(22) A New Method for Project Schedule Problem under Variable Resource-Constrained

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This paper proposes a new method for computing the total float (TF), critical activities and critical path and determining the resource dependent relations between activities in the project schedule under variable resource constrained. Traditional resources constraint scheduling (RCS) techniques cannot compute the correct floats and critical path(s). In addition to technological relations, there exist resource dependencies between activities in project schedule under resource constrained that are neglected in RCS techniques. Although, researchers have proposed many methods in order to overcome these shortcomings, most of them have assumed that the amount of resource is constant during the execution of the project. However, a real project might be under variable resource constrained in most situations. The proposed method will introduce a forward pass and a backward one of RCS technique for determining earliest times and latest times respectively in project schedule under variable resource constraints. The TF and critical path can be correctly computed afterwards. Furthermore, the proposed method identifies resource dependent relations between activities. Therefore this approach can produces a stable schedule with ability of updating.

Key words: *Construction project schedule, Critical path method, Resource dependent relation.*

1. Introduction

Critical Path Method (CPM) has been widely used as a construction project management tool to improve scheduling and project administration tasks. The CPM generates useful information about the project schedule such as, earliest start/finish times (ES/EF), latest start/finish times (LS/LF), total float (TF), critical path and so forth. TF is the most important concept among the all information. TF is the maximum amount of time that the finish date of an activity can be delayed without affecting the completion of the entire project. TF is calculated as the difference between LS and ES or between LF and EF of an activity.

However, CPM cannot produce realistic and practical schedule. Because it assumes that the amount of resource availability is unlimited during the project execution, while some resources are highly limited in practice[1)]. Although, resource constrained scheduling (RCS) techniques can consider to resource limitation, they are failed to provide the correct floats and identify the critical path(s). In traditional RCS technique, only the technological precedence relations between activities are considered in computing TF. However, there are resource dependency relations between activities in a resource constraint project that are ignored by backward pass of CPM in RCS technique[1),2),3),4),5)]. The project management software such as primavera p6 and MS-project cannot provide the correct float and critical path as well[1)].

Several studies are proposed to solve the RCS technique limitations. Kim and Garza[1)], Bowers[2)], Weist[3)], Woodword and Shanahan[4)], Lu and Li[6)] studied the methods of scheduling project to identifying the correct float and critical path(s) in the RCS. These studies provide useful information for the present problem. However, each method has significant shortcomings[5)]. In addition, the previous literatures assume that the amount of available resources is constant during the scheduling. While a real project is under variable resource constraints in different stages of the project period in most situations.

For the above reasons, this paper develops a new method for identifying the correct floats and the critical path(s) in resource constrained schedule with considering both variable and constant resource availability during the scheduling.

2. Problem definition and basic assumption

We consider a single project which consists of activity of j (or job j) (j=1,...,J) with a non-preemptable duration of d_j units of time. the activities are linked by two kinds of constraints, namely technological precedence constraints that force an activity j not to be started before all its predecessors i in set of P_j are finished and resource constraint which is arisen as follows. In order to be processed, activity j requires k_j units of resource during every period of its duration. Since resource available with limited amount of $K_{\rho i}$ units for each period, activity might not be scheduled at its earliest (precedence feasible) start time but later. The project is constrained by one renewable resource r which is variable during different scheduling stages. There are only finish-start technological precedence relations with minimal time lags. The scheduling unit is 1 day and it is assumed that the start of project is on afternoon of day 0. For each activity, the duration, the resource requests, and precedence relations are assumed to be deterministic and known in advance. Fenced bar chart is used to represent the project schedule and the activities' relation.

The schedule is divided into n_r periods based on the amounts of available resource r, i.e. $\rho_i = \rho_1, \ldots, \rho_{nr}$. The maximum available resource during the period of ρ_i is $K_{\rho i}$ units, where $K_{\rho i} \neq K_{\rho i+1}$, $\forall i \in \{1, 2, \ldots, n_r-1\}$. In order to make the maximum available resource constant all-through schedule period, n_r number of dummy activities, $\beta_i=\beta_1, \beta_2, \ldots, \beta_{nr}$, are inserted into schedule in which β_{i-1} preceded β_i. The duration of dummy activity β_i is $d_{\beta i} = \rho_i$. Let $ES_{\beta 1} = LS_{\beta 1} = 0$; $EF_{\beta i} = LF_{\beta i} = (ES_{\beta i}$ or $LS_{\beta i}) + d_{\beta i}$, for $\forall i \in (1, 2, \ldots, n_r-1)$; $ES_{\beta i} = LS_{\beta i} = EF_{\beta i-1} = LF_{\beta i-1}$, for $\forall i \in (2, 3, \ldots, n_r)$; and $EF_{\beta nr} = LF_{\beta nr} = T_r$. Where, $ES_{\beta i}$, $LS_{\beta i}$, $EF_{\beta i}$, $LF_{\beta i}$, and $d_{\beta i}$ are the earliest start time, latest start time, earliest finish time, latest finish time, and the duration of dummy activity β_i respectively. T_r is shortest project duration under resource constraints. It is assumed that the dummy activity β_i requires $k_{\beta i}$ units of resource: $k_{\beta i} = \text{Max}(K_{\rho 1}, \ldots, K_{\rho nr}) - K_{\rho i}$.

3. Algorithm description

The proposed method is developed as a heuristic method based on traditional CPM and RCS. The concept of algorithm is composed in 5 steps. For better demonstration of the algorithm, each step is described along a sample example. The sample example, as shown in **Fig.1**, consists of 5 activities and the maximum available resources are variable as: for first 4 days of project are 4 units, for days 5-9 are 8 units, and after day 9 to the end of project are 6 units.

(1) Step 1: Performing CPM

Traditional CPM forward and backward passes, assuming unlimited resources, are employed to find the ES, LS, EF, LF, and floats for each activities. The CPM result for sample example is shown in **Fig.2** in which the TF and critical path (bold line) are identified. However, if the maximum available resources are considered, there are over of the resource limited on days 1 to 4. Therefore, the RCS technique is needed to set the given resource limited.

(2) Step 2: Inserting dummy activities

This step aims to make the maximum available resource constant during the scheduling processes by inserting dummy activities into schedule. Thus, the schedule is divided into various periods based on resource variability along the schedule. Dummy activities are inserted into each period afterwards.

In **Fig.3** the sample example, for instance, is partitioned into three periods and the maximum of available resource and the duration of each period are defined as follows: $K_{\rho 1} = 4$, $K_{\rho 2} = 8$, $K_{\rho 3} = 6$; and $\rho_1 = 4$, $\rho_2 = 5$, $\rho_3 = T_r - 9$ respectively, so that three dummy activities, i.e. β_1, β_2, and β_3, are inserted into schedule. The duration and resource requirement for each dummy activity is determined as: $d_{\beta 1} = 4$, $d_{\beta 2} = 5$, $d_{\beta 3} = T_r - 9$ and $k_{\beta 1} = K_{\rho 2} - K_{\rho 1} = 8 - 4 = 4$, $k_{\beta 2} = K_{\rho 2} - K_{\rho 2} = 8 - 8 = 0$, $k_{\beta 3} = K_{\rho 2} - K_{\rho 3} = 8 - 6 = 2$ respectively. As shown in **Fig.3**, it can be assumed that the maximum available resource is constant during the

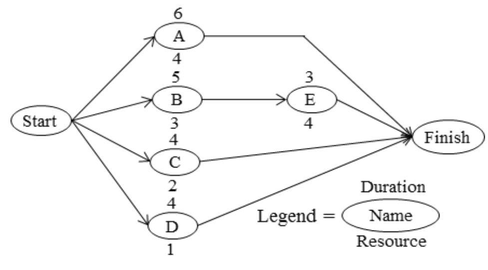

Fig.1 Sample example network

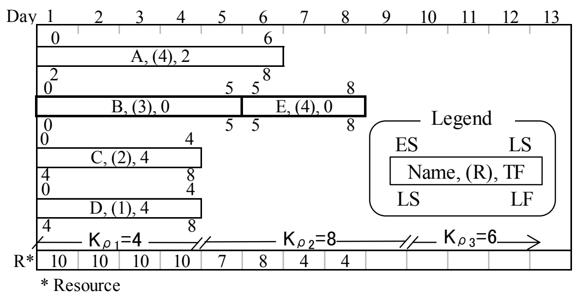

Fig.2 CPM schedule in Step 1

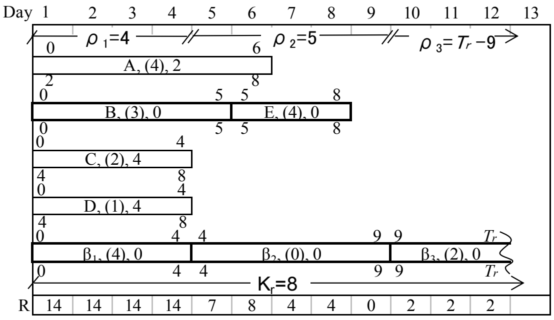

Fig.3 CPM schedule with dummy activities in Step 2.

scheduling, i.e. K_r=8, so that the RCS technique can be applied on the schedule.

(3) Step 3: RCS forward pass calculation

Based on the results of Step 1 & 2, similar to the serial heuristic of RCS technique [7] is employed to set the maximum available resource and omit the resource overloaded by delaying the ES_j of activities. The processes consists of at most g ($g = 1, \ldots, J + n_r$) stages, in each of that one activity is selected, allocated resource and scheduled. There are a schedule time t_g and three activity-sets in each stage. Activities which are completed up to t_g are in the completed set C_g. Activities which are already scheduled, but during the schedule time still active are in the active set A_g. Finally, activities which are eligible to be scheduled, i.e. those activities the predecessors of which are already completed, are in the eligible set E_g. Each stage is consists of two steps. (a) The new schedule time t_g is determined and it is nominated as the earliest finish time of activity(s) which is into active set and finished on the new schedule time. Afterward, the activity(s) is removed from the active set and put into the complete set. In this step, some activity(s) may get the eligibility to be scheduled and is placed into eligible set since all its predecessor activities are scheduled. (b) One or more eligible activity j_g, with considering highest priority and available resource feasibility, from set E_g is selected and scheduled. The priority is determined based on the characteristics of activity such as, least TF, the largest duration and so on. The schedule time t_g is determined as earliest start time of the selected activity. Afterwards, the selected activity is removed from the eligible set and put into the active set. Since the earliest start time of dummy activities β_i are fixed and cannot be delayed, they have the highest priority to be allocated by resource and scheduled in stage $t_g = ES_{\beta i}$. If there are not enough resources in K_r for β_i, one or more activity is removed from the active set and put into eligible set, so that its resources are allocated to the dummy activity. This process is repeated until all activities are scheduled and placed into the complete set.

Table 1 lists the processes of RCS forward pass in each stage. For instance, in stage $g = 1$ the schedule

Table 1 RCS forward pass process on sample example.

g	t_g	E_g	A_g	C_g	j_g
1	0	{β₁, A, B, C, D}	{ }	{ }	β₁, B, D
2	4	{β₂, A, C}	{B}	{β₁, D}	β₂, A
3	5	{C, E}	{β₂, A}	{β₁, B, D}	E
4	8	{C}	{β₂, A}	{β₁, B, D, E}	C
5	9	{β₃}	{A,C}	{β₁, β₂, B, D, E}	β₃
6	10	{ }	{β₃, C}	{β₁, β₂, A, B, D, E}	–
7	12	{ }	{ }	{β₁, β₂, β₃, A, B, C, D, E}	–

time is determined $t_1 = 0$; activities A, B, C, D and β_1, as shown in **Fig.3**, are eligible to be scheduled on day 0, so they are put into eligible set, i.e. E_1 = {A, B, C, D, β_1}. There is neither active activity nor complete one on day 0, so their sets are determined as empty. As shown in column 6 of Table 1, activities β_1, B and D are selected in order of their priority and resource feasibility, then they are allocated by their resource requirement, i.e. $k_{\beta1}$=4, k_B=3 and k_D=1. The process is terminated on stage 7, where all activities are placed into complete set. The project duration is determined as 12 days ($t_7 = T_r = 12$). As shown in **Fig.4**, the RCS forward pass set the limited available resource and determined the ES and EF for each activity successfully.

(4) Step 4: RCS backward pass calculation

The purpose of backward pass of RCS is to find LS and LF for each activity.

a) Step 4.1

The original schedule is reversed as all predecessor activities are changed to successor ones, and vice versa. Only forward pass of CPM is employed to find the ES and EF for each activity.

b) Step 4.2

Based on the ES and project duration of Step 3 constraint's LF (CLF for short) is calculated from Equation 1 and imposed on each activity in reverse CPM schedule. In this study the CLF is defined as a restriction time imposes on each activity that limits the late time in which can be finished on or before.

$$CLF_{Bj} = T_r - ES_{Fj} \tag{1}$$

Where, CLF_{Bi} is constraints latest finish time for activity j in reversed schedule, T_r is minimum project duration in RCS schedule in Step 3, ES_{Fi} is earliest start time of activity j in RCS schedule in Step 3.

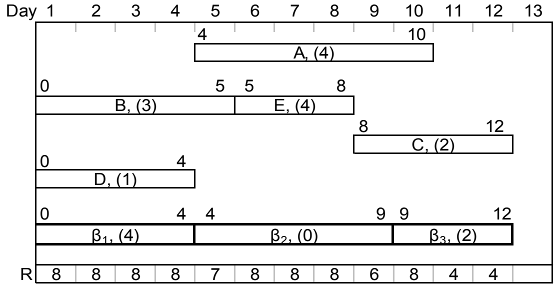

Fig.4 RCS schedule in Step 3.

The LS and TF are found by CPM calculation with considering CLF. Afterwards, based on serial heuristic of RCS technique, similar to RCS forward pass in Step 3, the maximum available resources are set by delaying activities within their TF. The activity is scheduled and allocated for resource in order of CLF ascending. Note that the project duration is fixed and equal to the result of Step 3. Therefore, activity cannot be delayed beyond its TF otherwise the project duration will be extended. Since the CLF for all activities are calculated based on RCS schedule in Step 3, there are enough TF for activities to set the maximum resource availability. The result of Step 4.2 on sample example is shown in **Fig.5**.

c) Step 4.3

Based on the results of Step 4.2, the CLF is calculated from Equation 2 and imposed on each activity in RCS schedule.

$$CLF_{Fj} = T_r - ES_{Bj} \tag{2}$$

Where, CLF_{Fj} is constraint late finish time for activity j for RCS schedule, ES_{Bj} is earliest start time of activity j in reverse RCS schedule.

Based on CPM calculation and considering the CLF, the LS and TF are found for each activity afterwards as shown in **Fig.6**.

(5) Step 5: Identifying resource dependency

There is resource dependent relation between activities a and b if: (i) there is no technological relation between them, (ii) $EF_a = ES_b$, (iii) $TF_a = TF_b$. A resource link is created between activities a and b If the aforesaid conditions are satisfied. For instance, there are resource dependency relation between activities D and A, also between activities E and C in **Fig.6**. As shown the final result of sample example in **Fig.6**, the proposed method identified resource dependency relations, the critical path and the TF reasonably.

Exceptionally, in some case a certain activity with positive TF may not use the full its TF because of resource constraints during the TF period. Therefore, the duration of each activity in the result of final step that has positive TF is checked by extension of its duration within the full its TF and one or more resource link is established if there is resource constraint.

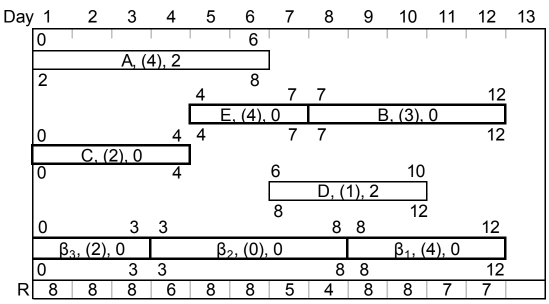

Fig.5 Reversed RCS schedule in Step 4.2.

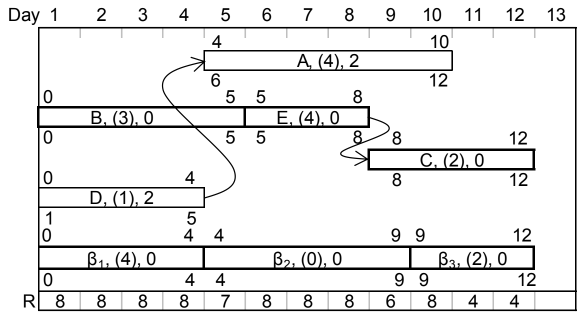

Fig.6 Sample schedule in Step 4.3 & 5

4. Conclusion

Traditional RCS techniques cannot provide the correct TF and the critical path(s). Though former researchers have developed methods to solve this problem, they assumed that the amount of available resources is constant during the scheduling. While a real project is under variable resource constraints in different stages of the project life cycle in most situations.

This paper provided a new method for RCS technique in which a backward pass computes the LS and LF correctly for each activity with considering both technological precedence and resource dependent relation between activities. The TF and critical path(s) can be calculated correctly afterwards. Finally, the proposed method can establish resource link between activities reasonably with considering both variable and constant resource availability during the scheduling. Therefore, this method can produce a realistic and stable schedule with ability of updating.

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