I − 3　ULTIMATE MOMENT CAPACITY AND INITIAL STIFFNESS OF EXTENDED END-PLATE CONNECTIONS

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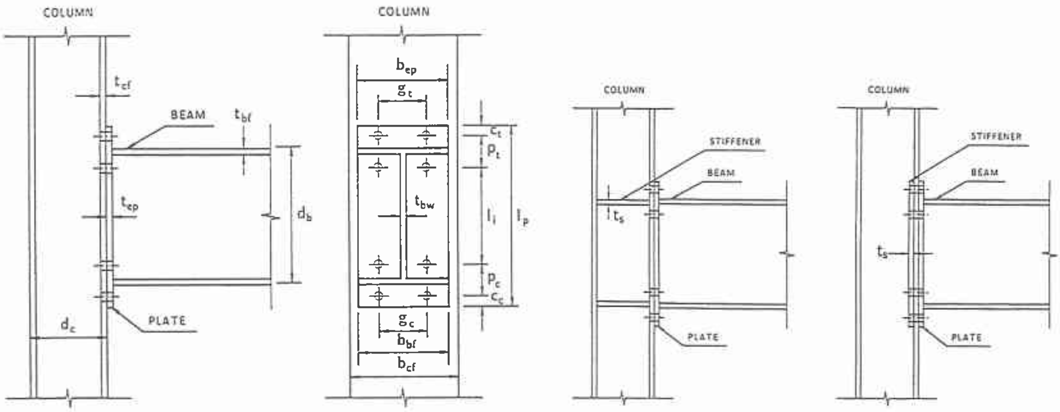
ABSTRACT

For evaluating moment-rotation characteristics of extended end-plate connections, the derivations of ultimate moment capacity and initial connection stiffness are of paramount importance. In this study, from a thorough review, a simplified methodology has been presented.



(a) unstiffened　(b) transversely stiffened　(c) longitudinally stiffened

Fig. 1 General configuration of extended end-plate connections

1. INTRODUCTION

The simplest form of extended end-plate connections consists of a steel plate profile welded to the beam end, bolted to the column flange and extended beyond the beam flange (Fig. 1a). For these connections, excessive column flange deformations often force the designers to stiffen the column flange either by transverse stiffener (Fig. 1b), or by longitudinal stiffener (Fig. 1c). The ultimate moment capacity and initial stiffness of both unstiffened and stiffened connections will be studied here.

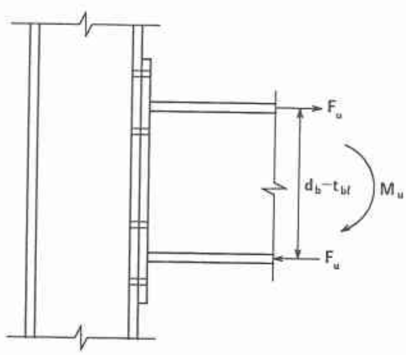


Fig. 2 Ultimate moment capacity

2. THE ULTIMATE MOMENT CAPACITY

The moment applied to a beam-to-column connection can be calculated from the idealization that an internal couple consisting of two flange forces F_u with a moment arm of $(d_b - t_{bf})$ equals the external moment M_u (Fig. 2), where d_b is beam depth and t_{bf} is beam flange thickness. The flange force playing the main role in connection failure is actually dependent on the mode of failure. The common failure modes can be listed as: (1) bolt failure, (2) column flange failure, and (3) end-plate failure. The minimum flange force among these failure modes will obviously be the governing one for the moment capacity determination. Thus, the ultimate moment capacity will be,

$$M_u = F_u(\text{min.}) \times (d_b - t_{bf}) \tag{1}$$

2.1. BOLT FAILURE

In this failure mode, the bolt elongation reaches to its yield point while end-plate and column flange are assumed to be comparatively stiff and their stress level remain far away from the yield stress. Stress equality for a four bolts tension cluster equally spaced on the top and below the beam flange will be a reasonable assumption.

On the other hand, in transferring the tensile flange force to the bolts end-plate deformation produces a bolt tension increment equals to q as shown in Fig. 8. This increment is commonly known as 'prying force'. Chasten, C.P. et al. (1992) assumed a distributed prying force on the projected end of end-plate and employed FEM to calculate it. Thus expression for flange force has two components, (i) contribution of bolt force and (ii) contribution of prying force.

$$F_u = 4A_{bo}\sigma_{yb} - \frac{b_{ep}t_{ep}^2\sigma_{yep}}{2.4c_t} \tag{2}$$

where A_{bo} is the cross-sectional area of the bolt shank, σ_{yb}, σ_{yep} are the yield stresses of the bolt and end-plate material respectively, and b_{ep}, t_{ep} and c_t are defined in Figs. 1.

2.2. END-PLATE FAILURE

The end-plate transfers the beam flange force to the column flange through fasteners. When the end-plate is not proportionated enough in comparison with other components of connection, the connection is expected to fail due to rupture of the end-plate. Surtees and Mann (1970) employed a linear yield line pattern (Fig. 3) and derived the following equation for flange force responsible for the end-plate failure.

$$F_u = \sigma_{yep}t_{ep}^2\left[\frac{2b_{ep}}{(p_t - t_{bf})} + \frac{1.2(d_b - t_{bf})}{(g_t - t_{bw})}\right] \tag{3}$$

where d_b is the bolt diameter, and p_t, t_{bf}, g_t and t_{bw} are defined in Figs. 1.

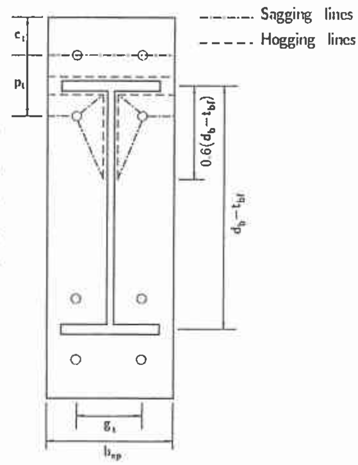


Fig. 3 End-plate yield line pattern

2.3. COLUMN FLANGE FAILURE

If the column flange is not stiffened, usually, it is very much susceptible to be deformed in the tension zone. To prevent this type of failure, column flange is sometimes strengthened by an addition of either longitudinal or transverse stiffener. Therefore both stiffened and unstiffened column flange failures will be studied here. Moreover, column flange failure can be associated with the simultaneous failure of bolt and/or end-plate. Four types of failure mechanisms are thus needed to be considered: (1) Mechanism A: simultaneous yielding of column flange and bolts (Fig 4a), (2) Mechanism B: yielding of column flange (Fig. 4b), (3) Mechanism C: simultaneous yielding of column flange, end-plate and bolts (Fig. 4c), and (4) Mechanism D: simultaneous yielding of column flange and end-plate (Fig 4d). Previous researchers offered formulae to calculate beam flange force for these four failure patterns which are listed in Tables 1 and 2. The parameters used in these tables are shown in Figs. 1, 4 and 5. All these formulae were developed by the analyses of column flange failure mechanisms by assuming some curved yield boundaries. Fig. 5 shows an example of yield line pattern of an unstiffened column flange.

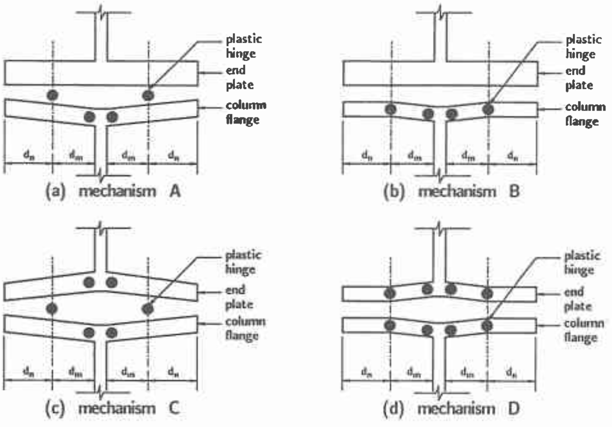


Fig. 4 Collapse mechanisms associated with column flange

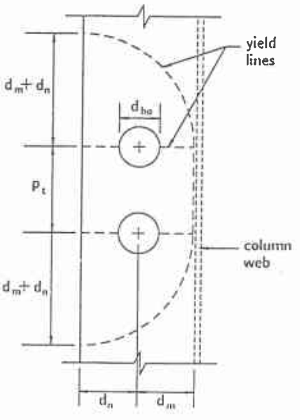


Fig. 5 Yield line pattern for mechanism A

Table 1 Flange Force for Column Flange Failure (Stiffened)

Reference	Expressions for flange force
Stiffened with transverse stiffener	
Packer, J.A. et al. (1977)	$F_u = \sigma_{ycf}t_{cf}^2\left\{\left(\frac{2}{p_t - t_s} + \frac{1}{w}\right)(2d_m + 2d_n - d_{ho})\right\} + \sigma_{ycf}t_{cf}^2\left\{\frac{p_t - t_s + 2w - d_{ho}}{d_m}\right\}$ $w = \sqrt{d_m(d_m + d_n - 0.5d_{ho})}$ (Mechanism B)
Stiffened with longitudinal stiffener	
Moore, D.B. et al. (1986)	$F_u = \sigma_{ycf}t_{cf}^2\left\{\frac{p_t + 2w}{2(d_m + d_n)} + \frac{2(d_m + d_n)}{w}\right\} + \frac{4A_{bo}\sigma_{yb}d_n}{(d_m + d_n)} + \frac{\sigma_{ys}t_s^2(d_m + d_n)}{w}$ $w = (d_m + d_n)\left[2 + \frac{\sigma_{ys}t_s^2}{\sigma_{ycf}t_{cf}^2}\right]^{0.5}$ (Mechanism A)
	$F_u = \sigma_{ycf}t_{cf}^2\left\{\pi + \frac{p_t + 2d_n - d_{ho}}{d_m}\right\} + \sigma_{ys}t_s^2\left\{\frac{p_t + 2d_n - 4d_{ho}}{2d_m} + 2\right\}$ (Mechanism B)

Table 2 Flange Force for Column Flange Failure (Unstiffened)

Reference	Expressions for flange force
Packer, J.A. et al. (1977)	$F_u = \sigma_{ycf} t_{cf}^2 \left\{ \pi + \frac{0.5 p_t}{(d_m + d_n)} \right\} + \frac{4 A_{bo} \sigma_{yb} d_n}{(d_m + d_n)}$ (Mechanism A)
	$F_u = \sigma_{ycf} t_{cf}^2 \left\{ \pi + \frac{2 d_n + p_t - d_{ho}}{d_m} \right\}$ (Mechanism B)
Witteveen, J. et al. (1982)	$F_u = \frac{\sigma_{ycf} t_{cf}^2 b_m + 8 A_{bo} \sigma_{yb} d_n}{2(d_m + d_n)}$ $b_m = 8d_m + 2.5d_n$ if $p_t > 4d_m + 1.25d_n$ $b_m = p_t + 4d_m + 1.25d_n$ if $p_t \leq 4d_m + 1.25d_n$ (Mechanism C)
	$F_u = \frac{\sigma_{ycf} t_{cf}^2 b_m}{d_m}$ $b_m = 8d_m + 2.5d_n$ if $p_t > 4d_m + 1.25d_n$ $b_m = p_t + 4d_m + 1.25d_n$ if $p_t \leq 4d_m + 1.25d_n$ (Mechanism D)

3. THE INITIAL STIFFNESS

The initial stiffness is the linear relationship between moment and connection rotation based on elastic response to loading. For calculation of initial stiffness, the principle followed by Yee and Melchers (1986) will be applied here with some modifications. The initial stiffness can be expressed as:

$$R_{ki} = \frac{M_i}{\theta_i} = \frac{F_i \, (d_b - t_{bf})}{\theta_i} \qquad (4)$$

in which M_i is initial moment, F_i is flange force and θ_i is initial rotation. The initial rotation can be related to the deformation of the beam flange as (Fig. 6):

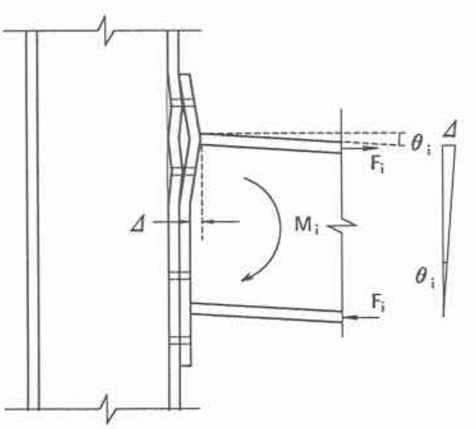


Fig. 6 Initial deformation of connection

$$\theta_i = \frac{\Delta}{d_b - t_{bf}} = \frac{\Delta_{ep} + \Delta_{cf} + \Delta_{bo}}{d_b - t_{bf}} \qquad (5)$$

where Δ = total deformation occurred at the connection, Δ_{ep} = deformation due end-plate flexure, Δ_{cf} = deformation due column flange flexure and Δ_{bo} = bolt elongation.

Therefore the initial stiffness can be written as:

$$R_{ki} = \frac{F_i \, (d_b - t_{bf})^2}{\Delta_{ep} + \Delta_{cf} + \Delta_{bo}} \qquad (6)$$

3.1. DEFORMATION DUE TO END-PLATE AND COLUMN FLEXURE

The elastic deformation of the end-plate at the tension flange of beam Δ_{ep} is obtained by a T-stub idealization in which two T-stubs, as shown in Fig. 7, are bolted to each other and the flange ends of the T-stub are being discontinuous. A simply supported T-stub flexural model is then idealized (Fig. 8), in which, the flange force F_i is being resisted by the bolt forces F_{bo} and the prying forces q become the reactions at the supports. The deformation at the tension flange is then determined from simple bending theory as:

$$\Delta_{ep} = \frac{F_i \, Z_{ep}}{E} \left[\frac{1}{8} - \frac{q_s}{2} \left(\frac{3}{4} \alpha_{ep} - \alpha_{ep}^{3} \right) \right] \tag{7}$$

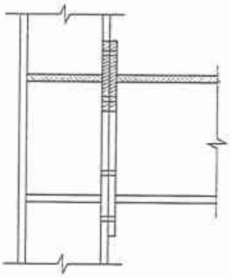


Fig. 7 The T-stub idealization

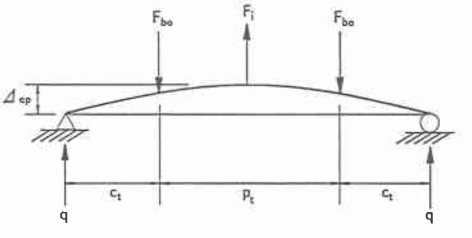


Fig. 8 Simply supported model for end-plate flexure

Similar T-stub idealization is applied for calculation of column flange deformation. In case of transversely stiffened connection, the web of the T-stub consists of the stiffener while in case of unstiffened and longitudinally stiffened connection, the web of the column itself constitutes the web of the T-stub.

$$\Delta_{cf} = \frac{F_i \, Z_{cf}}{E} \left[\frac{1}{8} - \frac{q_s}{2} \left(\frac{3}{4} \alpha_{cf} - \alpha_{cf}^{3} \right) \right] \tag{8}$$

where

$$q_s = \frac{Z_{ep}\, \alpha_{ep1} + Z_{cf}\, \alpha_{cf1}}{Z_{ep}\, \alpha_{ep2} + Z_{cf}\, \alpha_{cf2} + K/A_s} \tag{9}$$

$$\alpha_{ep1} = \frac{3\alpha_{ep}}{2} - 2\alpha_{ep}^{3} \tag{10}$$

$$\alpha_{ep2} = 6\alpha_{ep}^{2} - 8\alpha_{ep}^{3} \tag{11}$$

$$\alpha_{cf1} = \frac{3\alpha_{cf}}{2} - 2\alpha_{cf}^{3} \tag{12}$$

$$\alpha_{cf2} = 6\alpha_{cf}^{2} - 8\alpha_{cf}^{3} \tag{13}$$

The constant K used in equation (9) is originally expressed as a function of shaft, nut, thread and washer length of a bolt (Agerskov, H (1976)). Usually these kinds of minute details are not available in the reported experimental investigations. For the facilitation of computer implementation, this constant is expressed here as a function of grip of the bolt (Table 3).

For unstiffened and/or transversely stiffened connections:

$$\text{grip} = t_{ep} + t_{cf} \tag{14}$$

For longitudinally stiffened connections:

$$\text{grip} = t_{ep} + t_{cf} + t_s \qquad (15)$$

in which t_s is the thickness of stiffener.

Moreover, Yee and Melchers (1986) proposed two different expressions for K to account for snug tightened and pretensioned fabrication. Among the three components in the denominator of the right hand part of equation (9), the numerical value of the third part (K/A_s) is very low. In other way to say, pretensioning has little effect on the initial stiffness of a connection. Therefore, in this study, regardless of bolt tensioning method, only one expression for K is proposed.

The functions Z_{ep}, Z_{cf}, α_{ep}, α_{cf} and K, expressed in terms of different connection parameters, are listed in Table 3. All the parameters used in this table are described in Figs. 1.

Table 3 Expression for Z_{ep}, Z_{cf}, α_{ep}, α_{cf} and K

Type	Z_{ep}	Z_{cf}	α_{ep}	α_{cf}	K
Stiffened	$Z_{ep} = \frac{2\,(2c_t+p_t)^3}{b_{ep}\,t_{ep}^3}$	$Z_{cf} = \frac{(2c_t+p_t)^3}{b_{cf}\,t_{cf}^3}$	$\alpha_{ep} = \frac{c_t}{l_{ep}}$	$\alpha_{cf} = \frac{c_t}{l_{cf}}$	$1.25 \times \text{grip}$
Unstiffened	$Z_{ep} = \frac{2\,(2c_t+p_t)^3}{b_{ep}\,t_{ep}^3}$	$Z_{cf} = \frac{2b_{cf}^3}{(2c_t+p_t)\,t_{cf}^3}$	$\alpha_{ep} = \frac{c_t}{l_{ep}}$	$\alpha_{cf} = \frac{b_{cf} - gt}{2b_{cf}}$	

3.2. DEFORMATION DUE TO BOLT ELONGATION

In the calculation of Δ_{ep} and Δ_{cf}, the bolt elongation has already been taken into account. Hence separate calculation will not be needed here.

4. CONCLUSION

A simplified approach is proposed here to evaluate the ultimate moment capacity and the initial connection stiffness of extended end-plate connections. Preliminary comparison with some experimental investigations showed a satisfactory agreement. Full justification of the proposed approach needs a comprehensive comparison. Once this has been established, formulation of an analytical moment-rotation model would be facilitated. Study is being carried on with this eventual goal.

5. REFERENCES

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