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STORAGE FUNCTION MODEL BASED ON INFILTRATION THEORY

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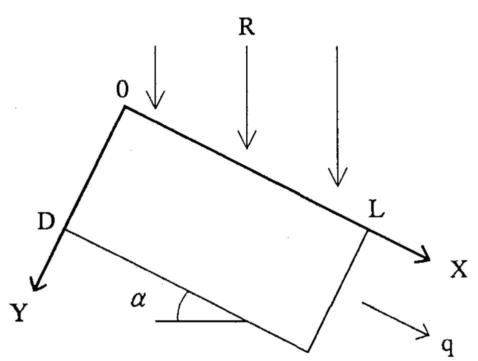
INTRODUCTION:

The storage of rainfall through the soil has a direct relationship with discharge (runoff). In this paper a process has been proposed by the numerical solution of infiltration flow which can be concluded into the storage function model. Basically, the phenomena of infiltration of rainfall through the soil particles can be described by well-known Richard's equation which can be adopted as a continuity equation. The solution of Richard's equation by difference method has been concentrated on an inclined rectangular soil column with the uniform and non-uniform rainfall on its surface as it is illustrated in Figure (1). Obviously, the storage of rainfall through the soil is directly effected with the soil's physical properties . Therefore, the effort has been made to include the physical properties of soil to develop the storage function model. For this reason, by integration of continuity equation through the depth of the soil column, it can be converted to the semi concentrated equation. And by further integration of semi concentrated equation through the length of the soil column, concentrated or storage function model has been achieved. However, the parameters of storage function model have been calculated through the steady state of storage-discharge relationship under the different rainfall and initial conditions. Finally, the validity of storage function model has been compared graphically.



Fiqure (1): Profile of soil column

STATEMENT OF THE PROBLEM:

The infiltration of rainfall into the soil particles can be expressed by Richard's equation in its two dimensional form as following:

$$\frac{\partial\theta}{\partial t} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) \tag{1}$$

Where (θ) is moisture content in the soil, (t) is time, (V_x) and (V_y) are velocities of water through the soil particles in (X) and (Y) directions. These velocities can be derived by Darcy's equation which are adopted as Momentum equations.

$$v_x = -k.\left(\frac{\partial\psi}{\partial x} - \sin\alpha\right) \tag{2}$$

$$v_y = -k.\left(\frac{\partial\psi}{\partial y} - \cos\alpha\right) \tag{3}$$

(Ψ) is suction in the soil, (α) is the angle of soil column with horizon and (K) is conductivity of soil which can be written as following:

$$k = k_s.\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{\beta} \tag{4}$$

Where (K_s) is saturated conductivity of the soil, (β) is depended on soil property, (θ_s) and (θ_s) are maximum (saturated) and minimum water content in the soil. The equation of water content can be derived by HaverKamp's equation.

$$\theta = \frac{a^2}{a^2+\psi^2}(\theta_s-\theta_r)+\theta_r \tag{5}$$

(a) is depended on soil property. For the purpose of solution of Richard's equation with the respect to momentum equations, following initial and boundary conditions have been considered.

$$\psi = (x-L).\sin\alpha + (y-D).\cos\alpha + c \qquad (6)$$

$$y=0 \qquad v_y = R.\cos\alpha \qquad (7)$$

$$y=D \qquad v_y = 0 \qquad (8)$$

$$x=0 \qquad v_x = 0 \qquad (9)$$

$$x=L \qquad \frac{\partial^2 \psi}{\partial x^2} = 0 \qquad (10)$$

By integration of Equation (1) along (Y), it can be converted to semi-concentrated equation.

$$\int_0^D \frac{\partial \theta}{\partial t}.dy = -\int_0^D \frac{\partial v_x}{\partial x}.dy - \int_0^D \frac{\partial v_y}{\partial y}.dy \qquad (11)$$

Clearly, each part of integration can be explained as following with the respect to boundary condition along (Y).

$$\frac{\partial}{\partial t}\int_0^D \theta.dy \qquad \text{where} \qquad \bar{\theta} = \frac{1}{D}\int_0^D \theta.dy \qquad (12)$$

$$\frac{\partial}{\partial t}\int_0^D v_x.dy \qquad \text{where} \qquad q(x,t) = \int_0^D v_x.dy \qquad (13)$$

$$\int_0^D \frac{\partial v_y}{\partial y}.dy \qquad \text{where} \qquad v_{y=D} - v_{y=0} = 0 - R.\cos\alpha \qquad (14)$$

Therefore Equation (11) which is a semi-concentrated equation can be written as:

$$D.\frac{\partial\bar{\theta}}{\partial t}+\frac{\partial q(x,t)}{\partial x}=R.\cos\alpha \tag{15}$$

By replacing Ψ's value from Equation (5) into Equation (2) and integration along (Y). The Equation (13) can be written as:

$$q(x,t)=f_1(\bar{\theta})-f_2(\bar{\theta}).\frac{\partial\bar{\theta}}{\partial x} \tag{16}$$

Where

$$f_1(\bar{\theta})=D.\sin\alpha.k_s.\left(\frac{\bar{\theta}-\theta_r}{\theta_s-\theta_r}\right)^{\beta} \tag{17}$$

$$f_2(\bar{\theta})=D.\frac{a}{2}.k_s.(\theta_s-\theta_r)^{1-\beta}.\frac{(\bar{\theta}-\theta_r)^{\beta-1.5}}{(\theta_s-\bar{\theta})^{.5}} \tag{18}$$

Equation (15) has been solved by following initial and boundary conditions.

$$\bar{\theta}_0=\frac{1}{D}\int_0^D\theta_0.dy \tag{19}$$

$$x=0 \qquad q(x,t)=0 \tag{20}$$

$$x=L \qquad \frac{\partial^2\bar{\theta}}{\partial x^2}=0 \tag{21}$$

The relation between storage (S) and discharge (q) can be achieved by further integration of Equation (15) along (X).

$$D.\int_0^L \frac{\partial \overline{\theta}}{\partial t}.dx+\int_0^L \frac{\partial q}{\partial x}.dx=\int_0^L R.\cos\alpha.dx \qquad (22)$$

AS we know

$$\int_0^L \overline{\theta}.dx= L.\hat{\theta}(t) \qquad (23)$$

or

$$S(t) = L.D.\hat{\theta}(t) \qquad (24)$$

Therefore, Equation (22) can be written as following which is a concentrated equation.

$$\frac{ds}{dt}+q_{x=L}= R.L.\cos\alpha \qquad (25)$$

According to unit Equation (25) is in its two dimensional form . In the present calculation it is converted to one dimensional form.

$$\frac{ds}{dt}+q_{x=L}= R.\cos\alpha \qquad (26)$$

Storage has a direct relation with discharge which can be described by the following equation.

$$s= k.q^{p} \qquad (27)$$

By replacing Equation (27) into Equation (26) following relation can be achieved.

$$\frac{dq}{dt}=\frac{R.\cos\alpha-q}{k.p.q^{p-1}} \qquad (28)$$

The Equation (28) has been used to calculate the discharge through the storage function model.

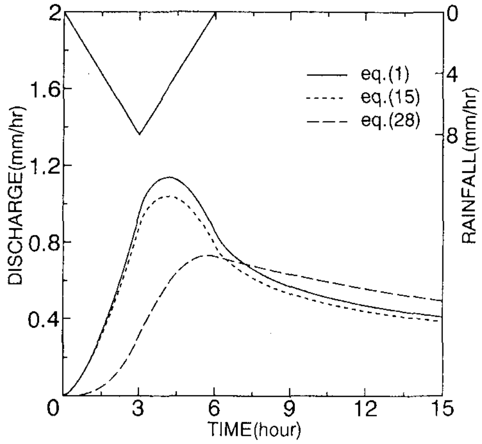


Figure (2):The Relationship Between Nonuniform Rainfall and Discharge with Initial condition C=0

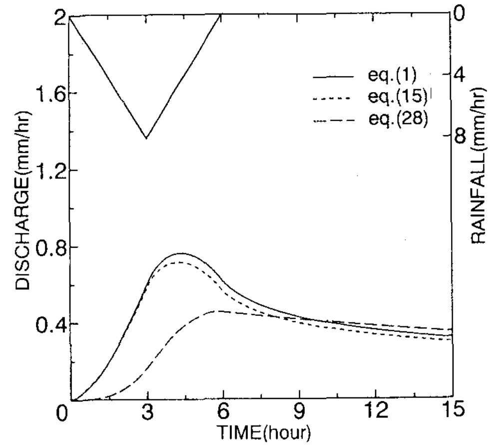


Figure (3):The Relationship Between Nonuniform Rainfall and Discharge with Initial condition C=-30

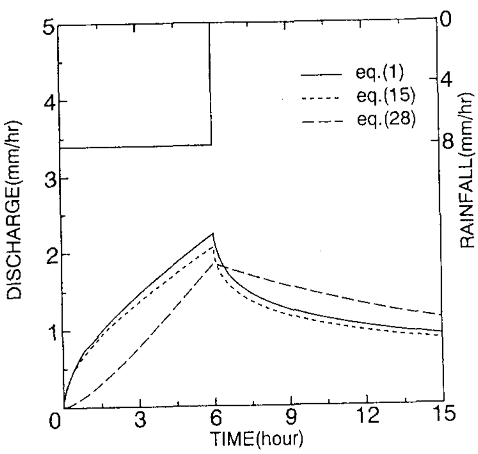


Figure (4):The Relationship Between Uniform Rainfall and Discharge with Initial condition C=0

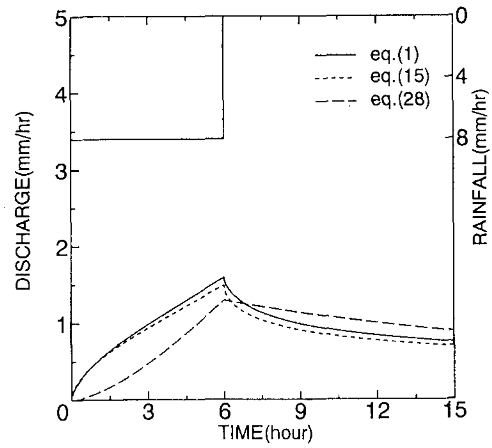


Figure (5):The Relationship Between Uniform Rainfall and Discharge with Initial condition C=-30

CONCLUSION:

The storage function parameters (k and p) have been calculated through the steady state of storage-discharge relationship of the Richard's equation under the different rainfall and initial conditions. This relationship creats a single line which is convenient to calculate the (k) and (p). However, two cases of initial condition (Equation (6)) with C=0 and C=-30 have been considered. For the case C=0, k=27.683, p=0.71956 and C=-30, k=38.0748, p= 0.64353 have been calculated. The Figures (2) and (3) show the results of calculation of Richard's, semi-concentrated and concentrated equations under the non uniform rainfall while the Figures (4) and (5) show the results under the uniform rainfall. As it is evident, the results of concentrated equation are not fitted properly. The (k) and (p) which are calculated through the steady state of storage-discharge relationship can not be adjusted along the rising and recession part of the actual hysterics of storage-discharge relationship. For further study , this defect can be reduced by a method to fit the actual hysterics.