{b}\left[V_{t2}\left(m_t + n_t - b\right) - M_{p,t}\right] \tag{27}$$

Thus, the tension resistance of fasteners is given by:

$$T_2 = V_{t2} + Q_2 \tag{28}$$

The ultimate moment capacity of connection for mechanism *type II*, taking moment about the center of rotation (Fig. 3(b)), can be obtained by:

$$M_{c2} = M_{p,s} + M_{p,b} + V_{t2}\,d_c \tag{29}$$

where $M_{p,b}$ is the pure plastic moment of fasteners which ignores the interaction with the axial force, is given by:

$$M_{p,b} = \frac{n_b\,\pi\,d_b^3}{16} f_{y,b} \tag{30}$$

π =3.142, d_b is the fastener's diameter, and $f_{y,b}$ is the yielding stress of fastener's material.

4. ASSESSMENT AND CONCLUSIONS

Several series of experiments of top- and seat-angle connection have been collected in up-dated data-bank program (SCDB) by Chen and Kishi (1989). The data-bank program is used to conduct a comparison among results of experiment, power model worked out by Kishi and Chen (1990), and proposed formulation using the subroutine developed by Chen and Kishi (1989). The procedures for assessing the moment-rotation curve are conducted in a direct and automatic manner. A comparison is depicted in figs. 5(a)-(c). It is observed from the comparison study that consideration of prying action in the proposed formulation decreases the ultimate moment capacity of some connections and best fitted with the experimental moment-rotation curves.

Therefore, it can be concluded that proposed approach is expanded the ability of power model (Kishi and Chen, 1990) by accounting for fasteners action in the formulation. And the formulation is capable of predicting the behavioral characteristics of top- and seat-angle connections with more accuracy becoming an effective and rational method.

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