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Runoff Analysis Based on Channel Networks: A Numerical Study

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Abstract

In the basin area, channel networks (that assumed by some scientists as a trivalent planted tree) have magnitudes, tributaries, exterior and interior links, junctions, and the outlet. This paper demonstrades the calculation of the dischrge in every subbasin, Q_s, and the total discharge in the outlet, Q_t, by using the numerical method, for 100 km^2 the total area of the catchment, 10 $m^3s^{-1}km^{-2}$ specific discharge, and 1 and 2 m/s flow velocities. According to the simulation results that the peak of total discharges are different for 4<= N =<30, yet the peak of total discharges are similar for 30< N =<100. In this paper, and then, we analyze linear correlation between the average probability of subbasins and network magnitude, N. The results show that the simulated computation are similar with the theoretical calculation, for the network magnitude 4 <= N =< 200.

Basic Concepts and Definitions

According to some scientists [Shreve, 1974; Choi and Molinas, 1990; and Tarboton et.al, 1991] channel networks is similar with trivalent planted tree. The channel networks have sources which called magnitudes, tributaries, exterior and interior links, junctions or nodes, and an outlet(see fig. 1).

We call magnitudes for the farthest upstream of points, exterior links for the segments of channel between a source and the first junction downstream; meanwhile the segments of channel between two successive junctions or a junction and the model outlet are called interior links.

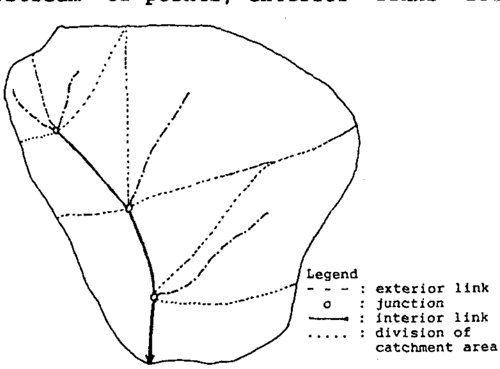

Figure 1. The model of channel networks in a catchment area [Boyd, 1982].

Channel Network Characteristics

In this method, for simplicity, we assume that not only catchment area have homogeneous porosity and density, but also exterior and interior link lengths,

and the average assodiated area of both links are independent and identically distributed in the channel networks with uniform environments [Hayakawa and Fujita, 1990]. We, thus, may develop Fujita (1982) theory. Fig. 2a shows as a simple network pattern in network magnitude N. From the figure, then, we can derive eq.(1) as follows,

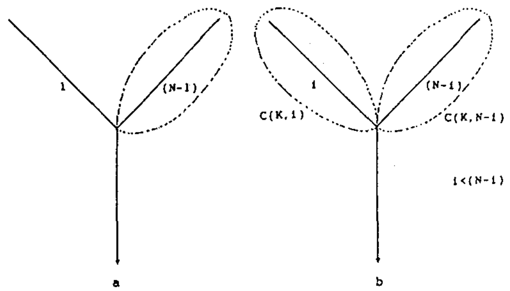

Figure 2. The simple network pattern for network magnitude N.

$$P(1,N)=\frac{N}{(2N-1)} \quad (1)$$

for $4<=N<=200$

where P(1,N) is occorence probability of network pattern, and (2N-1) is total number of subbasins.

For subbasins 2<=i<=[N/2], i.e. [ ] is Gauss notation, eq.(1) may write such as

$$P(i,N)=\frac{2(2i-3)!N!(N-2)!(2N-2i-2)!}{(i-2)!i!(N-i)!(2N-3)!} \quad (2)$$

N = odd number

and

$$P(m,2m)=\frac{8m}{(4m-2)!}\left\{\frac{(2m-1)!(2m-3)!}{m!(m-2)!}\right\}^2 \quad (3)$$

for N=2m, or N = even number

in which P(i,N) and p(m,2m) are occorence probability of network patterns for odd and even number N in the upstream subbasins of the outlet link have magnitude i and N-i, respectively, for network magnitude N (see fig. 2b).

Based on the fig. 2b, we define average probability of subbasins C(K,N), and mean probability of tributary, D(N). Both of them are expressed respectively as below,

$$C(K,N)=\sum_{i=1}^{[N/2]} P(i,N)\{C(k-1,i)+C(k-1,N-i)\} \quad (4)$$

$$D(N)=\sum_{i=1}^{[N/2]} P(i,N)\{D(N-1)+1\} \quad (5)$$

in which C(K-1,i) and C(K-1,N-i) are average probability of subbasins in subnetworks i and N-i, respectively; while {D(N-i)+1 is mean probability of a tributary in subbasin N-i.

For checking the result of C(K,N) at eq.(4), theoretically we may apply eq.(6) as follows

$$2N-1=\sum_{K=0}^{N-1} C(K,N) \quad (6)$$

Now let us see fig. 3 is the model of schematic channel networks for the network magnitude N=4. The figures show that 4/5 and 1/5 are the occurence probabilities for network patterns of fig. 3a and fig. 3b respectively.

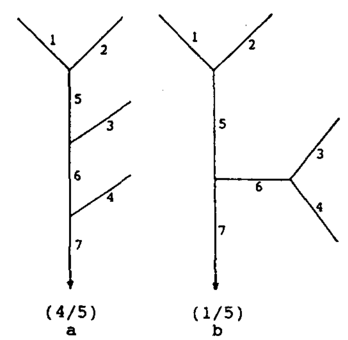

Figure 3. Schematic channel network for network magnitude N.

In the fig. 3a, the position of the subbasins 1 and 2 are at three links distance, subbasins 3 and 5 are at two links distance, and subbasins 4 and 6 are at one link distance from the outlet link 0. In the fig. 3b, whereas, the posotion of subbasins 1, 2, 3,and 4 are at two links distance, and subbasins 5 and 6 are at one link distance from the outlet link 0.

We, furthermore, may calculate the average probabilities of basins in the fig. 3, i.e.,

C(0,4) = {1(4/5)+1(1/5)} = 1
C(1,4) = {2(4/5)+2(1/5)} = 2
C(2,4) = {2(4/5)+4(1/5)} = 2.4
C(3,4) = {2(4/5)+0(1/5)} = 1.6

Transportation Process

Shreve (1974) supposed that mainstream length in the channel networks varies statistically in proportion to catchment area that a power decreases from about 0.6 for small to medium catchment areas (1 - 10^3 km^2) to near 0.5 for the largest in the world (nearly 10^7 km^2).

In this study, we use small catchment area for measuring mainstream length, L_m; so it is expressed theoritically, such as

$$L_m = HA_t^{0.6} \qquad (7)$$

where H is Hack`s constant, and the value of H depends on the map`s scale, here we use H=1.273 (for map scale 1:40,000). A_t is total catchment area.

In the simulation method, the mainstream length L_m is dividedby the tributaries, D(N), to become average distances, L_a. In this case we find the total number of tributaries in the mainstream, {D(N)+1}, see fig. 4.

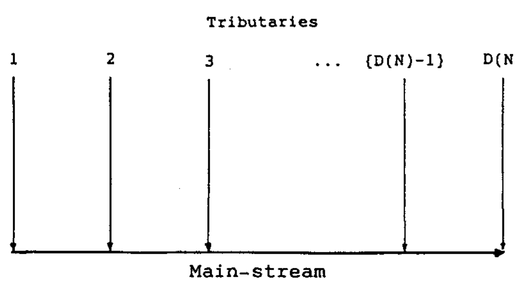

Figure 4. Mainstream length and one direction tributaries.

It, matematically, can be written as

$$L_m = \{D(N)+1\}L_a \qquad (8)$$

and then we may find mean area of tributaries or average associated area of links length or subbasins, A_a, in eq.(9).

$$A_t = (2N-1)A_a \qquad (9)$$

in which (2N-1) is total number of subbasins in the channel networks with network magnitude N.

When we substitute eq. (7) to eq.(8), we may find

$$L_a = \frac{1.273A_t^{0.6}}{\{D(N)+1\}} \qquad (10)$$

in which L_a is average length of mainstream. We, furthermore, can calculate the discharge at every junction, Q_s, and the total discharge at the outlet, Q_t, as expressed respectively in eq. (11) and eq.(12),

$$Q_s = A_a q_u(t-TP)\,C(K,N) \qquad (11)$$

$$Q_t = A_a \sum_{K=0}^{N-1} q_u(t-TP)\,C(K,N) \qquad (12)$$

where

- A_a : average associated area of links (km^2)
- q_u : specific discharge of subbasins (mm $hr^{-1}km^{-2}$)
- t : time (hour)
- K : position number of subbasins
- v : velocity of flow (m s^{-1})
- TP : propagation time (hour)

In the discharge calculation, we apply specific discharge, q_u, as a function of time, t. There are two kind of speci-

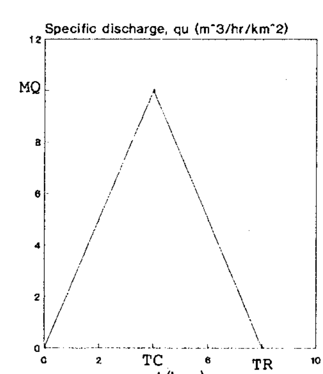

Figure 5. Relationship between specific discharge, qu and time, t.

fic discharge, and then the starting points of the q_u moves according to K value. For more detail let us see fig.5 Specific discharge - time relationship.

Gupta et.al (1982), theoritically, demonstrated to analyze subbasin discharges, Q_s, and total discharge, Q_t, for network magnitude N = 4. In the fig. 6 we can see that the starting point for subbasin discharge: C(0,4) = 1, is zero; C(1,4) = 2, is one; C(2,4) = 2.4, is three; and C(3,4) = 1.6, is four. In this case we use integer numbers for deciding starting points of subbasin discharges.

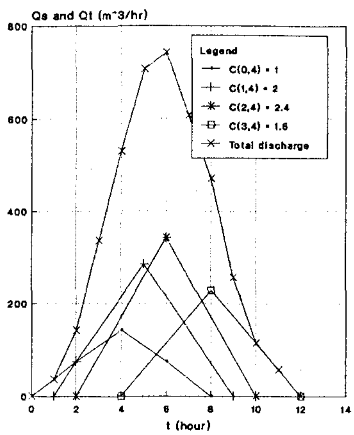

Figure 6. Analysis of subbasin discharges, Q_s, and total discharge, Q_t, function of of time, t. [Boyd, 1982].

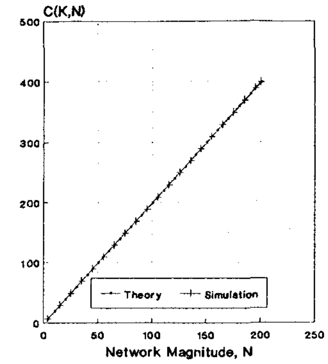

Figure 7. Correlation between average probability of subbasins, C(K,N), and network magnitude, N.

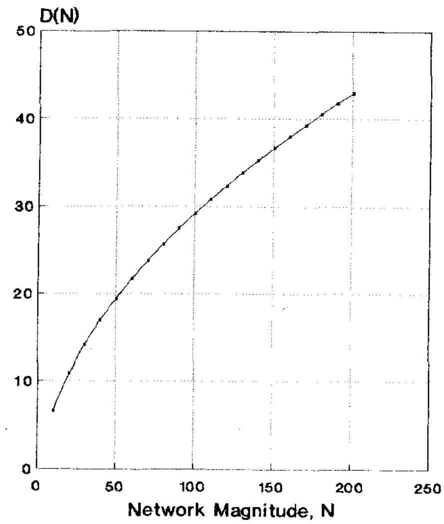

Figure 8. Relationship between mean probability of tributaries, D(N), and network magnitude, N.

Results and Discussions

First of all, we analyze the correlation between the average probability of subbasins, C(K,N), and the network magnitude, N, from eq.(4). The calculation results show that the average probability of subbasins is linear with the network magnitude. Fig. 7 shows that the simulation results is almost the same as theory, i.e., according to eq. (6).

Meanwhile, relationship between the mean probability of tributaries,D(N), and the network magnitude is not linear (see fig.8). However, if the network magnitude increases, the mean probability of tributaries increases too.

We, furthermore, calculate the subbasins discharge, Q_s, and the total discharge, Q_t, for network magnitude N=4, by using eq.(11) and eq.(12) respectively. In this calculation, we use simulation data, i.e, A_t=100 km^2, MQ=10 $m^3hr^{-1}km^{-2}$, v=1 m/s, TC=4 hr, and TR=8 hr.

The results show that there are four subbasin discharges. According to fig. 9 the subbasin discharge for C(0,4) only is the same as Gupta (1982) analysis, yet the another subbasin discharges are different. Although both of the total discharges are the same.

We, and then, analyze the total discharges for N = 10, 20, 30, ... 100 by

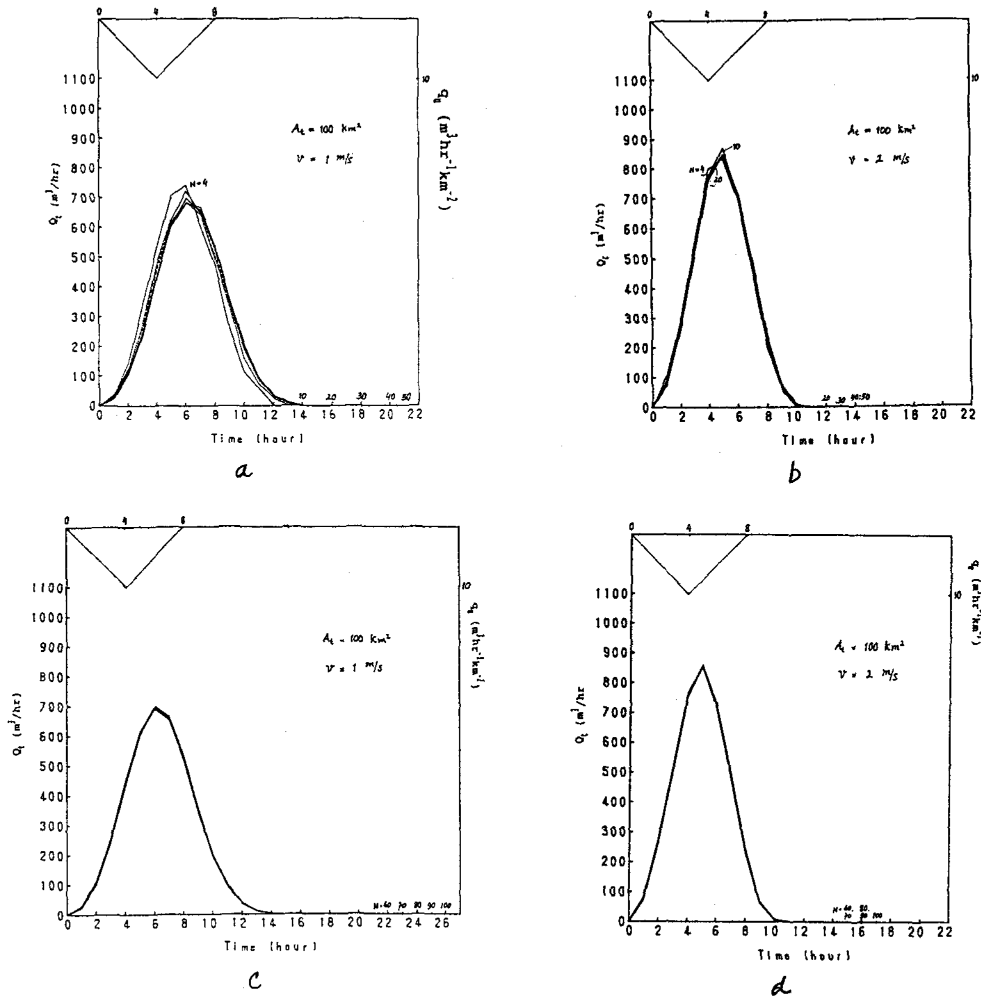

Figure 10. The total discharge, Q_t, function of time, t.

using variation of flow velocities 1 and 2 m/s. We find the results as shown in fig. 10a and fig. 10b. From the both figu res show that the peak of discharges for v=1 m/s are higher than the peak of discharges for v=2 m/s. However, all of the total discharges, for both v=1 m/s and v=2 m/s, are almost the same.

Of course, in this case the discharge for each subbasin is different one and anothers. In adddition, let us see fig.10c and fig. 10d. The total discharge - time relationship. It means we may differ among the total discharges for N = 4, 10, 20, and 30 in fig.10c. On the other hand, regarding fig.13 the total discharges for N = 40, 50, 60, ... 100, it is difficult to differ all of them, because they look like very similar.

Conclusions

There are some points for concluding this simulation method. Firstly, according to calculation mean probability of subbasin, C(K,N), is linear with increasing of network magnitude, N; and generally, all of the results are the same as theory (as shown in fig. 7). And then, we can say that correlation between mean probability of tributaries, D(N), and network magnitude is linear (eventhough it is not perfectly linear); in this case if we use line for drawing the grapic. While in fig. 8 it seems not linear, because we use curve to correlate one and another points.

Finally, in the same condition, the total discharges for 4<= N =<30 is different one and another; but they are similar for 30< N =<100, eventhough the sub basin discharges is different. It means that the subbasin discharges increase according to network magnitude, N. In another word, the calculation of total discharges are more detail for the network magnitude bigger and bigger (such as shown in fig. 10).

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