MODELING AND ANALYSIS OF PEDESTRIAN FLOW AT SIGNALIZED CROSSWALKS^{*}

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1. INTRODUCTION

The operational efficiency of vehicular traffic and pedestrian flow are considered as important concern especially at signalized crosswalks where both of them have to share the same space. A crosswalk is defined as a portion of roadway designated for pedestrians to use for crossing the street. Crosswalk geometry and configuration at signalized intersections directly affect the safety, cycle length and resulting delays for all users. Optimizing crosswalk configurations including width, position and angle is an important concern to improve the overall performance of signalized intersections. Pedestrian flow at signalized crosswalks can be unidirectional or bi-directional depending on pedestrian demand at both sides of the crosswalk. Pedestrian crossing time is basically a function of crosswalk length and walking speed. However when pedestrian demand increases at both sides of the crosswalk, crossing time increases due to the interaction between conflicting pedestrian flows. Quantifying the effects of bi-directional flow and crosswalk width on pedestrian crossing speed and crossing time is a prerequisite for improving the geometric design and configuration of signalized crosswalks.

A variety of methods have been developed for determining appropriate total pedestrian crossing times at signalized intersections. Although many of these methods have useful applications, most of them have shortcomings when considering the effects of bidirectional flow on crossing time. No consideration is given to deceleration or reduction in walking speed that results from the interaction between the conflicting flows.

The objective of this study is to develop a new methodology for modeling the bi-directional pedestrian flow at signalized crosswalks.

The structure of this paper is as follows: After introduction and literature review, total crossing time is modeled. Total crossing time is divided into discharge and crossing times. Discharge time is modeled by using shockwave analysis while crossing time is modeled by applying drag theory. The fundamental diagrams that represent the relationship between speed, flow and density of pedestrian flow are presented to show the sensitivity of pedestrian flow to bi-directional effects and crosswalk geometry. Then proposed models are validated and compared with existing models. Possible applications of the developed models such as the evaluation of lane-like segregation policy¹⁾ for pedestrian crossing are discussed. Finally, the paper ends up with summary of the results, conclusions and future works.

2. LITERATURE REVIEW

The walking speed and/or walking time of pedestrians are of prime importance in studying the operation and design of pedestrian facilities. Few studies addressed the issue of bi-directional pedestrian flow and its impact on crossing time at signalized crosswalks. Most of the existing works in this respect attempted to investigate the impact of bi-directional flow at other pedestrian facilities such as walkways and sidewalks. However the characteristics of the environment as well as the pedestrian arrival pattern at crosswalks is different from other pedestrian facilities.

Most crossing time estimations have been based on assumptions providing for start-up delay and a particular walking speed. The pedestrian chapter of the Highway Capacity Manual²⁾ and Pignataro³⁾ have formulations similar to Equation (1).

$$T = I + \frac{L}{S_p} + \left(x \frac{N_{ped}}{w}\right) \tag{1}$$

Where *T* is total time required for all the crossing process, *I* is initial start-up lost time, *L* is crosswalk length (*m*), S_p is walking speed (*m/sec*), *x* is average headway (*sec/ped/m*), N_{ped} is number of pedestrians crossing from one side of the crosswalk during an interval *p*, and *w* is crosswalk width (*m*). Equation (1) shows that the time spent on the crosswalk itself (*L/S_p*) is independent from the pedestrian demand, bi-directional effect and crosswalk width.

The Manual on Uniform Traffic Control Devices⁴⁾ depends on average walking speed (4*ft/sec*) and crosswalk length to estimate crossing time (clearance interval) which is similar to L/S_p in Equation (1).

The Japanese Manual on Traffic Signal Control⁵ proposes a formula similar to Equation (1), but the initial start-up lost time is included in the discharge time. This procedure does not consider the effect of bi-directional pedestrian flow.

Lam *et al.*⁶⁾ investigated the effect of bi-directional flow on walking speed and pedestrian flow under various flow conditions at indoor walkways in Hong Kong. They found that the bi-directional flow ratios have significant impacts on both the at-capacity walking speeds and the maximum flow rates of the selected walkways. However, the effects of different walkway's dimensions on walking speed and capacity of the walkway were not investigated.

Golani *et al.*⁷⁾ proposed a model for estimating crossing time considering start-up lost time, average

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walking speed, and pedestrian headways as a function of the subject and opposite pedestrian platoons separately. They found that the size of the opposite pedestrian platoon can cause a significant increase in the crossing time of the subject pedestrian platoon especially at high demands. The proposed model (Equation (2)) is based on HCM²⁾ model which was calibrated by using empirical data.

$$If _N_{ped1} \le 5$$

$$T = I + \frac{L}{S_{p}} + \left(2.09 \cdot \frac{N_{ped1}}{w} + 0.52 \cdot \frac{N_{ped2}}{w}\right)$$

$$If _N_{ped1} > 5$$

$$T = I + \frac{L}{S_{p}} + \left(0.81 \cdot \frac{N_{ped1}}{w} + 0.52 \cdot \frac{N_{ped2}}{w} + 6.4\right)$$
(2)

Where N_{ped1} is the size of the subject platoon and N_{ped2} is the size of the opposite pedestrian platoon. The proposed model relates the impact of bi-directional flow to the headway between pedestrians when they finish crossing. Therefore it is difficult to see how the interaction is happening and what the resulting speed drop or deceleration is.

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Virkler, et al.⁸⁾ collected data from some relatively low-volume and high-volume signalized crosswalks and recommended an equation for one-directional flow that also considers platoon size. However they did not consider the impacts of bi-directional pedestrian flows.

Virkler⁹⁾ introduced a method to estimate the required pedestrian crossing time under high-volume conditions and with bi-directional pedestrian flow. According to the data analysis, it was concluded that the opposing pedestrian platoon size does not add any significant effect on estimating the crossing time of the subject pedestrian platoon which is contradicting with the results of Golani et al.⁷⁾ and Lam et al.⁶⁾ Therefore the dispersion of the subject pedestrian platoon through the crossing process was the only considered factor in the proposed methodology.

This study aims to develop a rational methodology that can estimate total pedestrian crossing time as a function of crosswalk geometry, pedestrian demand at both sides of the crosswalk and signal timing.

3. TOTAL CROSSING TIME T_t

The total time needed by a platoon of pedestrians to cross a signalized crosswalk T_t from the beginning of the pedestrian green indication until the pedestrian platoon reaches the other side of the crosswalk can be divided into two main parts: discharge time T_d and crossing time T_c . Discharge time T_d is the necessary time for a pedestrian platoon to move from the waiting area and step inside the crosswalk. While crossing time T_c is the time which is necessary to cross the crosswalk:

$$T_t = T_d + T_c \tag{3}$$

The discharge time T_d is a function of pedestrian demand and crosswalk width. The definition of discharge time T_d is similar to that of vehicles waiting at the stop line of a signalized intersection. Shockwave theory is chosen for modeling pedestrian platoon discharge time.

Crossing time T_c is dependent on pedestrian crossing speed which is affected by the size of opposite pedestrian platoon and crosswalk width. This is analogous to a moving body facing a fluid which causes a reduction on its speed depending on its cross sectional area, the density of the fluid and the relative speed between them. This phenomenon is known as drag force theory and its analogy is used for modeling pedestrian platoon crossing time T_c .

For the purpose of this study, pedestrian demand is defined as the accumulated number of pedestrians at the edge of the crosswalk during the previous pedestrian flash green and red intervals.

4. MODELING DISCHARGE TIME T_d

Discharge time T_d basically depends on pedestrian arrival rate, pedestrian red interval, and crosswalk width. Shockwave analysis is used to estimate queue discharge time which is equivalent to the time necessary for a pedestrian platoon to discharge at the edge of the crosswalk.

(1) Model Assumptions

The following assumptions are made for modeling discharge time T_d :

- i) High pedestrian demand is assumed for the model development (Figure 1 a)). Later the developed model will be modified to consider low demand case.
- Pedestrian arrival rate A_1 is assumed to be uniform. ii) Therefore the accumulated pedestrian demand P_1 at the beginning of the pedestrian green interval is defined through Equation (4).

$$P_1 = A_1(C - g) \tag{4}$$

Where C is the cycle length and g is the pedestrian green interval.

iii) Pedestrian arrival unit is assumed as pedestrian row per second A_{rl} . Figure 1 a) shows how pedestrian rows are forming at high pedestrian demand. Assuming that the lateral distance which a pedestrian occupies along the crosswalk width is δ , then the maximum number of pedestrians that can fit through one row M_p along crosswalk width w is estimated through Equation (5).

$$M_p = \frac{w}{\delta} \tag{5}$$

To estimate pedestrian arrival rate in the unit of pedestrian row per second, arrival rate A_1 is divided by the maximum number of pedestrians that can fit in one row along the crosswalk width w.

$$A_{r1} = \frac{A_1}{M_p} = \frac{\delta A_1}{w} \tag{6}$$

Where A_{rl} is the arrival rate of the subject pedestrian demand in the unit of pedestrian rows per second. The number of accumulated pedestrian rows R_p at the start of pedestrian green interval is:

$$R_p = A_{r1}(C - g) = \frac{\partial A_1(C - g)}{w}$$
(7)

iv) The lateral distance that a pedestrian occupy δ is assumed to be a function of pedestrian demand and crosswalk width. However, for simplification,



C: cycle length, *g*: pedestrian green interval, *I*: pedestrian lost time, T_d : pedestrian platoon discharge time, *w*: crosswalk width, *D*: the longitudinal distance between waiting pedestrians, ω_1 : speed of stopping shockwave, ω_2 : speed of starting shock wave, A_{r1} : pedestrian arrival rate in pedestrian row per second, Q_d : discharge rate in pedestrian row per second, u_s : is pedestrian walking speed at sidewalks, u_o : is pedestrian free flow walking speed at crosswalks, K_i jam density in the unit of pedestrian row per meter.

Figure 1: Schematic formation of pedestrian rows at high demand and resulted shockwaves

longitudinal distance between waiting pedestrians D which is the same distance between pedestrian rows is assumed to be constant (**Figure 1** a)).

- v) Start-up lost time I is considered as a part of discharge time T_d , as shown in **Figure 1**.
- vi) To apply shockwave theory, speed of the arriving pedestrian rows at the sidewalk (upstream) is assumed to be equal to the pedestrian free-flow speed u_s at sidewalk. while speed of the discharging pedestrian rows at the crosswalk (downstream) is assumed to be equal to the pedestrian free-flow speed u_o at crosswalk, as shown in **Figure 1** b).

(2) Model Development

Figure 1 b) shows that two shockwaves are formed in the waiting area of the crosswalk. The first one is stopping shockwave which results from the continuous arrival of pedestrians to the waiting area that forms new rows. The second one is starting shockwave due to the discharge of the waiting pedestrian rows after pedestrian green indicator is displayed.

The speed of the stopping shockwave ω_1 is:

$$\omega_{1} = \frac{Q_{w} - Q_{u}}{K_{w} - K_{u}} = \frac{-A_{r1}}{K_{j} - A_{r1}/u_{s}} = \frac{-\delta A_{1}/w}{K_{j} - \delta A_{1}/wu_{s}}$$
(8)

The speed of the starting shockwave ω_2 is:

$$\omega_{2} = \frac{Q_{c} - Q_{w}}{K_{c} - K_{w}} = \frac{Q_{d}}{Q_{d}/u_{o} - K_{j}}$$
(9)

The number of arrived pedestrian rows should be equal to the number of discharged rows, therefore:

$$\omega_1((C-g)+T_d) = \omega_2 T_d \tag{10}$$

After substituting Equation (8) and (9) in Equation (10), the discharge time T_d becomes:

$$T_{d} = \frac{\left(\frac{-\delta A_{\rm l}/w}{K_{\rm j} - \delta A_{\rm l}/wu_{\rm s}}\right)(C-g)}{\left(\frac{Q_{\rm d}}{Q_{\rm d}/u_{\rm o} - K_{\rm j}}\right) - \left(\frac{-\delta A_{\rm l}/w}{K_{\rm j} - \delta A_{\rm l}/wu_{\rm s}}\right)}$$
(11)

(3) Parameter Estimation

The parameters included in Equation (11) are estimated as follows:

- i) To define a value for the pedestrian free-flow speed at crosswalks, a 1.5-hour video tape for the crosswalk at the east leg of Nishi-Osu intersection (6m wide × 25.4m long) in Nagoya City was analyzed. Nishi-Osu intersection is characterized by small pedestrian demand with a large crosswalk width. 102 samples of pedestrian's free-flow speeds were measured. All the considered pedestrians were leading pedestrians and they did not face any opposite flow or turning vehicles. The average free-flow speed for all the samples is 1.45 m/s. This value is assumed as the free-flow speed of pedestrians at crosswalks u_o.
 ii) Lam *et al.*¹⁰ studied pedestrian walking speed at
- ii) Lam *et al.*¹⁰⁾ studied pedestrian walking speed at different walking facilities and they found that pedestrian's free-flow walking speed at outdoor walkways is lower than that of signalized crosswalks by 17%. However, for the purpose of this study pedestrian free-flow speed at sidewalks u_s is assumed to be 20% less than the free-flow speed at crosswalks.
- iii) In order to define the jam density K_i , two 2-hour video tapes for the crosswalks at the east and west legs of Sasashima intersection (10m wide \times 19m long) in Nagoya City were analyzed. The data was collected in the morning peak hours, when long queues were formed due to the high pedestrian demand. The measured jam densities range between $0.9 - 1.4 \text{ ped/m}^2$, while the average is 1.1 ped/m^2 . The average measured value is assumed as pedestrian jam density K_i . It should be noted that the jam density which is used in the proposed model is in the unit of pedestrian row per meter. Therefore the minimum lateral distance δ_{min} that a pedestrian can occupy along the crosswalk width is assumed 1.0m, including the lateral clearance distance between waiting pedestrians. As a result, the jam density K_i in the unit of pedestrian row per meter (Figure 1 a)) is defined by Equation (12).

$$K_i = K_i (ped./m^2) * \delta_{\min} = 1.1 \quad ped.row/m$$
(12)

iv) Maximum discharge flow rate Q_d was observed at crosswalks with very high pedestrian demand. Two 2-hour video tapes for the crosswalks at the east and west legs of Sasashima intersection (10*m* wide× 19*m*)



a) Formations of pedestrian rows at low demand



b) Modeling occupied lateral distance δ

Figure 2: Low pedestrian demand case

long) in Nagoya City were analyzed. The average observed discharge rate at high pedestrian demand crosswalks is assumed as the maximum discharge flow rate Q_d . The average observed discharge rate is 0.45 *ped.row/sec*.

After estimating the jam density K_j and the discharge rate Q_d , Equation (11) can directly be used to estimate the discharge time T_d for high pedestrian demand. However at low pedestrian demand, some modifications should be considered, such as adjusting the lateral distance δ depending on pedestrian demand and crosswalk width.

(4) Modification for Low Pedestrian Demand Case

At low pedestrian demand many factors such as pedestrian origin and destination can affect pedestrian decisions regarding the waiting position which makes pedestrians form different rows in irregular patterns. Assuming a uniform pedestrian arrival rate, **Figure 2** a) shows how the pedestrian rows are forming when pedestrian demand is low. As pedestrian demand decreases, the number of pedestrians that forms a row decrease, which means that the lateral distance δ increases as pedestrian demand decreases.

Equation (11) is utilized by using the empirical data collected at Nishi-Osu and Sasashima intersections to estimate the average lateral distance that a pedestrian can occupy δ at different demand values. Then δ is modeled as a function of pedestrian demand per meter width of the crosswalk and is illustrated in **Figure 2** b).

A preliminary statistical analysis was performed to determine the best function to represent the relationship between δ and pedestrian demand per meter width of the crosswalk. The power function was found the best to describe this relationship (Equation (13)).

$$\delta = 2.5323 \left(\frac{P_1}{w}\right)^{-0.383} = 2.5323 \left(\frac{A_1(C-g)}{w}\right)^{-0.383}$$
(13)

Figure 2 b) shows that when pedestrian demand becomes high, δ becomes very close to 1.0 meter. This correspondes to the previous assumption of $\delta_{min} = 1.0 m$. After substituting Equation (13) in Equation (11), the resulted formula can be used to estimate discharge time T_d for any pedestrian demand volume and crosswalk width (**Figure 8** b)).

5. MODELING CROSSING TIME T_c

The force on an object that resists its motion through a fluid is called drag. When the fluid is a gas like air (**Figure 3** a)), it is called aerodynamic drag (or air resistance). While if the fluid is a liquid like water it is called hydrodynamic drag. Drag is a complicated phenomenon, and explaining it from a theory based entirely on fundamental principles is exceptionally difficult. Pugh¹¹⁾ described the relation of drag *D*, the relative velocity of the air or fluid and a moving body in terms of a dimensionless group, the drag coefficient *C_d*. The drag coefficient is the ratio of drag *D* to the dynamic pressure *q* of a moving air stream and is defined by Equation (14):

$$D = C_{d} q A_{p} \tag{14}$$

Where *D* is drag force $(kg \cdot m/sec^2)$, C_d is drag coefficient (dimensionless), *q* is dynamic pressure (force per unit area), and A_p is the projected area (m^2) .

The dynamic pressure q which is equivalent to the kinetic energy per unit volume of a moving solid body (Pugh¹²) is defined by Equation (15):

$$q = 0.5\rho u^2 \tag{15}$$

Where ρ is density of the air in kilogram per cubic meter, and *u* is the speed of the object relative to the fluid (*m/sec*). By substituting Equation (15) in Equation (14), the final drag force equation is:

$$q = 0.5C_d \rho u^2 A_p \tag{16}$$

(1) 'Drag Force' Caused by the Opposite Pedestrian Flow

To use the drag force concept to model the interactions between pedestrian flows, the following assumptions are made:

i) Opposite pedestrian demand is considered as a homogenous flow (**Figure 3** b)) with a density equal to the number of pedestrian waiting in the beginning of the green interval divided by an area equal to the width of the crosswalk multiplied by 1.0*m*.

$$\rho = \frac{P_2}{w*1} \qquad ped./m^2 \tag{17}$$

 The subject pedestrian flow is considered as one body moving against the opposite pedestrian flow. The interactions occur along the projected area of all pedestrians in the subject flow which is defined as



w: crosswalk width (m), L_0 : crosswalk length (m), P_2 : opposite pedestrian demand, P_1 : subject pedestrian demand, u_1 : speed of the subject pedestrian flow (m/sec), and u_2 : speed of the opposite pedestrian flow (m/sec).

Figure 3: Applying drag force concept on bi-directional pedestrian flows at crosswalk

the sum of the widths of all pedestrians in the subject flow:

$$A_p \approx \beta n \tag{18}$$

Where A_p is the projected area of the subject pedestrian flow (*m*), β is the average body width of one pedestrian, and *n* is the number of pedestrian in the subject pedestrian flow P_1 , shown in **Figure 3** b).

iii) The initial speed of the subject and the opposite pedestrian flow when they start crossing is assumed to be equal to their free-flow speed u_1 and u_2 respectively, therefore the relative speed *u* becomes:

$$u^{2} = (u_{1} - (-u_{2}))^{2} = (u_{1} + u_{2})^{2}$$
(19)

The initial speed is assumed to be constant value regardless of pedestrian platoon's density. Therefore in the case of uni-directional flow, crossing speed is always equal to free-flow speed.

After substituting Equation (17), (18), and (19) in Equation (16), then the drag force equation becomes:

$$D = 0.5 * C_d * \frac{P_2}{w} * (u_1 + u_2)^2 * \beta * n$$
 (20)

Assuming that the average width of one pedestrian body β is 0.6*m*, the drag force *D* becomes:

$$D = 0.5 * C_{Dadj} * \frac{P_2}{w} * (u_1 + u_2)^2 * n$$
(21)

Where C_{Dadj} is adjusted drag coefficient (dimensionless), and it is defined as:

$$C_{Dadj} = \beta^* C_d \tag{22}$$

(2) Deceleration of the Subject Pedestrian Flow

The net force on a particle observed from an inertial reference frame is proportional to the time rate of change of its linear momentum which is the product of mass and velocity:

$$F = \frac{dM}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma$$
(23)

Where *m* is the mass of the moving body and its equivalent to the subject pedestrian demand P_1 , and *a* is the average deceleration of the subject pedestrian flow. The final speed of a moving particle in a straight line with constant average deceleration according to the motion equations is:

$$u_f^2 = u_i^2 - 2aL$$
 (24)

Where u_i is initial speed (*m/sec*) which is assumed to be equal to the free-flow speed u_1 , u_f is final speed (*m/sec*), *a* is average deceleration (*m/sec*²), and *L* is travelled distance (*m*). **Figure 4** shows the projection of pedestrian flow trajectory from both sides of a crosswalk. A major assumption of this methodology is that both opposing flows will start walking with their free-flow speed on a straight line until they meet in the crosswalk. The meeting point is dependent on the speed of subject and opposite pedestrian platoons. The time when the two pedestrian platoons will meet is computed by Equation (25).

$$t = \frac{L_o}{u_1 + u_2} \tag{25}$$

The interaction distance l_i is assumed to be equal to the physical depth of the opposite pedestrian platoon. The physical depth of the opposite pedestrian platoon can be estimated by utilizing the methodology used for modeling the discharge time T_d . Therefore the physical depth l_i of the opposite pedestrian platoon is defined by Equation (26).

$$l_i = \frac{R_p}{K_i} = \frac{P_2 * \delta}{w * K_i} \tag{26}$$

Where R_p is the number of accumulated rows of the opposite pedestrian demand at the start of pedestrian green interval. **Figure 4** shows that resulting deceleration is averaged along the assumed interaction time. Therefore the final speed can be defined as:

$$u_{f} = \frac{l_{i}}{t_{i}} = \frac{l_{i}}{\left| \left(T_{c} - \frac{L_{o}}{u_{1} + u_{2}} (1 + \frac{u_{2}}{u_{1}}) + \frac{l_{i}}{u_{1}} \right) \right|}$$
(27)

Where T_c is crossing time of the subject pedestrian platoon and l_i is the physical depth of the opposite pedestrian platoon. If the physical depth of the opposite pedestrian platoon is longer than the remaining crossing distance (L_o - u_1t), the interaction distance will be equal to L_o - u_1t . Therefore when estimating the interaction distance, the physical depth of opposite pedestrian platoon should be compared with the remaining crossing distance for the subject pedestrian platoon and the smaller one should be considered as the interaction distance. By substituting Equation (27) in Equation (24), the average deceleration of the subject pedestrian platoon

$$a = \left[u_1^2 - \left(l_i / \left(T_c - \frac{L_o}{u_1 + u_2} (1 + \frac{u_2}{u_1}) + \frac{l_i}{u_1} \right) \right)^2 \right] / 2l_i \qquad (28)$$

The net force $(ped.m/sec^2)$ that causes the deceleration of the subject pedestrian platoon is defined as the average deceleration *a* (Equation (28)) multiplied by the mass of the subject pedestrian platoon *M* which is assumed to be equal to the subject pedestrian demand P_1 .



 L_o : crosswalk length, T_c : crossing time, t: time from the beginning of crossing until the subject and the opposite pedestrian platoons meet in the crosswalk, t_i : interaction time, l_i : interaction distance, u_1 and u_2 : free-flow speed of the subject and the opposite pedestrian flow respectively.

Figure 4: Time-space diagram of the conflicting pedestrian flows

(3) Model Development

The drag force caused by an opposite pedestrian flow should be equal to the force that causes the deceleration of the subject pedestrian flow. By equating the two forces, and solving them for the crossing time T_c , the net equation becomes:

$$T_{c} = \frac{l_{i}}{\sqrt{u_{1}^{2} - \frac{C_{Dadj}P_{2}l_{i}(u_{1}+u_{2})^{2}}{w}}} + \frac{L_{o}}{u_{1}+u_{2}}(1+\frac{u_{2}}{u_{1}}) - \frac{l_{i}}{u_{1}}$$
(29)

Pedestrian demand is defined as the number of accumulated pedestrians during pedestrian red and flash green signal indications, and those who arrive during the discharge time. Therefore opposite pedestrian demand can be presented as:

$$P_2 = A_2 * (C - g + T_d)$$
(30)

Where P_2 is opposite pedestrian demand, A_2 is arrival rate of the opposite pedestrian demand (*ped/sec*), and T_d is discharge time of the opposite pedestrian platoon. After substituting Equation (30) in Equation (29), the average crossing time and walking speed of the subjected pedestrian flow are given by Equations (31) and (32) respectively.

$$T_{c} = \frac{l_{i}}{\sqrt{u_{1}^{2} - \frac{C_{Dadj}A_{2}l_{i}(u_{1} + u_{2})^{2}(C - g + T_{d})}}}{w} + \frac{L_{o}}{u_{1} + u_{2}}(1 + \frac{u_{2}}{u_{1}}) - \frac{l_{i}}{u_{1}}$$

$$u_f = \sqrt{u_1^2 - \frac{C_{Dadj}A_2l_i(u_1 + u_2)^2(C - g + T_d)}{w}}$$
(32)

Equations (32) and (31) are final equations which represent how walking speed and crossing time vary with pedestrian demand combinations of bi-directional flow and crosswalk geometry.

(4) Estimating the Drag Coefficient C_{Dadi}

The value of adjusted drag coefficient C_{Dadj} according to aerodynamic drag is dependent on the



Figure 5: Modeling drag coefficient as a function of directional split ratio *r*

kinematic viscosity of the fluid, projected area and texture of the moving body. In the pedestrian's case, this value can here be assumed to be dependent on the pedestrian demand at both sides of the crosswalk and their split ratio.

To utilize Equations (32) and (31) to estimate speed drop and crossing time, C_{Dadj} was first estimated from empirical data. A 2-hour video tape for the crosswalk at the east leg of Imaike intersection (9.6m wide \times 21.5m long) in Nagoya City was analyzed. The pedestrian demand in each cycle at each direction, the average pedestrian trajectory length, and the average pedestrian crossing time in the same cycle were extracted from the video tape. Then by using Equation (31), C_{Dadj} was estimated for 38 data point where the total pedestrian demand was ranging from 5 - 30 pedestrians per cycle (one cycle is 160sec). In any case when pedestrians encounter a turning vehicle that causes a reduction on their speed or a change on their trajectory, the whole cycle was neglected and removed from the data base. Furthermore if pedestrians walk outside the crosswalk, that cycle was also neglected. After analyzing the available data, C_{Dadj} was modeled in terms of the split ratio r which is the ratio of the subject pedestrian demand to the total pedestrian demand. Equation (33) defines the directional split ratio r.

$$r = P_1 / (P_1 + P_2) \tag{33}$$

Figure 5 shows the relationship between directional split ratio and C_{Dadj} . As directional split ratio increases the drag coefficient also increases due to increasing pedestrian demand. After estimating the drag coefficient, Equations (32) and (31) can be used to estimate the average crossing speed and crossing time T_c for different demand volumes under different crosswalk dimensions.

6. FUNDEMENTAL DIAGRAMS

As an important step to validate how the proposed methodology describes the behavior of pedestrian flow, the fundamental diagrams of directional and bi directional pedestrian flow are drawn.

(1) For Subject Directional Pedestrian Flow

Figures 6 a) and b) represent the speed-flow and speed-density relationships for the subject pedestrian flow. The density of subject pedestrian flow k_1 is defined by Equation (34).



Figure 6: Fundamental diagrams for subject directional pedestrian flow at signalized crosswalks



Figure 7: Fundamental diagrams for bi-directional pedestrian flow at signalized crosswalks

$$K_1 = P_1 / (l_1 * w) \tag{34}$$

Where K_1 is the density of subject pedestrian platoon, P_1 is subject pedestrian demand, l_1 is physical depth of subject pedestrian platoon (Equation (26)) and w is crosswalk width. Figure 6 a) shows that as directional split ratio increases the maximum subject pedestrian flow increases. Figure 6 b) shows the drop in average walking speed due to the effects of bi-directional pedestrian flow. As the density of subject pedestrian flow increases either due to decreasing crosswalk width or increasing subject pedestrian demand, the interactions increase causing reduction in the average walking speed. The drop in the walking speed continues until a point where the speed drops drastically. This tendency is reasonable if we assume that pedestrian cannot walk outside the crosswalk. Therefore it is expected that the average walking speed will drop as the demand increases for a specific crosswalk width, until it reaches almost zero where every pedestrian cannot walk any more. Figure 6 b) shows that the crossing speed of the subject pedestrian flow also increases at a specific density when directional split ratio increases.

(2) For Both Bi-directional Pedestrian Flows

Figures 7 a) and b) represent the speed-flow and speed-density relationships for both directions of flow at the crosswalk. The total pedestrian density K_t at the crosswalk is defined according to Equation (35).

$$K_{t} = K_{1} + K_{2} = (P_{1}/(l_{1} * w)) + (P_{2}/(l_{2} * w))$$
(35)

Where K_1 and K_2 are the density of subject and opposite pedestrian platoon respectively, P_1 and P_2 are subject and opposite pedestrian demand respectively, l_1 and l_2 are physical depth of subject and opposite pedestrian platoons respectively and w is crosswalk width. Figure 7 a) shows that directional split ratio has significant impact on the maximum flow rate of a crosswalk. It shows that the interactions between opposing pedestrian flows increase as directional split ratio reaches 0.5 where the lowest maximum flow rate qoccurs. Furthermore at directional split ratio of 0.9 or 0.1 which is very close to uni-directional flow, the maximum total pedestrian flow q that can be achieved is 1.75 ped/m/sec. However Highway Capacity Manual²⁾ (EXHIBT 11-3) assumes a maximum uni-directional pedestrian flow rate of 1.66 ped/m/sec for commuters



Figure 8: Validations of the proposed models

which is between the total maximum bi-directional flow rates at directional split ratios of 0.9 or 0.1 and 0.8 or 0.2 according to the proposed methodology. This can be referred to the lower assumed free-flow speed and the consideration of aged pedestrians by HCM^{2} .

Figure 7 b) shows how the average crossing speed varies with total pedestrian density K_t . When total density K_t decreases either due to reducing pedestrian demand or increasing crosswalk width, crossing speed increases until it becomes almost constant (free-flow condition). But when total density K_t increases, crossing speed decreases, as the interactions between the opposing flows increases, until it reaches a point where the opposing flows block each other causing a drastic decrease in crossing speed.

7. VALIDATION AND COMPARISION

To validate the proposed models, average crossing speed was measured under different directional demand ratios and compared with the estimated speed from the proposed model. **Figure 8** a) illustrates the differences between measured and estimated crossing speeds. A paired t-test was performed and the result showed that the estimated values were not significantly different from the observed values at the 95% confidence level. The proposed model produces a mean absolute percentage error of 3.54%. According to the proposed methodology estimated speeds are expected to be lower than observed values. This tendency is logical since the developed model estimates the speed directly after the interaction with the opposite pedestrian flow while observed speed is the average speed through all the crossing process and it is measured by dividing pedestrian trajectory length to crossing time. However **Figure 8** a) shows that estimated speeds for some data points are higher than observed values. These points were extracted from the video tapes collected at Imaike intersection when pedestrian demand was low. If pedestrians walk slowly at low demand (limited interactions), this can be referred to their dresired speed which is not considered in the proposed methodology.

The discharge time T_d estimated by Equation (11) is compared with the observed data and the estimated discharge time from the existing formulations in HCM²) and Japanese Manual on Traffic Signal Control⁵, as shown in **Figure 8** b). By comparing the mean absolute percentage error and root mean square error, it is clear that the proposed model produces more accurate and reliable results. Furthermore the existing formulations always tend to undersestimate the necessary discharge time for large pedestrian platoons. The tendency of the proposed discharge time model is more consistent with the observed data.



Assumptions: Crosswalk length L_o is 20*m*, free flow speed of subject and opposite pedestrian flows (u_1 and u_2 respectively) is 1.45*m/s*, pedestrian speed at sidewalk u_s is 1.16*m/s*, pedestrian demand at each side of the crosswalk is 25*ped/cycle*, directional split ratio *r* is 0.5, jam density K_i is 1.1*ped.row/m*, and maximum discharge flow rate Q_d is 0.45*ped.row/sec*.

Figure 9: Comparison between lane-like segregation and mix-lane policies

8. MODEL APPLICATIONS

The developed models can be utilized for wide range of applications such as the evaluation of pedestrian flow at signalized intersections, assessing pedestrian signal timing and improving the geometric design of signalized crosswalks.

Teknomo¹⁾ proposed a lane-like segregation policy as an effective tool to improve pedestrian flow at signalized crosswalks. It aims to change the bidirectional flow into uni-directional as shown in **Figure 9** a). He concluded that lane-like segregation policy (unidirectional) is superior to mix-lane policy (bi-directional) in terms of average speed, uncomfortability, average delay and dissipation time. However in his analysis, the necessary discharging time for a pedestrian platoon and the resulted delay were not considered.

In order to evaluate the performance of the lane-like segregation policy, Figure 9 b) is presented with assumed values. It compares the crossing and discharging times of the lane-like segregation and mix-lane policies for the same subject pedestrian demand volume. If we assume that the existing policy is mix-lane where opposing pedestrian flows share the same space and the existing crosswalk width is 8m, then the total estimated crossing time T_t is 25sec (Figure 9 b)). However if lanelike segregation policy is implemented and the available crosswalk width is divided equally between the conflicting pedestrian flows (directional split ratio is assumed as 0.5) the total estimated crossing time is 28sec (Figure 9 b)). After the implementation of lane-like segregation policy, crossing speed increases and crossing time decreases. While discharging time increases because of dividing the available crosswalk width which will compensate the saving in crossing time and leads to higher total crossing times and delays. Therefore providing wider crosswalks for each pedestrian flow at both sides of the crosswalk is necessary in order to maintain the same total crossing time. However as crosswalks become wider, cycle length will increase because of all-red time requirement. Longer cycle lengths will cause longer delays and deteriorates the overall mobility levels of signalized intersections. Moreover

lane-like segregation policy may create new conflicts at both sides of the crosswalk due to the desired destination of pedestrians when they exit the crosswalk. According to the previous conclusions, it is not recommended to apply the lane-like segregation policy unless pedestrian level of service at a specific intersection is in a higher priority than other user's level of service.

Existing methodologies for the estimation of minimum required pedestrian green interval depend on constant average walking speed and crosswalk length to estimate total crossing time. Therefore developed models can be utilized for better estimation of the minimum required pedestrian green at signalized crosswalks with bi-directional pedestrian flow. However the main desired application behind the proposed methodology is to rationally define the required crosswalk width under different pedestrian demands considering the reduction in walking speed due to an opposite pedestrian flow.

9. CONCLUSIONS

A new methodology for modeling crossing time considering bi-directional pedestrian flow and discharge time necessary for a pedestrian platoon at signalized crosswalks is proposed in this paper. Shockwave and drag force theories are successfully utilized for modeling the discharge and crossing times respectively. The proposed methodology does not consider the speed variation between individuals inside the platoon. Meanwhile, such phenomenon can be considered by assuming a speed distribution for the pedestrian platoon instead of average constant speed.

The final formulation of crossing time T_c provides a rational quantification for the effects of crosswalk geometry and bi-directional pedestrian flow on walking speed and crossing time. However, the proposed models do not consider the effects of age, trip purpose and different ambient conditions on crossing and discharge times. Therefore calibrating the developed models will expand their applicability to different cases such as signalized crosswalks at school zones or crosswalks with aged pedestrian activities.

In the modeling methodology, it was assumed that the initial speeds of the subject and the opposite pedestrian platoons are constant values regardless of their densities. This assumption should be modified to consider the reduction in the initial speed due to the increasing in pedestrian platoon's density. The proposed crossing time model is validated with a limited data which does not cover many pedestrian demand combinations, therefore more concrete validation is required.

The developed models produce reasonable fundamental parameters of speed, flow and density. Lane-like segregation policy eliminates the conflicts between opposing pedestrian flows which increase crossing speed; however discharge time will significantly increase due to dividing the available crosswalk width. This leads to an increase in the total crossing time and the resultant total delay. Thus the implementation of wider crosswalks is required to reduce the total crossing time and total delay which causes longer cycle lengths and may negatively affect on the overall mobility levels of signalized intersections.

The proposed methodology will be utilized to rationally define the required crosswalk width under different pedestrian demand volumes considering the bidirectional flow effects.

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MODELING AND ANALYSIS OF PEDESTRIAN FLOW AT SIGNALIZED CROSSWALKS^{*}

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Pedestrian's crossing speed and time are of prime importance in studying the operation and design of signalized crosswalks. Existing methodologies do not consider the effects of bi-directional flow or crosswalk geometry on crossing speed and the resultant crossing time. In this study a new methodology is proposed for modeling the total time necessary for a platoon of pedestrians to cross a signalized crosswalk, considering the effects of bi-directional flow and crosswalk geometry. The fundamental relationships between speed, flow and density of bi-directional pedestrian flow are presented. An evaluation for the lane-like segregation policy for pedestrian crossing is included. The final formulation of crossing time T_c provides a rational quantification for the effects of crosswalk geometry and bi-directional pedestrian flow on crossing time and speed.

信号交差点における横断歩行者交通流のモデル分析^{*}

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歩行者の横断速度および横断時間は,信号制御された横断歩道の設計運用の検討に重要であるが,既存 手 法では双方向歩行者交通流や横断歩道幅員が横断速度および横断時間に与える影響について考慮されて いない.本研究では,これらの影響を考慮した横断歩行者の横断完了に必要な総時間をモデル化する新しい 手法を提案した.双方向歩行者交通流の速度,交通量,密度の基本的な関係を示すとともに,横断歩行者が レーン状に分離して横断する場合についても評価した.モデル化により,横断時間 T_cは横断歩道の幾何構 造と双方向の歩行者交通流が,横断時間および横断速度に与える影響を良好に再現可能であることが示され た.