

An interaction model of rock bolt and rock mass in underground support

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On the basis of a refined Shear-lag model, a constitutive equation is proposed to estimate the axial force of rock bolt according to the displacement of surrounding matrix. Coupling and decoupling behaviors of reinforcement in pullout tests, around an underground opening and opening joints are discussed respectively. For pullout test, the distribution of axial force and shear stress is compared with former research and test results, and a back analysis method is suggested to estimate the shear strength of the interface and the ultimate capacity of the rock bolt. As an example, the behavior of rock bolts intersecting multizone around a tunnel is analyzed, and the neutral location of rock bolts is discussed further. For a rock bolt intersecting joints, more than one neutral point may appear according to analysis.

Key Words: Rock bolt, interaction, pull out, tunnel, joint

1. INTRODUCTION

The reinforcements such as rock bolts, anchors, soil nails, grids metal straps and geosynthetics included in geo-matrix, have been widely used in civil and mining engineering. Despite of extensive researches both experimentally and theoretically, only little has been understood on the interaction behaviour of reinforced system on rock/soil at present. Understanding the interaction behaviour is very important because the working status of rock bolt is related to its supporting effect directly. Pull out test is often used to evaluate the strength, integrity and effectiveness of the reinforcement system nowadays, while the pullout test itself remains vagueness sometimes. It is well known that there is a “neutral point”, a “pick-up length” and an anchor length” on the rock bolts in-situ according to field monitoring data ¹⁾. The displacement of rock mass and rock bolt is considered to be same, and shear stress on interface is zero at neutral point while axial load of the reinforcement reaches maximum. Based on the suggested position of neutral point ²⁾, Indraratna ³⁾ established the analytical model of grouted rock bolts. However, the neutral point is not valid at least when decoupling occurs ⁴⁾. According to past numerical studies, interface properties between reinforcement and matrix are focused on. The relationship of shear force τ and displacement u is often simplified as $\tau=ku$, where, k is defined as the stiffness of interface. In order to satisfy the pullout experiment results, k is idealized by bilinear⁵⁾ or hyperbolic idealization ⁶⁾. Generally, it is difficult to obtain ideal test results especially the initial shear stiffness k , because tested data is not only the properties of interface, in most cases but also the properties of reinforcement and the surrounding matrix. As a result, the behaviour of rock bolt in numerical simulations remains arbitrary sometimes. In order to evaluate the supporting effect of rock bolt in underground excavation quantitatively in theory, an interaction model is introduced based on Shear-lag model (SLM) or fiber-loading theory in the following. The SLM theory was originally developed by Cox ⁷⁾, which has been widely used by material scientists and structural geologists as a powerful

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analytical method. Unfortunately, it is not suitable for rock bolt because of their different boundary conditions. In this paper, the traditional SLM has been improved and brief theoretical summary is presented. Coupling and decoupling behaviours of reinforcement system in pullout test, uniform deformed rock mass and hard rock with joints are discussed.

2. INTERACTION MODEL OF ROCK BOLT AND ROCK MASS

2.1. IMPROVED SHEAR-LAG THEORY AND COUPLING BEHAVIOR OF ROCK MASS

In order to describe the interaction behavior between the linear reinforcement such as fibre, Cox⁷⁾ suggested a model as followings,

$$\frac{dP(x)}{dx} = H(u_b - u_m) \quad (1)$$

where, u_b is displacement of reinforcement while u_m is the displacement of matrix at the edge of influence radius R , and x is the distance from original point of the coordinate, as shown in Fig. 1; $P(x)$ is pullout force at the position of x , and H is material parameter.

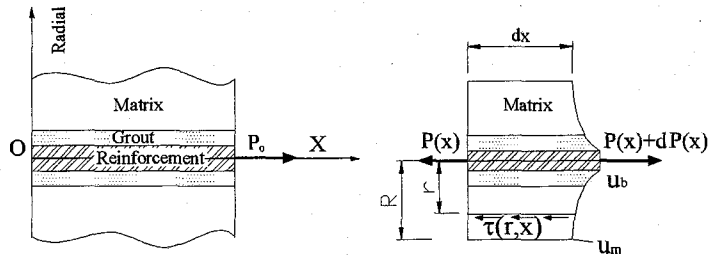


Fig. 1 coordinate of reinforcement

Original SLM is based on the idealized assumption that there is no slip on interface medium^{7, 8, 9, 10)}. In order to get the parameters assumed in the model, shear stress distribution in matrix $\tau(r, x)$ is often assumed as Eq. (1)^{9, 11)}:

$$\tau(r, x) = r_b \tau(r_b, x) / r \quad (r_b \leq r \leq R) \quad (2)$$

where r_b is the radius of reinforcement; x is the coordinate along reinforcement; r is the distance to reinforcement. The parameter H is estimated as followings according to Eq.(2).

$$H = 2\pi G_m \ln(R / r_b) \quad (3)$$

where R is the influence radius of rock bolt; G_m is the shear modulus of matrix around fibre. During a last half century, it has been developed from elastic to plastic system for different purpose in fibre composite material field. However, pressure initiated by reinforcement to rock mass/soil tends to be zero according to the assumption above. Obviously, it is not suitable for reinforcement system. In other words, the interaction behavior should be analysed again, and the assumption of shear stress distribution in original Shear-Lag should be substituted by the equilibrium of infinitesimal elements in rock bolting system. The coordinate system of rock bolt and rock mass is also shown in Fig.1, and the equilibrium of infinitesimal elements are drawn in Fig.2. Before slippage, not only the rock bolt but also the rock mass and the composite of the both keep in balance. According to the analysis of infinitesimal elements such as rock bolt, surrounding matrix and rock bolt together with rock mass, equilibrium Eq. (4) can be established in the cylinder coordinate system,

$$\frac{dP(x)}{dx} = -2\pi r_b \tau_b \quad (4a)$$

$$\frac{\partial \sigma_m(r, x)}{\partial x} + \frac{\partial \tau(r, x)}{\partial r} + \frac{\tau(r, x)}{r} = 0 \quad (4b)$$

$$\frac{dP(x)}{dx} + 2\pi \int_{r_b}^R r \frac{d}{dx} \sigma_m(r, x) dr = 0 \quad (4c)$$

where, $\sigma_m(r, x)$ is the normal stress parallel to rock bolt at (r, x) ; $\tau(r, x)$ represents the shear stress at (r, x) .

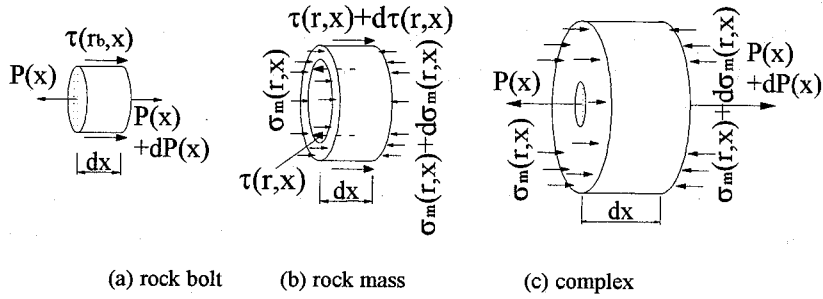


Fig. 2 Equilibrium of infinite elements in rock bolting system

In order to simplify the analysis and satisfy boundary conditions, uniform stress distribution is assumed in the influence area of reinforcement. Neglecting the normal stress on grout paralleled to reinforcement and assuming $R \gg r_b$, parameter H is expressed as followings:

$$H = \frac{2\pi G_g G_m}{[\ln(R/r_b) - 1/2]G_g + \ln(r_g/r_b)G_m} \quad (5a)$$

where G_g is the shear modulus of grout. Generally, the thickness of the grout is much smaller than the influence radius R . It is assumed that properties of the grout are the same as rock mass, and this assumption implies that the thickness of the grout is omitted and there is $r_g = r_b$. Consequently, the Eq. (5a) is simplified to Eq. (5b).

$$H = 2\pi G_m / (\ln(R/r_b) - 1/2) \quad (5b)$$

2.2. DECOUPLING BEHAVIOR OF ROCK BOLT

Different types of reinforcement have different decoupling behaviors. Shear strength at interface is made up of three parts such as adhesion, inter lock and friction toward axial direction. They lost in sequence as the compatible of deformation losing along the interface. After decoupling, shear stress at interface becomes residual strength at slipping part. Reinforcement behaves differently because of the difference of residual shear strength. For friction type's reinforcement, residual strength equals its strength, while for a grouted rock bolt, residual strength is usually smaller than its interface shear strength. It has been revealed that confining pressure influences the strength of interface dramatically¹³⁾.

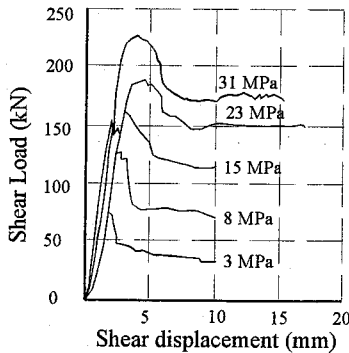


Fig. 3 Influence of confining pressure on shear stress in direct shear test (Moosavi et al., 2001).

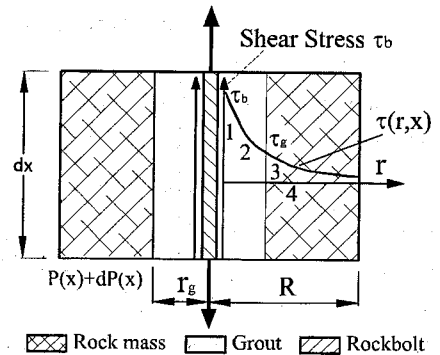


Fig. 4 Possible decoupling position of rock mass with rock bolt

Fig. 3 shows the test result of interface behavior of deformed reinforcement under different confining pressure. Although it is declared that full mechanism of bond failure during axial slip in a deformed bar can be explained by the shearing mechanism

in cement annulus¹², failure may occurs at bolt-grout interface, in the grout medium, at grout-rock interface or in rock mass. It depends on which one is the weakest, as shown in Fig. 4. It should be noted that when shear stress exceeds the strength of rock mass, decoupling occurs interior rock mass even though interface is strong enough. On the basis of suggested model, Mohr-Coulomb law is recommended here to describe the decoupling behavior of reinforcement and rock mass, and shear strength can be calculated with Eq. (6).

$$\tau_m = c + \sigma_{nb} \tan \phi^* \quad (6)$$

where, τ_m , c and ϕ^* are the shear strength, cohesion and internal friction angel of interface. σ_{nb} is the normal stress perpendicular to reinforcement. Interface properties such as friction angle and cohesive value can be obtained by direct shear tests or pullout test.

3. APPLICATION OF THE MODEL

3.1. INTERACTION BEHAVIOR IN PULLOUT TEST

Based on the proposed model, pullout behavior of reinforcement in laboratory is discussed. The boundary condition of the pullout test can be written as followings,

$$x=0, P(x)=P_o \quad (7a)$$

$$x=L, P(x)=0 \quad (7b)$$

The pull out force P_o and displacement at the end of reinforcement u_o are known, and axial force at the other end of reinforcement is zero. If normal stress caused by reinforcement is assumed to be uniform on specimen, matrix strain at the edge of specimen, then the distribution of the axial force in rock bolt can be expressed as Eq.(8) according to the boundary condition Eq.(7) and constitutive Eq.(5),

$$P(x) = P_o \sinh[\alpha(L-x)] / \sinh(\alpha L) \quad (8a)$$

$$\alpha = \sqrt{H(1/(A_b E_b) + 1/(E_m S))} \quad (8b)$$

where, E_b and A_b are Young's modulus and cross section area of reinforcement. E_m is deformation modulus of rock mass and S is influence area of single reinforcement, and it is the area of the cross section of specimen in this case; L is the embedded length of reinforcement. If L is long enough, pullout force along reinforcement can be simplified as Farmer's formulas¹³. The total pullout force may not be maximum if residual strength of interface exists at debonding section. Ultimate pull out force after debonding is expressed as:

$$P_{\max} = 2\tau_{mo}\pi y r_b + 2\tau_m \tanh(\alpha(L-y))A_b / (\alpha r_b) \quad (8c)$$

where τ_m and τ_{mo} are the shear strength and residual strength of interface respectively; y is the debonding length. For friction types reinforcement such as Swellex bolt, the residual stress equals the shear strength of interface. Therefore, debonding length $y=L$ when it reaches ultimate pullout load. Shear strength of frictional reinforcement is calculated with Eq. (10).

$$\tau_{mo} = P_{\max} / 2\pi r_b L \quad (9)$$

The theoretical prediction is compared with the pullout model⁴, as shown in Fig. 5. The calculation parameters are as followings; $P_{\max}=180\text{kN}$; Reinforcement: embedded length $L=1.5\text{m}$, radius $r_b=10\text{mm}$; $E_b=210\text{GPa}$; Grout: radius $r_g=17.5\text{mm}$, Young's modulus $E_g=35\text{GPa}$, Poisson's ratio $\mu_g=0.25$; Matrix: $E_m=45\text{GPa}$, Poisson's ratio $\mu_m=0.25$; $R=35r_b$; $\tau_{mo}/\tau_m=0.1$. It is obtained that peak shear stress is 13.8MPa by back analysis, which is a little bigger than that of pullout load model 12.8MPa. The pullout model is just based on the pullout concept and the deformation of rock mass is ignored. This is the reason of the difference. Furthermore, since it is just based on the pullout concept, it is difficult to analyze the interaction behavior of rock bolt and rock mass by using the pullout model when the deformation of the rock mass is complicated.

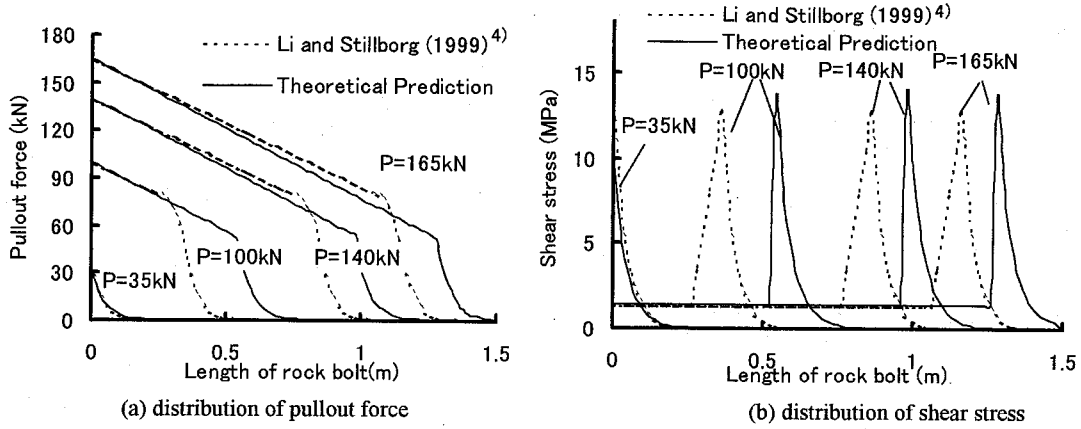


Fig. 5 Axial load distribution of reinforcement

3.2. REINFORCEMENTS AROUND AN UNDERGROUND OPENING

Coupling behavior of reinforcement and rock mass could be described by Eq. (6) in different boundary conditions. Considering the supporting effect of reinforced system, the strain of rock mass at the edge of influence radius R is expressed as Eq. (10), and constitutive equation (3) can be written as equation (11) after differential.

$$\varepsilon_m = \varepsilon_{ini} - \Delta\varepsilon_m; \quad \Delta\varepsilon_m = P/(SE_m) \quad (10)$$

$$\frac{d^2 P(x)}{dx^2} = H \left[\frac{P(x)}{E_b A_b} - \varepsilon_m \right] \quad (11)$$

where, ε_{ini} is the rock mass strain without reinforcement. E_m is the deformation of rock mass; S is the influencing area of single reinforcement. If displacement is time dependent, time dependent axial force can be calculated. Obviously, the initial strain of rock mass determines the axial force in reinforcement. If the displacement formula of rock mass is simple, for example Eq. (12a), like the elastic displacement style, the axial force distribution can be calculated according to Eq. (12).

$$u = A/(x+B) \quad (12a)$$

$$P(x) = -[Ei(1, \xi)e^\xi + Ei(1, -\xi)e^{-\xi}] A/2 + C_1 e^{\alpha x} + C_2 e^{-\alpha x} \quad (12b)$$

$$\xi = \alpha(x+B) \quad ; \quad Ei(1, \xi) = \int_0^\infty \frac{e^{-t\xi}}{t} dt$$

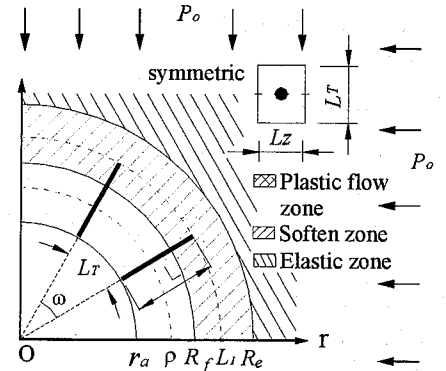


Fig. 6 Reinforcement around tunnel.

where, A , B are constant parameters of the rock mass displacement, which can be determined by empirical approach, experiments or theory analysis; C_1 and C_2 are parameters which can be determined by boundary condition. The force at both ends of reinforcement is considered zero if the effect of face plate is ignored. Displacement of rock mass around tunnel in-situ is relatively complicated especially for soft rock¹⁴⁾. Fig. 6 shows ground condition around tunnel and one case of rock bolt in-situ, and r_a is the radius of tunnel and ρ is the neutral position of rock bolt.

The rock bolt intersects plastic flow zone, soften zone and elastic zone, and it is not easy to get theoretical expression of axial force distribution. Numerical method is favorable in this case. For example, if hydraulic pressure P_o is 1.0MPa, axial compress strength of rock mass is 1.5MPa, typical distribution of axial force along reinforcement installed around circular tunnel is shown in Fig. 7

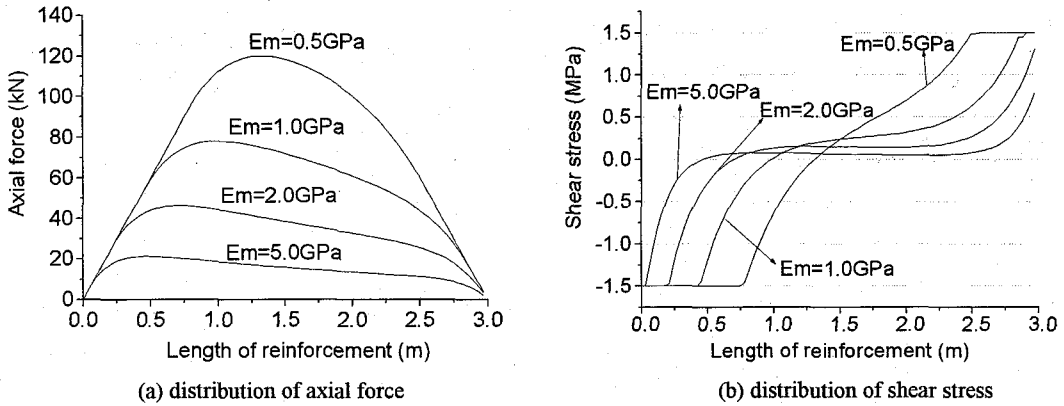


Fig. 7 Axial distribution along reinforcement installed around circular tunnel

In Fig.7, other parameters for calculation are as followings; $L=3.0\text{m}$, $r_b=12.7\text{mm}$; $E_b=210\text{GPa}$; $r_g=25\text{mm}$, $E_g=23.5\text{GPa}$, $\mu_g=0.3$; Rock mass: $\mu_m=0.25$; $S=1.2\text{m}^2$; shear strength 0.75MPa . When E_m changes from 0.5GPa to 5.0GPa . According to Fig. 7, resultant axial force of rock bolt in-situ can be divided into pullout part, anchor part at neutral point. At neutral point, the axial force reaches maximum and shear stress at interface becomes zero. The position of neutral point is very important because it influences the supporting effect of reinforcement. In the part between the neutral point and the tunnel wall, the deformation of rock mass is restrained by the rock bolt, while it is pulled out in the part between the neutral point and the other end of the rock bolt. The relationship between the neutral point and the rock bolting effect have been discussed by Jiang et al^[5]. Obviously, the neutral point depends on displacement distribution of rock mass. By using the proposed model, it is easy to predict it easily. According to Eq. (11) and neglecting the effect of the face plate, the neutral point calculated with proposed model is compared with Tao and Chen's model, as shown in Table 1.

In the above table, the neutral position suggested by Tao and Chen is calculated with the following formula,

$$\rho = L / \ln(1 + L / r_a) \quad (13)$$

where ρ is the neutral point's position from central of circular tunnel and L is the length of reinforcement. Assuming deformation modulus of rock mass changes from 0.5GPa to 5GPa , displacement of rock mass changes sensitively, and resulted position of neutral point changes from 1.3m to 0.48m correspondingly.

Table 1 Distance of neutral point from tunnel wall

E_m (GPa)	Proposed model (m)	Tao and Chen's ²⁾ (m)
0.5	1.3	1.38
1	0.98	
2	0.72	
5	0.48	

3.3. REINFORCEMENT INTERSECTING JOINT

Reinforcement intersecting the joints can be divided into continuous adhesive part and joint opening part. The basic interaction constitutive equation is also valid for continuous adhesive part and its interface behavior is the same as that described before. Deformation of intact rock can be neglected when it is small enough compared to the opening of joint. Axial load and shear stress is mainly determined by the opening distance of rock joint, which applies a tensile load to both sides of reinforcement intersecting the joints. The concept of one reinforcement intersects two joints is shown in Fig. 8. D is the distance between two joints; σ_1 and σ_2 are axial stress at position 1 and position 2 respectively, and they are not the same

in some conditions. Boundary condition is the opening displacement of joint in this case.

Axial stress decreases in adhesive part because joint is easy to open, which suggests that there may be more than one neutral point along a reinforcement installed in jointed rock mass. This result is consistent with Björnfort and Stephansson's work¹⁶(1984). Before decoupling, axial load and shear stress in section is expressed as,

$$\sigma_b = \{\sinh[\alpha(x-D)]\sigma_1 + \sinh(\alpha x)\} / \sinh(\alpha D) \quad (14a)$$

$$\tau_b = \{\alpha r_b [\cosh[\alpha(x-D)]\sigma_1 - \cosh(\alpha x)\sigma_2]\} / [2 \sinh(\alpha D)] \quad (14b)$$

where, α is the same as that in Eq. (7). If the distance between two joints D is long enough compared to the diameter of rock bolt and $\sigma_1 = \sigma_2$, Eq. (14) can be simplified as Eq. (15), which is similar to the pullout load model (Li and Stillborg, 1999).

$$\sigma_b(x) = \sigma_1 e^{-\alpha x} \quad (15a)$$

$$\tau_b(x) = 0.5 r_b \alpha \sigma_1 e^{-\alpha x} \quad (15b)$$

After decoupling, only residual shear stress remains at interface. When the length D is long enough (assuming $D \gg r_b$), axial load becomes zero at a distance to opening joint. Before decoupling, the opening of joint is only the displacement followed by the strain of reinforcement between the joints. It is very small because Young's modulus of reinforcement such as rock bolt is very large. In other words, the interface of rock bolt starts decoupling at a very small opening displacement of the joint. This confirms the finding achieved by other studies¹⁷⁾. Axial stress distribution of reinforcement intersecting three joints is compared with Li and Stillborg's (1999) model, as shown in Fig. 9. In this example, three joints named as a , b , c have opened 50, 20 and 5 μm at the position of 0.4m, 0.6m and 0.8m, respectively. Young's modulus and radius of reinforcement is assumed as 210GPa and 10mm respectively. The Young's modulus of surrounding matrix is assumed as 45GPa, and it is considered very hard and its deformation is omitted. Tensile force in rock bolt calculated by this model is only a little bigger than that of pullout load model. In short words, the pullout model can be thought as a specific case of the proposed model. It also should be pointed out that the boundary condition in pullout model is only the pullout force at one joint while in the proposed model, the boundary condition is the force at two joints which are intersected by the rock bolt.

4. CONCLUSIONS

On the basis of the improved Shear-lag model, a constitutive equation has been established according to the relationship between the displacement of rock mass and the resultant axial force in rock bolt. The balance of infinitesimal elements such as rock bolt, rock mass and the complex are all considered in the model. The coupling and decoupling behavior of a rock bolt in pullout test, under the condition of the deformed rock mass, and the hard rock with joints have been analyzed and discussed. According to this model, axial stress of the reinforcement and corresponding shear stress at interface has quantitatively determined based on the displacement of the rock mass. Physical properties of the rock mass influence the

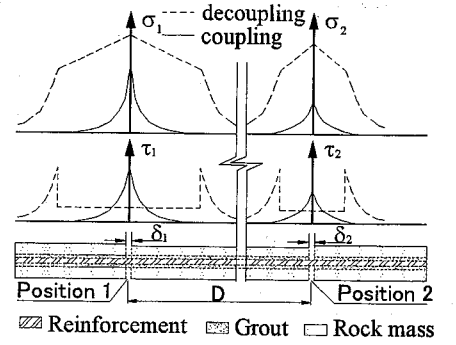


Fig. 8 Distribution of axial stress σ and shear stress τ when reinforcement intersecting

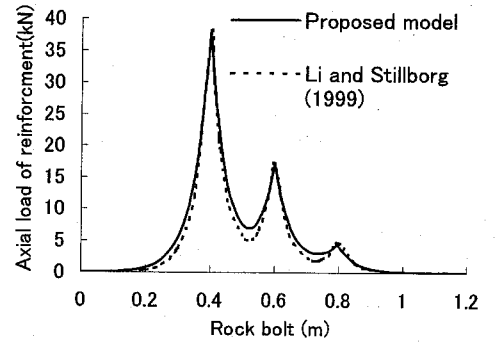


Fig. 9 Axial stress of reinforcement intersecting three joints.

axial load and corresponding shear stress at interface. In the pullout test, the shear stress decreases exponentially with increasing the distance from the decoupling front. The pullout load may not become maximum if residual stress exists. The analysis of grouted rock bolt around circular tunnel suggests that a rock bolt may be positioned in both plastic zone and elastic zone, which is very important to its performance, and it has a pullout part and anchor part divided by neutral point. The analysis of a reinforcement intersecting joints shows that there may be more than one neutral point in this condition. The opening displacement of rock joints may induce axial stress peaks in the rock bolt. The behavior of the reinforcement is described from the viewpoint of displacement in proposed model, which makes it possible to evaluate the effect of the reinforcement quantitatively.

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