Ⅳ－66

A Variant-Weighting Factor Method for Estimating of Traffic states on Freeways

N. POURMOALLEM, T. NAKATSUJI, T. FUJIWARA
Faculty of Engineering, HOKKAIDO University

Abstract: This paper describes improvement to Cremer's variant of Payne's macroscopic freeway traffic flow model, with the objective of using the improved model to drive an extended Kalman filter for estimating traffic states. We extend the flow rate and the time mean speed equations to flow density under the influence of responsive traffic states. We make a method-orientated approach for optimization of the model parameters. Using traffic data of the Metropolitan Expressway, the effectiveness of the improvement was investigated. The extended model the estimation errors greatly, comported with the original Cremer model.

Keywords: *Macroscopic traffic flow model, Payne model, Cremer model, Kalman Filter, Box's model*

1. Introduction

Macroscopic traffic flow models, based on a hydrodynamic theory (compressible fluid) of traffic flow contain several traffic variables, the variables defined by state equations and observation equations.

State equations describe the density and space mean speed variables. The concept of treating traffic macroscopically as a continuum was proposed by Lighthill, et al.*(1)*. After that, the model was revised, and another equation including the dynamic behavior of space mean speed was introduced by Pipes *(2)*, Payne *(3)*, Philips *(4),etc.*. Payne formulated the equilibrium hypothesis for density-speed.

Observation equations describe the flow rate and time mean speed variables. As a direct consequence of the other continuum variables' of the macroscopic traffic flow models, flow rate products the density and the space mean speed variables at end each segment by Wardrop *(5)*. After that flow rate and time mean speed improved by Lebacque *(6)* and Cremer *(7)*. They assumed, a constant weighting factor in observation equations, that defines the flow rate at a point as the linearly-weighting average of the product of density and space mean speed in both adjacent segment. Lebacque described the weighting factor, that obtained by application should be limited to the capacity of segments. Cremer optimized the weighting factor with other model parameters, which is solved by non-linear programming *(8)*. In both the Lebacque and Cremer models, when traffic is in free-flow states, flow rates are influenced by the upstream traffic states. On the other hand, when traffic is congested, they are influenced by the downstream states, a constant weighting factor cannot reflect these phenomena.

To cope with this problem, we made improvement on the observation equations based on the Cremer model. We introduced a weighting factor that is defined as an exponential function of density. By providing a the weighting factor for each segment, this made it possible to apply the modified observations equations to extensive traffic situations.

The paper is organized as follows: In the next section, we provide basic of macroscopic traffic flow modeling. In the third section, we address how we improved the observation equations. In the forth section, we have segmentation of road from the Metropolitan Expressway. In the firth section, we presented a method-oriented approach for identification model parameters. In the sixth section, we present the experimental results, we examine how effective the improvement is. In seventh section, we evaluate the new observation equations. In the eighth section, a summary with conclusions is given. Finally, we provide basic of Kalman filter *(9)* and Box's model *(10)* in appendix.

2. Macroscopic Traffic Flow Modeling

2.1 Model Variables

Macroscopic traffic flow models describe traffic phenomena aggregately based on traffic variables, such as density, average speed, and flow rate. Consider the section of freeway shown in Fig.1. The section may have on- or off ramps and may contain a change in the number of lanes. The freeway is now considered to be formally divided into several sections, each section of freeway is subdivided into several segments.

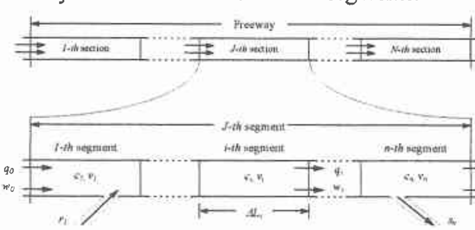


Fig.1 Discretized section of a freeway.

With respect to this configuration the following macroscopic variables are introduced:

$c_i(k)$: density in segment i at time k *(veh/km)*

$v_i(k)$: space mean speed of the vehicles *(km/hr)*

$q_i(k)$: flow rate from segment i into segment $i+1$ during time interval k to $k+1$ *(veh/hr)*

$w_i(k)$: time mean speed, harmonic individual vehicle speed which are measured at cross-section *i-th* segment during time k to $k+1$ *(km/hr)*

$r_i(k)$ and $s_i(k)$: flow rate of on-ramp and off-ramp during time interval k to $k+1$ *(veh/hr)*

ΔL_i : length of segment i

2.2 Model Equations

a) States Equations

- density

Modeling traffic as flow of a compressible hydrodynamic as proposed by Lighthill, et al. *(1)*. In that model, the simple hydrodynamic equation employed only the conservation equation which has general consideration of the dynamic behavior of density:

$$\frac{\partial c}{\partial t} + \frac{\partial q}{\partial x} = r - s \qquad (1)$$

for our purposes, it is more convenient to consider differential equations. With these variables, a space discretization form of Eq.(1) can be given:

$$c_i(k+1) = c_i(k) + \frac{\Delta t}{\Delta L_i}\left[q_{i-1} - q_i + r_i - s_i\right]_{(k)} \qquad (2)$$

- space mean speed

After that, the model was criticized and another equation representing the dynamic behavior of space mean speed was added by Pipes *(2)*, Payne *(3)*, Philips *(4)*, *etc.* The ideas of Payne the mean speed is influenced by three terms: a relation term, a convection term and a density gradient term. This is expressed by the follows equations:

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{T}\left[v - V(c)\right] - \frac{\upsilon}{c}\frac{\partial c}{\partial x} \qquad (3)$$

Papageorgiou *(11)*, Cremer *(7)*, *etc.* improved the Payen-type macroscopic model as Eq.(3). The density gradient term reflects the drivers' anticipation to a foreseen relative density change, and the a density parameter takes into account that this effect become negligible for low density values, which are improved by Cremer as follows.

$$v_i(k+1) = v_i(k) + \frac{\Delta t}{\tau}\left[V(c_i) - v_i\right]_{(k)} + \frac{\Delta t}{\Delta L_i}\left[v_i\left(v_{i-1} - v_i\right)\right]_{(k)} + \frac{\nu}{\tau}\frac{\Delta t}{\Delta L_i}\left[\frac{c_i - c_{i+1}}{c_i + \kappa}\right]_{(k)} \qquad (4)$$

where Δt is the time interval of simulation. τ is time constant, ν is sensitivity factor ,and κ is density parameter. $V(c_i)$ in the above dynamic behavior of space mean speed is the steady-state speed, which is defined by a density-speed characteristic *(k-v)* curve by May et. al *(12)* as follows;

$$V\left(c_i(k)\right) = v_f\left[1 - \left(\frac{c_i(k)}{c_{max}}\right)^{(l-1)}\right]^{\left(1/(1-m)\right)} \qquad (5)$$

where v_f is free speed, c_{max} is jam density. m and l are positive real parameters ($l > 1$ and $0 \le m < 1$).

b) Observation Equations

- flow rate

A simple identity provides the basic relationship between flow rate, space mean speed, and density by Wardrop *(5)*:

$$q = cv \qquad (6)$$

In one way, substituting the density Eq.(2) and the speed $V(c_i(k))$ Eq.(5) into Eq.(6), we get a flow-density relationship $Q[c_i(k)]$ that is broadly known as the fundamental diagram of traffic engineering. The flow rate is as follows:

$$q_i(k) = Q\left[c_i(k)\right] \qquad (7)$$

which we will call model A. Other dictionaries way, substituting the density Eq.(2) and the space mean speed Eq.(4) into Eq.(6), we get a flow-speed-density relationship as follows:

$$q_i(k) = c_i(k)v_i(k) \qquad (8)$$

which we will call model B. In some models , the flow rate can be expressed as a linearly-weighted average of products of the density and the space mean speed on the adjacent segments as shown in Fig.1 with the models A and B. The flow rates were linearly described as follows:

$$q_i(k) = \alpha Q\left[c_i(k)\right] + (1-\alpha)\left[Q\left[c_{i+1}(k)\right] - r_{i+1}(k)\right] - s_i(k) \qquad (9)$$

$$q_i(k) = \bar{\alpha} Q\left[\min\left\{c_i(k), c_{cr}\right\}\right] + (1-\bar{\alpha})\left[Q\left[c_{i+1}(k)\right] - r_{i+1}(k)\right] - s_i(k) \qquad (10)$$

$$q_i(k) = \alpha c_i(k)v_i(k) + (1-\alpha)\left[c_{i+1}(k)v_{i+1}(k) - r_{i+1}(k)\right] - s_i(k) \qquad (11)$$

where α and $\bar{\alpha}$ are the weighting factor. In Eqs.(9) and (11), the weighting factor is as follows;

$$\alpha = constant, \quad renege\ of\ 0 \le \alpha \le 1 \qquad (12)$$

In Eq.(10) $\bar{\alpha}$ defined by Lebacque *(6)* as follows:

$$\bar{\alpha} = \begin{cases} \alpha & if \quad c_{i+1}(k) < c_{cr} \\ 1-\alpha & if \quad c_{i+1}(k) \ge c_{cr} \end{cases} \qquad (13)$$

- time mean speed

The time mean speed $w_i(k)$ as a linearly-weighted average of the space mean speed:

$$w_i(k) = \left[\dot{\alpha} v_i + (1-\dot{\alpha})v_{i-1}\right]_{(k)} \qquad (14)$$

where $\dot{\alpha}$ can be same as the above value of the weighting factors in each Eqs.(12) and (13).

We start with the version by the original Cremer model (OC model)*(7)*. The Cremer model first estimate traffic states using the state Eqs.(2) and (4) and the observation Eqs. (11) and (14) then adjusts them using observed detector data, the model is combined with a Kalman filter *(9)*.

3. Variant-Weighting Factor Method

As shown in Eqs.(9), (10), (11), and (14) assumes a constant weighting factor as Eq.s(12) and (13). It states that the upstream segment i contributes to flow rate and time mean speed by a linear relationship α, and the downstream segment $i+1$ contributes by $(1-\alpha)$. However, it is a bit unreasonable that the factor α does not depend on traffic states: When traffic is in a free-flow state, traffic dynamics are mainly dominated by the traffic states in the upstream segments. On the other hand, when traffic is in a heavy state, they are strongly influenced by the states in the downstream segments, since some growing congestion generated in a downstream segment would propagate upwards. A constant weighting factor cannot describe such phenomena. To deal with this problem, we introduced a weighting factor that was dependent on traffic states:

$$\alpha = f\left[c_i(k)\right] \qquad (15)$$

where f denoted an negative exponential function play role relationships between weighting factor and traffic states as density $c_i(k)$ as follows:

$$\alpha\left(c_i(k)\right) = e^{-\beta c_i(k)} \qquad (16)$$

where β is a curvature in the range of $0 \leq \beta \leq 1$. By providing a weighting factor for each section, we can apply the OC model to extensive traffic states. We call this method a variant-weighting factor method (VWF method).

Since this function decreases monotonously with density, it can represent the above-mentioned traffic flow phenomena very well. It should be noted here that the introduction of such a factor would make the structure of both the state and observation equations complicated as the feed-forward concept. Consequently, it would become burdensome to differentiate the equations and derive the state and observation matrices in Kalman filter (see App.(a)) as the feed-back concept.

4. Data Collection

The observed data used here came from a road section on the Metropolitan Expressway (Shuto Kosoku, Yokohama-Haneda Line) as shown Fig.2.

Traffic detected were installed on both nearside and passing lanes. Date variables were flow rate, time mean speed, and occupancy. As shown in Fig.2, we divided the road section into 3 sections S1, S2, and S3, that is, we assumed that there were 11 segments whose lengths ΔL_i ranged from 400 to 600 meters as shows Table 1. Then we assumed that traffic data were observed only at four points of OP1 (entrance), OP2, OP3, and OP4 (exit), although the data were actually observed at all the boundary points. We used the traffic data from Oct. 28 to Nov. 1 in 1993. Considering the balance with the segment length, we used the time interval of 10 seconds for the macroscopic simulation.

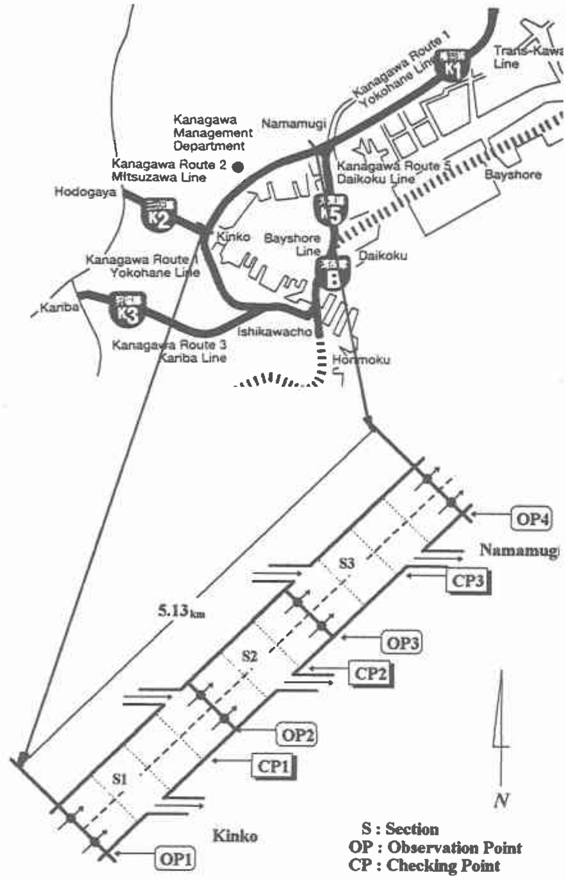


Fig.2 Geometric of the Metropolitan Expressway.

Table.1 Configuration of sections.

section	length (km)	segments	on-ramp	off-ramp
1	1.92	4	1	1
2	1.23	3	-	1
3	1.98	4	1	1

5. Identification Model Parameters

We mentioned earlier that the OC model is very sensitive to the variation of the model parameters. Cremre presented a model for estimation model parameters. They examined the sensitivity. They stated that the parameters of the density-speed curve, such as v_f, c_{max}, m, and ℓ in Eq.(5), were more sensitive than the other parameters of the traffic flow model, such as τ, ν, and κ in Eq.(4) (see App.(b).

For estimation model parameters, we defined a method-orientated approach. In the approach, Box's compels algorithm has a role optimization of the model parameters (see App.(c)). In this way, Box's complex algorithm combined with the traffic flow model and observed data as Fig.3.

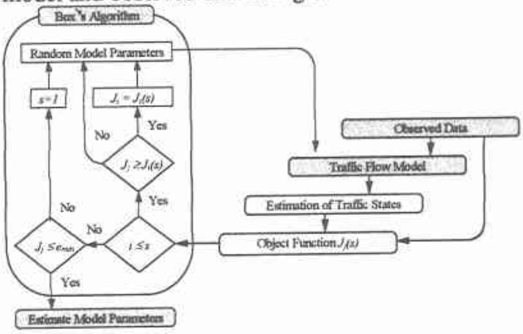


Fig.3 Model-orientated approach for estimation of model parameters.

We optimized the parameters so as to minimize the following objective function $J_j(s)$ Eq.(17) at each the checking point (CP1, CP2, CP3):

$$J_j(s) = \left(\frac{1}{h} \sum_{k=1}^{n} \frac{1}{\sigma_{q_j}^2} \left(\hat{w}_j - w_j \right)_k^2 + \frac{1}{\sigma_{w_j}^2} \left(\hat{q}_j - q_j \right)_k^2 \right)^{1/2} \quad (17)$$

where s is number training patterns, and $Jj(s)$ is estimation errors in s-th pattern. $q_j(k)$ and $\hat{q}_j(k)$ are observed and estimated flow rates at checking point j at time k, respectively. $w_j(k)$ and $\hat{w}_j(k)$ are time mean speed. h is the total number of time steps.

$\sigma_{q_j}^2 = 62500\ (veh/h)^2$ and $\sigma_{w_j}^2 = 49\ (km/h)^2$ are the standard deviation of flow rate $q_j(k)$ and time mean speed $w_j(k)$. Table.2 is given ones for extensive traffic states.

Table.2 Optimum parameters at each section by model-orientated approach using the OC model.

section	v_f km/h	c_{max} veh/km	$l-1$	$\frac{1}{(1-m)}$	α
1	84	328	1.7	4.9	0.75
2	84	288	1.4	4,9	0.73
3	87	198	1.9	4.7	0.74

To investigate the effect of traffic states using the weighting factor, we optimized it for shorter time periods: Subdividing the time periods into 32 shorter ones, we re-optimized the weighting factor so as to minimize the differences between the estimated and the observed ones at the checking points, while fixing the other parameters at constant values. Fig.4 shows the relationship between the optimized weighting factor and the averaged density of the upstream segments of the checking points. We can see that the factor definitely decreases as the density increases. In this way, the weighting factor is strongly dependent on traffic states and it is unreasonable to adopt a constant weighting factor. This is why we introduced the variant-weighting factor method (VWF method).

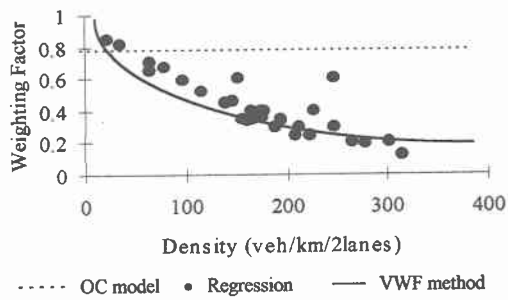


Fig.4 Weighting factor versus density.

We established the model-orientated approach for estimating model parameters that was based on the VWF method combining Box's complex algorithm. Table.3 presents the estimated parameters at each section. The parameters of the traffic flow model, such as τ, ν, and κ were omitted, except for the weighting factor α defined using Eq.16.

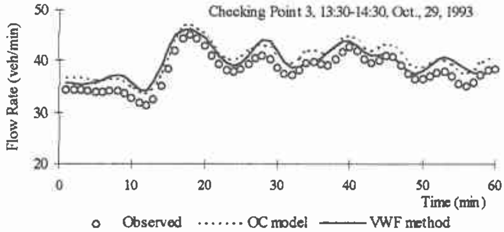
Table.3 Optimum parameters at each section by model-orientated approach using the VWF method.

section	v_f km/h	c_{max} veh/km	$l-1$	$\frac{1}{(1-m)}$
1	84	330	1.8	4.9
2	84	284	1.3	4.3
3	89	176	1.9	4.7

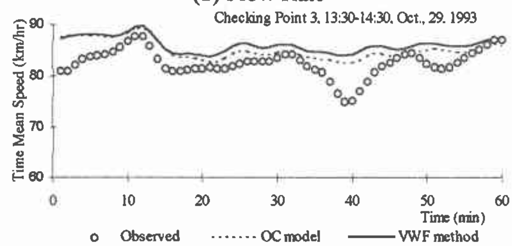
6. Estimation of Traffic States

To examine how effective the improvements we made were for estimating traffic states, we applied the two approaches, which were linear relationship as the OC model and non-linear relationship as the VWF method, to a road section shown in Fig.2. Although it was a bit tough to find any time period in the Metropolitan Expressway that includes simultaneously both light and heavy traffic states, by taking such a time period from 13:30 to 14:30 on Oct. 29 on sections S1 S2, and S3, we investigated how precisely the VWF method could estimate the flow rate and the time mean speed at CP1, CP2, and CP3. Since it was not adequate to use the parameters that were identified by the traffic data on the same day, we used the optimal ones in Tabs.2 and 3, excluding the ones on the same day. Fig.5(1) shows the flow rate estimates by the VWF method at CP3, compared with those produced by the OC model. The estimates were compiled every minute to compare with the observed ones. We can see that the flow rates by the OC model differ somewhat from the observed ones. On the other hand, the ones by the VWF method more or less agree with the observed data except for the initial time period. Similarly, Fig.5(2) shows the estimates of time mean speed of the same time period as in Fig.5(1).

Both methods are not successful in describing the abrupt change at around 40 minutes.



(1) Flow Rate



(2) Time Mean Speed

Fig.5 Estimates by the OC model and the VWF method compared with those observed at CP3.

7. Evaluation of Variant-Weighting Factor Method

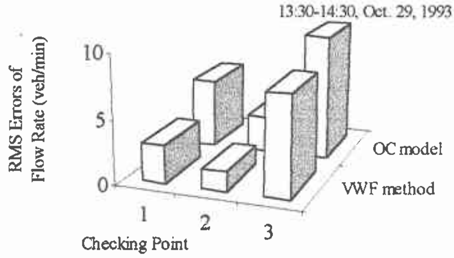
To evaluate the effectiveness of the VWF method in more detail, we applied those methods to traffic data from other days. That is, picking out a time period per day from Oct. 28 to Nov. 1, we estimated the flow rate and the time mean speed at three checking points using these methods and compared them with the observed ones. Fig.6 shows the comparison of the estimation errors of flow rate (1) and time mean speed (2) at three checking points of the same time period for the OC model and the VWF method. The estimation errors were evaluated by the root mean square errors (RMSE) as follows;

$$\text{RMSE of flow rate} = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\hat{q}_j - q_j\right)^2\right)^{1/2} \quad (18)$$

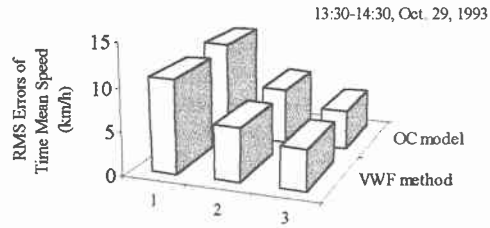
$$\text{RMSE of time mean speed} = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\hat{w}_j - w_j\right)^2\right)^{1/2} \quad (19)$$

Consequently, in this case, the VWF method decreased the estimation error 20 to 46% for the flow rate and 4 to 6% for the time mean speed, compared with the OC model.

Fig.7 shows how many times each method estimated most precisely for all the time periods in Fig.6. The VWF method was best for 13 of 2 sets of traffic data for flow rate, and for 8 of 7 for time mean speed. Adding the numbers in which the VWF method was best to those of the OC model, the estimation precision was improved for almost all traffic data. In other words, the improvements we made here were effective in accurately estimating both the flow rate and the time mean speed.



(1) Flow Rate



(2) Time Mean Speed

Fig.6 Comparison of estimation errors at three checking points evaluated by the OC model and the VWF method.

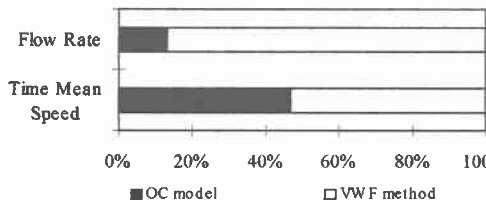


Fig.7 Comparison of how many times each method estimated best flow rate and time mean speed (Oct. 28 to Nov. 1).

8. Conclusions

The original Cremer model, in which the traffic states defined by density and space mean speed are first simulated by a macroscopic traffic flow model and then they are corrected by the actually observed data using the Kalman filter, is very useful for estimating traffic states on a freeway. However, because the model is very sensitive to the variation of model parameters, we have to identify them carefully. In this paper, we tried to improve the observation equations based on the original Cremer model: We established a weighting factor that was responsive to traffic states. Applying the modified method to a road section on the Metropolitan Expressway, we evaluated how precisely they could estimate the traffic states and compared them with those by the original Cremer model. We introduced the variant-weighting factor method, that was responsive to traffic states. In many examples, the method consider ably decreased the RMSE of the flow rate and the time mean speed.

Appendix

a) Kalman Filter Technique

Choosing $c_i(k)$ and $v_i(k)$ as the state variable vector x_k, and as the observation variable vector y_k, we defined the following Kalman filter (KF)[2]:

$$x_{k+1} = f(x_k) + \xi_k \qquad (a.1)$$

$$y_k = g(x_k) + \zeta_k \qquad (a.2)$$

We linearize these equations as following:

$$\Delta x_{k+1} = \Phi_k \Delta x_k + \xi_k \qquad (a.3)$$

$$\Delta y_k = \Psi_k \Delta x_k + \zeta_k \qquad (a.4)$$

where Δ *is* the difference of vectors, ξ_k and ζ_k are the noise vectors. $\Phi_k = \partial f / \partial x$ is the state matrix and $\Psi_k = \partial g / \partial x$ is the observation matrix. Calculating Φ_k and Ψ_k step by step, we can correct the state variables every time we obtain the newly observed data y_k: and y_k, and $\hat{x}_k$ as the filtered estimate of x_k. K_k is Kalman gain matrix.

$$\hat{x}_k = \tilde{x}_k + K_k(y_k - \tilde{y}_k) \qquad (a.5)$$

where $\tilde{x}_k = f(\hat{x}_{k-1})$ and $\tilde{y}_k = g(\tilde{x}_k)$. The vector $\tilde{x}_k$ and $\tilde{y}_k$ are referred to as the one-step predictor of x_k

b) Sensitivity with Respect to Parameters Change

Cremer and Papageorgiou presented a method for estimating of the model parameters. In addition, the sensitivity of the model with respect to parameter changes and structural changes was investigated. The results are listed when each parameter is changed by ±5% successively as presented Table a.1.

Table a.1 Sensitivity index (value of parameter ± 5%) for small parameter changes.

parameter	-5%	+5%
v_f	9.0%	3.0%
c_{max}	4.0%	1.0%
l-1	6.0%	1.0%
$1/(1-m)$	0.0%	2.5%
α	1.0%	0.0%
κ	1.0%	0.5%
υ	0.6%	0.0%
τ	0.2%	0.3%

c) Box's Complex Algorithm

The constrained Complex (Simple) model for finding the maximum (or minimum) general non-linear (or linear) value of a function

$$\bar{Y} = F(\bar{X}, X_1, \cdots, X_I \cdots, X_m) \qquad (a.6)$$

where $\bar{X}$ = *control variables*

$\bar{Y}$ = *states variables*

X_I = *unknown variables*

(I = 1, 2, ……, M)

By obtaining sets of observed data $(\bar{X}_K, \bar{Y}_K)$ *(k=1, 2, …,N)*, one can identify the variables by a regression technique. Since Eq.(a.7) is a in non-linear form and is subject to some constraints given by Eq.(a.8), the problem here reduces to a linear constrained least mean square problem. The known variables are estimated so as to minimize the objective function J as follows:

$$J = \sum_{K=1}^{N} \left[\bar{Y}_K - F(\bar{X}_K - X_1, \cdots\cdots, X_M) \right]^2 \qquad (a.7)$$

subject to M constraints of the form

$$G_K \le \bar{X}_K \le H_K \qquad (a.8)$$

$$K = 1, \ldots, N($$

where $X_{M+1}, \cdots\cdots, X_N$ are functions of $X_1, \cdots\cdots, X_M$ and the lower and upper constraints G_K and H_K are either constants or functions of $X_1, \cdots\cdots, X_M$. (To find a minimum, $-F$ is maximized.) It is assumes that an initial point $X_1^0, \cdots\cdots, X_M^0$, which satisfies all the M constraints is available.

References

[1] Lighthill J. J., Whitham G. B.: *On Kinematic Waves II: A Theory of Traffic Flow on long crowded road*, Proc. R. Soc. Lond. Ser. A299, pp.317-345, 1955.

[2] Pipes L. A.: *An operational Analysis of Traffic Dynamics*, Journal of Applied Physics, Vol. 24, No. 3, pp. 274-287, 1953.

[3] Payne H. J.: *Model of Freeway Traffic and Control, Simulation Councils Proceeding Series*, Vol. I, No. I, Mathematical Model of Public System, pp.51-61, 1971.

[4] Phillops W. F.: *Kinetic Model for Traffic Flow*, Pub. No. DOT/RSPD/DPB/50-77-17, U.S. Dept. of Transportation, Washington D.C., 1978.

[5] Wardrop J. G. : Some Theoretical Aspects of Road Traffic Research, Proc. Instn. Civ. Engrs., pt. II, 1, 325-365, 1952.

[6] Lebacque J. P. : *Simulation Semi-macroscopic des reseaux Urbains.* Internal Report, INETS, Arcueil, 1983.

[7] Cremer M.: *Der Verkehrsfluβ auf Schnellstraβen*, Springer-Verlag Berlin Heidelberg New York, 1979.

[8] Cremer, M. Papageorgiou M.: para*rameter Identification for A Traffic Flow Model*, Automatica, Vol. 17, No. 6, pp.837-843, 1981.

[9] Sorenson H. W.: *Kalman Filtering Theory and Application*, IEEE PRESS. New York, 1986.

[10] Kuester L. J., Mize H. J.: *Optimization Techniques with FORTRAN*, McGraw-Hill, pp. 368-385, 1973.

[11] Propageorgiou M.: *Application of Automatic Control Concept to Traffic Flow Modelling and Control*, Springer, Berlin, 1983.

[12] May A. D., Keller H. E. M.: *Non-linear Car-Following models*, Highway Research Board Record 199, HRB, Washington D. C., pp. 19-32, 1967.