トンネル工学研究論文・
報告集第13巻2003年11月
論文（2）

Analytical model for grouted rock bolt in tunnel design

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An analytical model for the grouted rock bolt in NATM tunneling design is proposed. Typical distributions of axial loads are presented considering the debonding at the interface between rock bolt and rock mass. The parameters study shows that: 1) The position of maximum axial force is not only related with the length of rock bolt and internal radius of tunnel, but also strongly influenced by physical properties of rock mass. 2) Maximum axial load and its position in rock bolt tends to be constant when anchor length of bolt is long enough, which means it is not good only to increase the length of rock bolt under certain condition. 3) Young's modulus of rock mass influences the working of bolt significantly.

Key Words : rock bolt, model, tunnel, interface , rock mass

1. INTRODUCTION

Rock bolts are widely used as an effective supporting member in tunneling construction nowadays. Although there are many researches about rock bolting in theory or in-situ monitoring, the coupling mechanism of the grouted rock bolt is still not clear. The distribution of the axial force shows the working condition of the rock bolt, and the supporting effect can be evaluated by the distribution to some degree. Monitoring axial force is required generally. However, it is difficult to estimate the axial force of rock bolts around tunnel in design step. Freeman[1)] monitored both the loading process of the bolts and the distribution of stresses along the bolts, and pr oposed the concept of "neutral point", "pick-up length" and anchor length". At the neutral point, the displacement of rock mass and rock bolt is consi dered as the same, and the shear stress at interface is zero while the axial load of the bolt reaches maximum. Considering the position of the neutral point, Indraratna [2)] established an analytical model for the design of grouted rock bolt system according to Elasto-plastic constitutive law. However, the position of neutral poin t is not easy to predict in fact, and it changes when a decoupling phenomenon occurs[3)]. It also should be pointed out that the decoupling behavior of the interface between the rock bolt and the grout or the grout and the rock mass is not considered in Indraratna's model. The neutral point is strongly influenced by the type s of rock deformation. Some methods were proposed to predict the distribution of axial force of the rock bolt. Fundamental study about rock bolt was studied by Saito T. and Amano S. (1982) [4)], and a force distribution along the rock bolt was presented in the 2-D coordinate. Considering the interaction of rock bolt and rock mass, Aydan [5)] (1989) proposed a constitutive equation including the displacement of rock mass for the predicting of ax ial force, and further discussion was carried out based on the Elasto-plastic law. The shear stress was assumed and the anti-pressure from the rock bolt, which is important for the design of rock bolt, was neglected in the model. Li and Stillborg [3)] (1999) proposed a pullout model to describe the rock bolting behavior. A method was provided to predict the distribution of axial force of the rock bolt in tunnel design. However, only

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elastic displacement of the rock mass was discussed, and the prediction becomes difficult when the displacement of the rock mass is complicated, for example, the rock bolts in plastic zone or intersecting zones with different displacement pattern. According to paper reviews, only the balance of rock bolt itself was focused and the stress condition of surrounding rock mass was neglected in most cases. A model is proposed by the authors to analyze the interaction behavior between the rock mass and rock bolt. This paper is to introduce an example of its application in the design of tunnel.

2. BEHAVIOR OF SOFT ROCK AROUND CIRCULAR TUNNEL

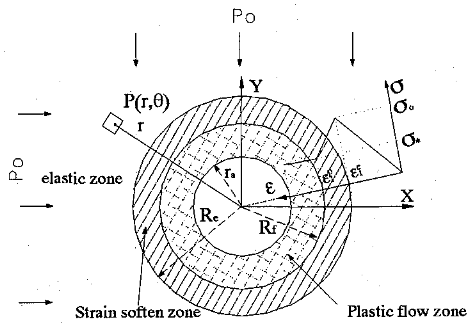


Fig.1 Soft rock mass behavior around tunnel

It has been confirmed that soft rocks have the features of transition from strain softening to residual state and large dilation under lower confining pressure. Hence, it is necessary to take the effect of such behavior into account on the stability analysis and support design of tunnels. Obviously, the post failure behavior of rock mass should be considered in rock bolt design. Under the hydraulic pressure condition, typical characteristic behavior of soft rock around a tunnel has been discussed[6]. **Fig.1** is the cross section of circular tunnel after excavation under hydraulic pressure *Po*. According to strain-softening law, three zones may appear in the rock mass around a tunnel such as plastic-flow zone, strain soften zone and elastic zone after excavation. The radius of tunnel is defined as r_a, and R_f is the radius of plastic-flow zone and R_e is the elastic radius. If the rock bolts intersect different zones, the continuous boundary condition should be considered for the prediction of axial force. By using the pullout model proposed by Li and Stillborg (1999)[3], it is difficult to obtain the solution.

3. ANALYSIS OF ROCK BOLT AROUND CIRCULAR TUNNEL

(1) Theory background

According to the proposed model, coupling behavior of rock bolt and rock mass is able to be described by Eq. (1) together with different boundary conditions[7]. Considering the effect of rock bolt system, the strain of rock mass at the edge of influence radius R can be calculated with Eq. (1d).

$$\varepsilon_m = \varepsilon_{ini} - \Delta\varepsilon_{mr} \quad (1a)$$

$$\frac{d^2P(x)}{dx^2} = H\left[\frac{P(x)}{E_bA_b} - \varepsilon_m\right] \quad (1b)$$

$$\Delta\varepsilon_{mr} = P(x)/(SE_m) \quad (1c)$$

$$S = \pi R^2 \quad (1d)$$

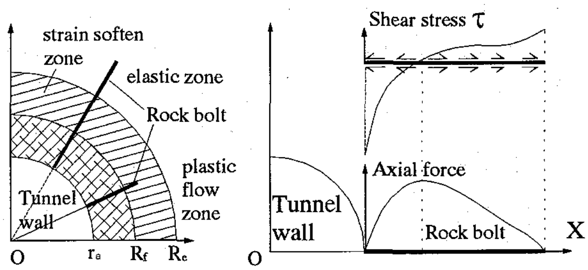


Fig.2 Rock mass and rock bolt behavior around tunnel

where, ε_{ini} is the strain of rock mass without bolt. E_b and A_b are Young's modulus and section area of rock bolt respectively; $\Delta\varepsilon_{mr}$ is calculated according to Eq.(1c). Obviously, the initial strain of rock mass determines the axial force in rock bolt.

In order to simplify analysis, circular tunnel section and hydraulic pressure is assumed here for further analysis. Based on the strain soften constitutive law, the theoretical displacement function of soft rock mass around circular tunnel can be deduced. Since the displacement functions of are relatively complicated, it is not easy to get the closed solution of stress distribution or the position of neutral point. Numerical method is favorable in this case. According to strain-soft model, rock mass is divided into three parts around tunnel as the plastic flow region, strain soften region and elastic region. Correspondingly, the rock bolt may be installed in single or multiple regions in -situ, as shown in **Fig.2**. Rock bolt installed in one region is taken as an example to demonstrate the application of the model. It is assumed that bolt spacing is Lt in transverse section and is *Ls* in longitudinal direction, influence area of single bolt S can be calculated as $S=Ls\cdot Lt$, correspondingly, influence radius R is computed with Eq.(1d). When rock bolt is installed in elastic region, for example in case 1 as shown in **Fig.2**, the displacement is expressed as Eq.(2).

$$u_m = p_o(1+\mu_m)r_a^2/[E_m\cdot(r_a+x)] \tag{2}$$

where r_a is the radial of tunnel; P_o is the hydraulic pressure; μ_m is the Poisson's ratio and E_m is the deformation modulus of rock mass. Correspondingly, differential equation and axial force $P(x)$ in rock bolt can be expressed as Eq.(3) and Eq.(4).

$$\begin{aligned} &\frac{d^2P(x)}{dx^2}-\alpha^2P(x)+\frac{m}{(r_a+x)^2}=0 \\ &\alpha^2=\left[(A_bE)^{-1}+(SE_m)^{-1}\right]H\ ,\quad m=(1+\mu_m)p_or_a^2H/(E_m) \end{aligned} \tag{3}$$

$$\begin{aligned} &P(x)=-\left[Ei(1,\xi)e^{\xi}+Ei(1,-\xi)e^{-\xi}\right]m/2+Ae^{\alpha x}+Be^{-\alpha x} \\ &\xi=\alpha(x+r_o)\ ,\quad Ei(1,\xi)=\int_0^\infty\frac{e^{-\xi t}}{t}dt \end{aligned} \tag{4}$$

where A and B in Eq.(4) are parameters which can be determined by boundary conditions.

(3) Neutral point of rock bolt in continuum rock mass around circular tunnel

The location of neutral point is important for the analytical model of rock bolt system. At neutral point, the axial force reaches maximum and shear stress at interface becomes zero. Tao and Chen[8] independently investigated the interaction mechanism of fully grouted bolts around circular tunnel and gave the neutral point along bolt as Eq.(5).

$$\begin{aligned} &\rho=L/(\ln[1+(L/r_a)] \\ &L=(40r_b\sim 60r_b) \end{aligned} \tag{5}$$

where L is the length of bolt ; r_b is the radius of rock bolt. According to the proposed model, neutral point depends on the displacement of rock mass without rock bolt and rock mass physical properties. Not considering the effect of external fixture, the boundary conditions of the single rock bolt can be written as Eq.(6)

$$x=0,\quad \mathrm{P(x)}=0 \tag{6a}$$

$$x=L,\quad \mathrm{P(x)}=0 \tag{6b}$$

Once the displacement of rock mass is known, it is easy to calculate the resultant axial force and neutral point position according to the constitutive law and boundary conditions. When rock bolt in elastic zone, the position of neutral point is the solution of Eq.(7)

$$\left[2/\xi+Ei(1,-\xi)e^{-\xi}-Ei(1,\xi)e^{\xi}\right]m/2+Ae^{\alpha x}-Be^{-\alpha x}=0 \tag{7}$$

The following presents an example to compare the difference of proposed model and Tao 's suggestion.
It is assumed that radius of tunnel r_a is 4.0m; deformation coefficient of rock mass $E_m=1.0GPa$; deformation coefficient of grout $E_g=30.0GPa$; Poisson's ratio of rock mass and grout $\mu_g=\mu_m=0.3$; hydraulic pressure $Po=1.0MPa$; rock bolt distance $Lt=Lz=1.2m$; radius of rock bolt $r_b=1.0cm$; radius of grout $r_g=2.0cm$. The neutral point of rock bolt around tunnel are listed in **Table 1** together with that calculated with Eq.(5). According to the example above, when the rock bolt is relatively short, the neutral point of proposed model is near to that of Tao and Chen' suggestion. But when the length of rock bolt exceeds 2.0 meter, the neutral point tends to be constant according proposed model, while it departs from the tunnel wall according to

Tao and Chen's model.

4. APPLICATION OF THE PROPOSED MODEL

Table-1 Neutral point of rock bolt in different condition s

Length of rock bolt (m)	0.5	0.75	1.0	1.5	2.0	3.0	4.0
Proposed model	0.24	0.35	0.45	0.61	0.67	0.68	0.69
Tao and Chen[9]	0.25	0.36	0.48	0.71	0.93	1.36	1.77

Axial force along grouted rock bolt is measured in -situ at Holland Zaka Tunnel in Nakasaki. The tunnel is located at the depth of 18.35m. Rock mass around tunnel is classified as DI, which belongs to soft rock. The section of the tunnel and the position of measured rock bolts are shown in **Fig.3**. The tested rock bolt named as RB2 is taken as an example to demonstrate an application of the proposed model. According to investigation and te st data, the parameters used in the example are listed in **Table-2.**

Table-2 Physical and mechanical properties in analysis

Radius of Tunnel, r_a	4.75m
Hydraulic pressure, *Po*	0.375MPa
Axial strength of rock mass, σ_c	0.5MPa
Deformation modulus of rock mass, E_m	1.0GPa
Poisson's ratio of rock mass, μ_m	0.25
Length of rock bolt, *L*	4.0m
Young's modulus of rock bolt, E_b	210GPa
Radius of rock bolt, r_b	12.7mm
Distance between rock bolts, $Lt*Lz$	1.2m*1.4m
Shear strength on interface, τ_m	0.38MPa

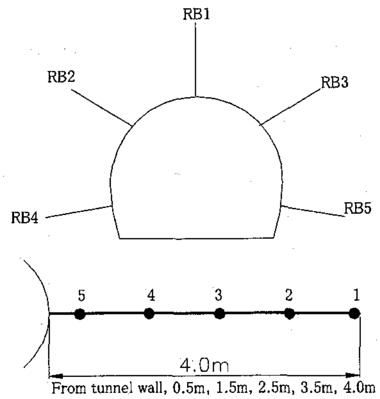


Fig.3 Section of Holland Saka tunnel (No.68+28, Section

The distribution of predicted axial load agrees with tested data except the maximum axial force, as shown in **Fig.4.** There are only five measuring points along one rock bolt, and it is possible that the position of maximum axial force was not measured. The comparison states that it is possible to predict axial force in rock bolt.

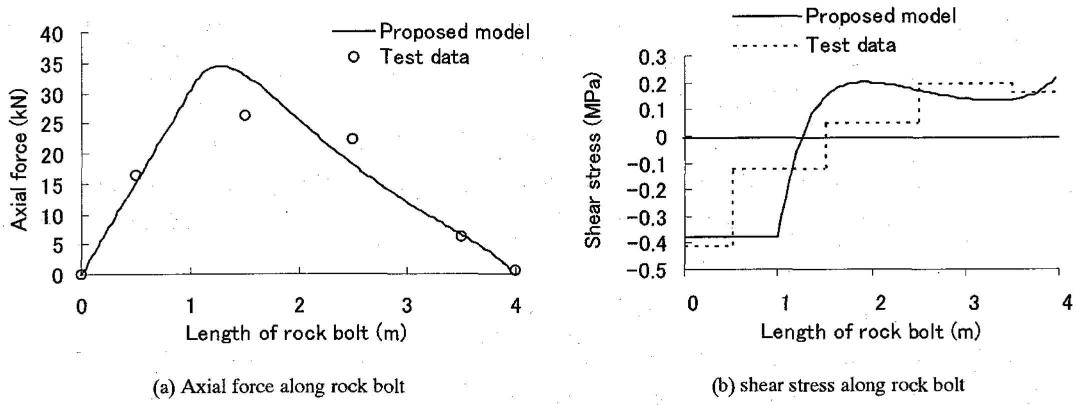
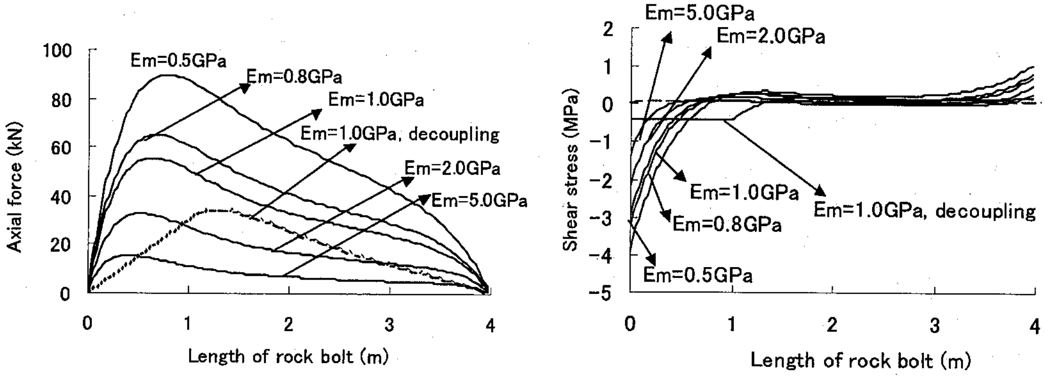


Fig.4 Theoretical prediction (No.68+28, Section C)

5. DISCUSSION

(1) Deformation modulus of rock mass

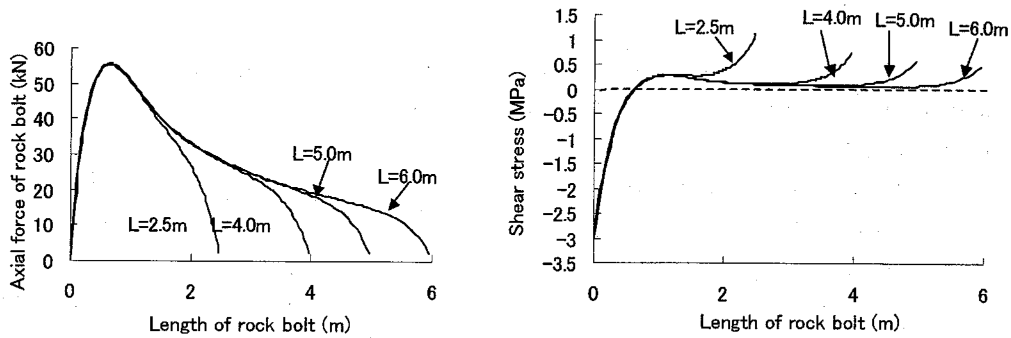
Rock mass is classified in order to simplify supporting design in practice. The supporting pattern is empirical and key factors influencing the initial force in rock bolt remains vagueness. In most cases, the physical properties of rock mass such as strength or deformation modulus ranges widely. The strength and deformation modulus of rock mass determine the displacement of ground, which controls the resultant forc e in rock bolt. Therefore, discussion becomes important during the design step for rock bolt. Soft rock is usually classified as C and D class. Its deformation modulus under 5.0GPa and distributes a wide range. Due to Eq.(1), larger deformation modulus of rock mass will initial larger axial force in rock bolt for same displacement. Supposing the deformation modulus of rock mass ranges from 0.5GPa to 5.0GPa, and other parameters are the same as those in **Table 2.** The axial force and shear stress along rock bo lt is shown in **Fig.5.**



(a) Axial force along rock bolt (b) shear stress along rock bolt

Fig.5 Influence of deformation coefficient of rock mass on axial force distribution

When the deformation modulus of rock mass changes from 0.5GPa to 5.0GPa, the neutral point of the rock bolt changes from 0.76m to 0.40m. If the rock mass is strong enough, no decoupling will take place at the interface, and maximal axial load increases with the decreasing of de formation modulus. In-situ investigation shows that the shear strength of rock mass would not be more than 0.38MPa. Hence, decoupling will take place in situ, and it reaches a ba lance position from 0.62m to 1.26m, as shown in **Fig.5**. Correspondingly, the ma ximum axial force decreases from 50kN to 34kN.



(a) Axial force along rock bolt (b) shear stress along rock bolt

Fig.6 Distribution of axial force along rock bolt of different rock bolt length

(2) Length of rock bolt

The basic function of grouted rock bolt is to adjoin the rock mass work together especially in jointed rock mass. Hence, it is more efficient when it is longer. At the same time, it has a pick up part and anchor part in-situ, anchor length has to be prevented from decoupling at interface. The following is to discuss the length influence to the axial load in rock bolt. Rock bolt length ranges from 2.5m to 6.0m, and other parameters are the same as those in **Table-2**. The axial force and shear stress is shown in **Fig.6**. According to the calculation results, s hear stress at the end toward tunnel changes a little for different rock bolt's length, while shear stress at the anchor end decrease obviously with extension of rock bolt. When rock bolt length equals 2.5m, about 1/4 of tunnel diameter, maximum axial load is 49.0kN and neutral point locates at 0.55m from tunnel wall. It keeps almost constant as 51.2kN after the length of rock bolt exceeds 2.5m, which means the supporting effect of rock bolt does not increase dramatically once it is longer than 3.0m in this case.

6. CONCLUDING REMARKS

A model is proposed to analysis the behavior of grouted rock bolt in tunneling design. According to the model, a xial force in rock bolt and shear stress at the interface between bolt and rock mass is strongly r elated with the displacement of rock mass and boundary condition s. Prediction of proposed model agrees with the tested axial force in-situ. The analysis of grouted rock bolt around circular tunnel con firms that: (i) Rock bolt in-situ has a pick-up part and anchor part and a neutral point. With the increasing of rock bolt's length *L*, while shear stress at inte rface decreases, maximum axial force of rock bolt installed in continua rock mass does not incr ease significantly when L is long enough. (ii) The position of the neutral point depends on the displacement fun ction of rock mass. It is the function of rock bolt length and certain ph ysical properties of rock mass such as deformation modulus. Its position is near to that calculated with Tao 's model when rock bolt is short, within 1.0m. However, it turns to be constant after rock bolt exceeds certain length before decoupling. (i ii) Deformation modulus of rock mass influences the stress in rock bolt. For th e same geometry of tunnel, the larger the deformation modulus, the smaller axial force in rock bolt because larger deformation modulus of rock mass will decrease displacement under certain boundary conditions.

The proposed constitutive law describes interaction mechanism of rock bolt and rock mass, which make it possible to considering the decoupling behavior of rock bolt in-situ. The model is more reasonable because it is established from the viewpoint of displacement. The proposed analytical method prese nts a way to predict resultant axial force in rock bolt at designing step of tunneling, which is helpful for optimum and economic design in NATM tunneling, and the evaluation for the supporting effect of rock bolting in quantity becomes reality together wi th the model of rock bolt system.

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