OPTIMIZATION OF DHAKA'S MASS TRANSIT SERVICES FOR MINIMUM TRAVEL TIME AND COST.

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1. Introduction

In Dhaka Metropolitan Area (DMA), buses and minibuses are the primary mass transit modes and provide the cheapest and only affordable service to the urban poor. However, there are extreme irregularities in operation, scheduling, headway and fleet assignment, the number of stops, stop spacing and dwell time and access and egress from bus stops. Buses depart terminals at fixed intervals, however, headways become increasingly irregular as the vehicles move along their routes because of randomness in the schedule, drivers irregular stoppings, dwell time, en route interactions with slow moving motorized and non-motorized transport, and traffic congestion. Inevitably, prospective passengers are deterred from using bus services because the services are extremely irregular, unreliable, inconvenient and uncomfortable in terms of scheduling, waiting time, vehicle travel time and users travel time. Many passengers move from buses to less efficient and more expensive paratransit modes, which in turn reduces the total revenue of bus services.

The total cost to a bus service is the sum of the users travel time cost and the system operating cost. The users travel cost depends on the access/egress times and modes, waiting times at stops, in-vehicle times and transit fares. System operating cost depends on the fleet size requirements. The number of stops, stop spacing and traffic congestion have a remarkably affect on vehicle running speed and travel time, which in turn affects the fleet size requirements, and hence the operating cost of the service. Users travel time includes access/egress time, waiting time, in-vehicle time, which depends on the number of stops, headway, traffic congestion, and vehicle running speed. Therefore, we considered several tradeoff relationships within the basic transit parameters that are significantly interrelated in order to derive optimum models. The number of stops makes a tradeoff between the user access/egress time and the running speed of the vehicles. As the number of stops increases users access/egress time decreases, the time loss associated with acceleration and deceleration for stopping increases, and vehicle running speed decreases, which in turn influences the users travel time, vehicle travel time and cycle time (i.e., fleet sizes). Therefore, there must exist a number of stops for which user travel time and the travel cost are minimized. Another parameter, headway, makes a trade-off between the users travel costs and the fleet size requirements (i.e., operating costs). User travel time cost increases with longer headways because waiting time and boarding/alighting time both increases. Operating cost decreases with longer headways because fleet size is inversely related to headway. Therefore, there must be an optimum headway for which user travel time and travel costs are minimized. Furthermore, en route traffic congestion and interaction with motorized and non-motorized vehicles reduces running speeds and increases the average passenger waiting time and travel time. Traffic congestion causes delayed arrival of buses, and so more passengers accumulate at the stop, which increases the stopping and standing time and the average passengers waiting and travel time. Since, the number of stops, headway, traffic congestion and vehicle interactions all have a composite effect on vehicle travel time, waiting time, fleet size requirements, operating cost and capital cost, the independence relationships between these parameters must be carefully examined.

The basic equations for users travel time and vehicle travel time for local (stop at all stops), call-on (stop only when hailed or passengers are alighting), request-stop (stop anywhere along the route on passenger demand), accelerated (skips different sets of predetermined stops) and express (limited stop) services are derived in a previous study [7]. In this paper only a brief derivation is provided. Additionally, the optimum users travel time models have already been partially analyzed [6], and so emphasis is placed on analyzing the minimum travel cost models in this study, with less attention to users travel time models.

The optimization of various physical and operational aspects of public transportation systems has been the subjects of many studies. Vuchic (1966) analyzed optimal station locations for two different criteria. Byrne and Vuchic (1972) analyzed the problem of finding minimum-cost line positions and headway. Lesley (1976) analyzed bus stops spacing for minimum user cost and minimum total cost. There have been a number of studies on the mass transit of the DMA, of which Firdus (1984), Ahsan (1993), DITS (1993), and Zahir (1997) are worth mentioning. Most of the studies point out the overall problems suffered by the DMA mass transit and passengers transport system. However, none of these previous studies adequately considered the optimum parametric interdependencies that can be used to minimize users travel time and total travel cost. Therefore, there exists a need to present the spectrum of optimum stopping policies, optimum number of stops and optimum headway models with respect to minimum travel cost and users travel time objectives, and system parameter combinations.

In this study we developed a methodology for determining the optimum headway, number of stops, fleet size, optimum stopping and scheduling policies for local, call-on and request-stop services in order to minimize users travel time and travel cost. We also performed the

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sensitivity test and determined the effect of small change of the basic parameters to the transit performance and their mutual influences based on the field survey. We conducted four types of field surveys to collect bus data in DMA bus routes in July 1999. Theoretical guidelines for the selection of transit stopping policies, number of stops, headway under different transit services and operating conditions were presented through simulation by using the realistic data. From these variations, individual operators and passengers will be able to enjoy regular, reliable, scheduled services through mutual maximization of their respective benefits by reducing the users travel time and operation cost, and in the process maximizing revenue by increasing patronage.

2. The Problems

A wide variety of methods of ownership and operations, levels of control, regulations and competition exist in DMA's mass transit system. The private sector consists of an extremely fragmented ownership patterns (average two buses per owner) are dominating and proving the monopoly (almost 95% of total services)[3] services in all bus/minibus routes. The public sector having very small fleet size and proving transit services in few routes. In private sector, the individual owner groups into a number of route associations. The central owner association monitors, controls and operates the total fleet size and headway, and allocates the bus fleet in different transit routes as per demand and period of operation through the route associations. Therefore, the route fleet size of DMA's transit route is adjustable and re-allocable to any route without capital investment. However, this extremely fragmented ownership pattern in private sector prevents professional management, unified control, coordination, collective policy making, capital accumulation for investments, ability to make industry-wide strategic decisions, scope for optimizing resources utilization and company based mass transit operation. Individual owner sees his or her own interest in micro sense and engage in wasteful aggressive competition including overloading, haphazard boarding, and alighting rather than thinking the interests of whole industry in macro sense. That result leads an extremely irregular, unreliable, uncomfortable and inconvenience services in terms of levels of service, time scheduling, fleet size assignment, traffic congestion, stop spacings and drivers unusual behavior on en route. Buses move with slow moving mixed traffic of small motorized and non-motorized vehicles that create multiple interactions between the vehicles and cause several slow-downs on en route and reduce the average transit speed that increases the time loss associated with acceleration and deceleration and the users travel time. Moreover, in peak-period buses become fully loaded in the beginning of journey and drivers stopped vehicles mostly at the major stops and skipped small stops even the passengers are waiting for boarding. But in off-peak, buses stop almost at all stops even there are no passenger for boarding/alighting and the buses wait for passenger arrivals. It is clear that drivers stop longer at some stops for boarding/alighting and waiting for passenger arrivals if the number of on-board passenger is not enough to earn satisfactory revenue, otherwise skip or make short stop in less demanded stops. These situations increase the users travel time, vehicle travel time and passenger-waiting time at skipped stops as well as longer stopped stops, which adversely affect to the safety and traffic congestion around bus stops and stimulate unhealthy competition among bus drivers. These optimization models in this paper would reduce those existing irregularities in peak and off peak periods and enhance the levels and reliability of transit services that would reduce the average users travel times and travel cost to a great extent.

3. Survey and Data

From July 10 to July 30, 1999, we conducted four types of field surveys at four important bus routes namely, route no.1, route no. 8, route no.9, and route no. 13 in DMA for local and express service in peak and off-peak periods for the both directions. Firstly, at the bus stop survey, we collected data on bus and passenger arrival and departure times; the number of passenger boarding and alighting and left behind; bus stopping and standing time; and passengers waiting times at stops along the routes and the service interval times at terminals. Secondly, in the passenger interview survey at bus stops, we collected data on the passengers access and egress modes and times; average in-vehicle travel time and travel distance; fares paid to bus and to access/egress modes; and levels and reliability of services. Thirdly, we obtained data on the vehicle travel time, route length, stop spacings, terminal time, the number and duration of the vehicle facing congestion and the average boarding and alighting time per person. Fourthly, in order to determine vehicles dynamic characteristics (speed, acceleration and deceleration rates etc.), we used "YAZAKI IN-VEHICLE SPEED DETECTOR" that read the practical speeds, acceleration and deceleration rates directly through the speed and clock pulse signals of the vehicle at 0.5sec intervals along the route length. Therefore, we could determine the actual transit travel pattern, stopping time and standing time at stops; the number and duration and locations of traffic congestion, and the drivers' behavior in peak and off-peak periods along the route. At present, there does not exist any call-on, accelerated and request stop types of transit service in DMA. Therefore, we could not be able to collect field data for those services. However, since the number of stops, route length, passenger generation rates and other parametric values for local service are similar to those for call-on service except stopping criteria, we used the same data collected for local service for analyzing the call-on service.

4. Model Development

The development of these models are to determine the types of operation, number and locations of stops, headway, fleet size, vehicle capacity, speed and other related operational aspects for a given transit line that minimize the total travel cost and users travel time. Since, the total DMA's transit fleet is monitored and controlled by a central owner association, route fleet size is adjustable to any route as per demand without capital investment. We considered that the route fleet is large enough, thus, the problem becomes to determining of the headway and the number of vehicles to be used for the

transit line for the different services. This is the most common problem faced by DMA's private operator for their short-range decisions. However, if the fleet size is limited, the selection of right combination of the number of stops and headway should be made from the feasible combinations of these two parameters for a given fleet size. If the fleet size is fixed, the system operating cost and capital cost is unchanged. However, the number of stops and stopping criteria, and en route traffic congestion affect the vehicle operating speed, access and egress time, in-vehicle riding time, vehicle stopping time and acceleration and deceleration time loses which in turn affect the vehicle cycle time. For a given fleet size the increases in the number of stops and traffic congestion increase the feasible headway that increases the average passenger waiting time. Furthermore, with the increasing of the number of stops, user access/egress time decreases and users in vehicle riding time increases. Therefore, there is a combination of the number of stop and headway that minimizes the users travel time for a given fleet size. In the following sections we formulated and derived the optimum headway models, number of stop models, combination of the number of stops and headway models that minimize the system operation cost and users travel time for local and call-on for large and limited fleet size for $S = L/n > S_c$. I.e., when the distance between two consecutive stoppings is longer than the critical stop distance S_c that allows a vehicle to accelerate into a constant cruising speed V before decelerate into stop. Where, S is the average spacing distance between two consecutive stops, L is the entire route length, and n is the number of stoppings.

(1) Cycle Time and Fleet Size

Cycle time is the mean time for a vehicle to complete a round trip including the terminal time T_t that spent at each terminal. Assuming that the vehicle travel time T_v for entire route length L and terminal time are the same in each direction, so we could express the cycle time θ is twice the sum of the vehicle travel time and terminal time as follows:

$$\theta = 2(T_v + T_t) \quad \text{----------(1)}$$

The fleet size N is defined as the number of vehicles serving the route. The average headway h, between the vehicles can be expressed by $h = \theta/N$. Therefore, the fleet size is,

$$N = \frac{\theta}{h} = \frac{2(T_v + T_t)}{h} \quad \text{----------(2)}$$

(2) Number of Stoppings for Different Services

Local Service: The vehicle stops at all prefixed stops whether there is passenger demand or not. The number of stoppings n is equal to the number of stops provided, i.e., $n = (s-1)$. Where, s is the total number of equidistant stops including terminals.

Call-on Service: The vehicle stops at prefixed stops only where the passengers' demand for boarding or alighting exists, otherwise skipped. The number of stoppings for the one way vehicle trip depends upon the passenger generation rate and trip origin/destination pattern. It is less or equal to the number of stops provided. Passenger demands assume to be independent of the stopping policy and number of stops provided. The probability of passenger arrival at stops and boarding and alighting demand along the route follow a Poisson distribution. The sum of the boarding and alighting demand at any point along the route is constant per unit time. The bus arrival at stops follows Binomial distribution. The mean number of passengers using a stop for boarding and alighting is $m = 2p\theta/N(s-1)$, where, p is the mean number of trips generated per unit time. This mean number of passenger m can not use all buses if the bus does not stop in some bus stops. So, the probability of r passengers boarding and alighting at a stop $P(r)$ can be express as under:

$$P(r) = e^{-m}\frac{m^r}{r!} = e^{\frac{-2p\theta}{N(s-1)}}\frac{\left\{\frac{2p\theta}{N(s-1)}\right\}^r}{r!} = e^{\frac{-2ph}{(s-1)}}\frac{\left\{\frac{2ph}{(s-1)}\right\}^r}{r!} \quad \text{----------(3)}$$

It is also seldom happen that a stop is skipped because the vehicle is full and nobody wishes to alight. When the bus has skipped, it means that the number of boarding and alighting is zero. The probability of skipping a bus stop $P(0)$ is obtained by substituting $r = 0$ in equation (3), which is equal to $P(0) = e^{\frac{-2ph}{(s-1)}}$. Hence, the probability of stoppings a bus at a stop, $P_s(s) = 1 - e^{\frac{-2ph}{(s-1)}}$ ----------(4)

Therefore, the average number of stoppings for a one-way trip for call-on service, $n^c(s)$ = Total number of stops in one way trip × the probability of a bus stoppings at a stop.

$$n^c(s) = (s-1)\times P_s(s) = (s-1)\times\left\{1 - e^{\frac{-2ph}{(s-1)}}\right\} \quad \text{----------(5)}$$

Request-stop Service: The vehicle stops anywhere at the passenger's origins and destinations along the route whenever there is request for boarding or alighting. Theoretically, the number of stoppings is infinite, however, each stopping is the purpose for at least a single boarding or alighting i.e., maximum number of stoppings are twice the number of total trip generated along the route for one-way trip, $2p\theta/N$. Moreover, there is a possibility of simultaneous boarding and alighting and also more than one passenger may board or alight in a single stopping. The number of stoppings will be less than the maximum number of stoppings $2p\theta/N$. In that case, the number of stoppings for request service, $n^r = 2p\theta/(cN)$, where, c is the average number of passengers board or alight simultaneously during a single stopping.

(3) The Vehicle Travel Time and Users Travel Time

The vehicle travel time T_v for making a one way trip is the sum of the all the travel times between two consecutive stops along the entire length. The travel time between two consecutive stops is the sum of acceleration and deceleration time and time loss associated with traffic congestion/obstacles, constant speed state time, and stopping time for boarding and alighting passengers at stops. When the vehicle faces m times traffic congestion or obstacles at equidistant within two adjacent stoppings for a moment, vehicle travel times T_v^l, T_v^c & T_v^r for local, call-

on and request-stop services are derived [7] respectively as under:

$$T_v^l = n^l\sqrt{2(m+1)S_c(a+b)/ab} + 2ph\mu + n^l(S-S_c)/V$$

$$T_v^l = n^l\xi + L/V + 2ph\mu \text{ ----------(6)}$$

$$T_v^c = n^c\sqrt{2(m+1)S_c(a+b)/ab} + 2ph\mu + n^c(S-S_c)/V$$

$$T_v^c = n^c\xi + L/V + 2ph\mu \text{ ----------(7)}$$

$$T_v^r = n^r\xi + L/V + 2ph\mu \text{ ----------(8)}$$

Where, $\xi = \sqrt{2(m+1)S_c(a+b)/ab} - S_c/V$;

And, the total acceleration and deceleration time and congestion time loss are $n^l\sqrt{2(m+1)S_c(a+b)/ab}$, the cruising speed state travel time $n(S-S_c)/V$ and the boarding and alighting time $2ph\mu$ for the entire route. n^l, n^c and n^r are the respective number of stoppings for local, call-on and request stop service; a and b are the linear rate of acceleration and deceleration; μ is the average boarding or alighting time per person, and l is the average users travel distance.

The users travel time T_u consists of users access and egress time $T_e = L(3-2x)/\{6V_a(s-1)\}$, waiting time at stops $T_w = h/2(1+2q)$ and average in-vehicle riding time $T_m = l/L(n\xi + L/V + 2p\mu h)$, where, V_a is the walking speed, x is the portion of passenger access to and egress from the stops by walking, and q is the probability of two successive full vehicles. The users travel times T_u^l, T_u^c & T_u^r for local, call-on and request-stop service while the vehicle faces m times traffic congestion or obstacles at equidistant within two adjacent stoppings for a moment is derived [7] respectively as under:

$$T_u^l = l/L\{n^l\xi + L/V + 2ph\mu\} + L(3-2x)/\{6V_a(s-1)\} + h/2(1+2q) \text{ ----------(9)}$$

$$T_u^c = l/L(n^c\xi + L/V + 2ph\mu) + L(3-2x)/\{6V_a(s-1)\} + h/2(1+2q) \text{ ----------(10)}$$

$$T_u^r = l/L\{n^r\xi + L/V + 2ph\mu\} + h/2(1+2q) \text{ ----------(11)}$$

By using equation (2), (6), (7) and (8) we can express the fleet size for local N^l, call-on N^c and request-stop service N^r as under:

$$N^l = 2(n^l\xi + L/V + 2ph\mu + T_t)/h \text{ ----------(12)}$$

$$N^c = 2(n^c\xi + L/V + 2ph\mu + T_t)/h \text{ ----------(13)}$$

$$N^r = 2(n^r\xi + L/V + 2ph\mu + T_t)/h \text{ ----------(14)}$$

(4) Total Cost

The total cost C_t per unit time is assumed to consist of the users travel time cost per unit time C_u and the system operating cost per unit time C_o, which can be expressed as: $C_t = C_u + C_o$ ----------(15)

However, the users travel time cost consists of four elements of costs; access and egress time cost, waiting time cost, riding time cost and fare to access/egress modes and transit service. We could define the values of unit access/egress time ψ_e, waiting time ψ_w and riding time ψ_m, and the total users travel time cost per passenger per unit time could be expressed as:

$$C_u = T_e\psi_e + T_w\psi_w + T_m\psi_m + F_b + F_r \text{ ----------(16)}$$

Where, T_e, T_w and T_m are the average users access and egress time, waiting time and in-vehicle riding time and F_b and F_r are the average individual fare paid to transit and rickshaw service respectively.

For the simplification, we convert the all users trip time elements of unit values into an equivalent uniform monetary unit value ψ_c taka/hour (1 taka=0.02 US$) to calculate the average users time value and add up the fare paid to transit and rickshaw to determine the average total users travel time cost per hour.

$$C_u = P\psi_c(T_e+T_w+T_m) + PF_b + 0.6PF_r \text{ ----------(17)}$$

$$C_u = P\psi_c\left[\begin{matrix} l/L(n\xi + L/V + 2p\mu h) + h/2(1+2q) \\ + L(3-2x)/\{6V_a(s-1)\} \end{matrix}\right] + PF_b + 0.6PF_r \text{ ----------(18)}$$

Where, P is the average passenger volume per hour. We found [3] 30% of the total passenger access/egress to and from transit service by rickshaw. Therefore, the total rickshaw fare per hour is accounted for $0.6PF_r$ for access and egress.

The system operating cost is the cost per hour for the operation of transit services. It consists of fixed-cost (head office cost), semi-variable cost (deports) and variable cost (fuel, crew, maintenance etc.). We added up these two variable costs and defined the operating cost as fixed cost and variable cost. The variable cost per hour is the product of the fleet size and the average operating cost per vehicle per hour V_c. Therefore, the total system operating cost per hour of a particular bus route is made up the fixed cost (F) per hour plus the total variable cost per hour and can be expressed as:

$$C_o = F + NV_c = F + 2V_c(n\xi + L/V + 2ph\mu + T_t)/h \text{ --(19)}$$

5. Models Formulation and Optimization

In this section we formulate models to determine the optimum conditions and interrelations among the basic transit factors by correlating performance parameters, vehicle dynamic characteristics, en route traffic congestion and users travel time cost and system operating cost for the minimization of total travel cost and users travel time. Since the number of stops, fleet size and headway are the basic parameters of a transit service and others can be expressed in terms of them, we formulate the models in terms of headway, number of stops and fleet sizes. The optimization is performed through the minimization of the objective function subject to given constraints. The models are formulated as under:

(1) Headway Model (Min. Travel Cost)

The purpose of this model is to determine the optimum headway, fleet size requirement and vehicle capacity that minimize the total cost for a given number of stops, passenger generation rates, vehicle dynamic characteristics etc. and the model is formulated as:

Minimize total travel cost, i.e., Min. $C_t = \{C_u + C_o\}$ ---(20)

Where, $C_u = P\psi_c T_u(h) + PF_b + 0.6PF_r$ and

$$C_o = F + V_c N(h)$$

Given Parameters: P, p, L, l, a, b, V, V_a, ψ_c, V_c, μ, $(s-1)$, F, F_b, F_r, m.

Optimized parameters: headway h and fleet size N.

(a) Optimization

After substituting the values of C_u and C_o from the equation (18) and (19), differentiate the objective function (20) with respect to headway h, and set to zero for differential for optimization, and solve it for h, we will gain the optimum headway h_l^* that minimizes the total travel cost for local service as:

$$h_l^* = \left\{2V_c\left(n^l\xi + L/V + T_t\right)/\left[P\psi_c\left\{2p\mu l/L + (1+2q)/2\right\}\right]\right\}^{1/2} \text{-(21)}$$

From the equation (21), we could see that the optimum headway h_l^* is proportional (but not linearly) to the number of stops and inversely proportional to the square root of passenger demand p and user's time value ψ_c.

By substituting the value of the optimum headway h_l^* from equation (21) into (12) we could find the corresponding optimum fleet size N_l^* for local service as:

$$N_l^* = 2\left(n^l\xi + L/V + T_t + 2p\mu h_l^*\right)/h_l^* \text{ ----------(22)}$$

Similarly, we derived the optimum headway function equation for call-on service that minimizes the total travel cost as:

$$P\psi_c\left[\frac{l}{L}\left\{2pe^{\frac{-2ph}{(s-1)}}.\xi + 2p\mu\right\} + \frac{1}{2}(1+2q)\right] + \frac{2V_c}{h}\left\{2pe^{\frac{-2ph}{(s-1)}}\xi + 2p\mu\right\} - \frac{2V_c}{h^2}\left\{n^c\xi + L/V + 2ph\mu + T_t\right\} = 0 \text{----------(23)}$$

Since it is a non-linear equation, we can not derive an expression of optimum headway h_c^* as a function of other parameters. It is not possible to solve this equation for the optimum headway h_c^* analytically. Therefore, we used the Newton's method to solve the systems of non-linear equation to determine the optimum headway h_c^* that minimizes the travel cost for a given number of stops, passenger generation rates and vehicle dynamic characteristics. The corresponding optimum fleet size N_c^* is calculated from equation (13).

(2) Number of Stops Model (Min. Travel Cost)

This model is to determine the optimum number of stops for a given headway, passenger generation rates, vehicle dynamic characteristics, and the type of operation that minimize the total travel cost can be formulated as:

Minimize the total travel cost, i.e., Min. $C_t = C_u + C_o$ -(24)

Where, $C_u = P\psi_c T_u(s-1) + PF_b + 0.6PF_r$ and

$$C_o = F + V_c N(s-1).$$

Given parameters: P, p, h, L, l, a, b, V, V_a, V_r, m, ψ_c, n, μ, F, F_b, F_r

Optimized parameter: $(s-1)$ and N.

(a) Optimization

Similarly, after substituting the values of C_u and C_o from equation (18) and (19), differentiate the objective function equation (24) with respect to $(s-1)$and set to zero for optimization and solve the optimum number of stops $(s-1)_l^*$ for local service that minimizes the total travel cost as:

$$(s-1)_l^* = \left[P\psi_c L(3-2x)/6V_a\xi\left(P\psi_c l/L + 2V_c/h\right)\right]^{1/2} \text{---------(25)}$$

Similarly, we derived the optimum relationships between the optimum number of stops and transit parameters for call-on service that minimizes the total travel cost as:

$$\xi\left\{1 - e^{\frac{-2ph}{s-1}}\left(1 + \frac{2ph}{s-1}\right)\right\}\left(\frac{P\psi_c l}{L} + \frac{2V_c}{h}\right) - \frac{P\psi_c L(3-2x)}{6V_a(s-1)^2} = 0 \text{-(26)}$$

We solved this equation by using Newton's method for $(s-1)$, which is the optimum number of stops $(s-1)_c^*$ for call-on service for given headway and vehicle dynamic characteristics that minimize the total travel cost. The corresponding optimum fleet size N_c^* for call-on service is determined from the equation (13).

(3) Number of Stops Model (Min. Users Travel Time)

The purpose of this model is to determine the optimum number of stops for local and call-on service, which minimize the users travel time for a given headway and traffic congestion and vehicle dynamic characteristics and can be formulated as:

Minimize user travels time: $Min\{T_u(s-1)\}$----------(27)

Optimized parameter: Number of stops, $(s-1)$.

Given Parameters: p, L, l, h, a, b, V, V_a, μ, m.

Local service: After substituting the value of users travel time T_u from equation (9) into the objective function equation (27), differentiate it with respect to $(s-1)$and set to zero for optimization, and solve the optimum number of stops $(s-1)_l^{**}$ that minimizes the user travel time for local service as:

$$(s-1)_l^{**} = L\left\{(3-2x)/6lV_a\xi\right\}^{1/2} \text{----------(28)}$$

From equation (28) it is clear that the optimum number of stops for minimum users travel time is independent of passenger generation rate p and headway h. Therefore, $(s-1)_l^{**}$ is not influenced by the vehicle capacity. It is rather a function of vehicle dynamic characteristics, traffic congestion, the average users travel distances l and access/egress speed V_a. In comparison with the equation (25) and (28), the optimum numbers of stops for the minimum users travel time $(s-1)_l^{**}$ is greater than the optimum number of stops for minimum total travel cost $(s-1)_l^*$, i.e., $(s-1)_l^{**} > (s-1)_l^*$. And $(s-1)_l^{**}$ is independent of h and p, whereas $(s-1)_l^*$ is a function of headway and passenger generation rates.

Call-on service: Similarly, after putting the value of T_u from equation (10) into the objective function equation (27), differentiate it with respect to $(s-1)$and set to zero and derived the optimum relationship between the number

of stops and others parameters for call-on service that minimize user travel time as:

$$\frac{l}{L}\xi\left\{1-e^{\frac{-2ph}{s-1}}\left(1+\frac{2ph}{s-1}\right)\right\}-\frac{L(3-2x)}{6V_a(s-1)^2}=0 \text{ ----------(29)}$$

From this equation, it is noticed that p and h always appear in the product (ph) form, which is also equal to the average total passenger generation for one-way trip. Therefore, the optimum number of stops $(s-1)_c^{**}$ for call-on service is a function of passenger generation rates for one-way trip that minimize the users travel time. Since this is a non-linear equation, we solve this equation for $(s-1)$by Newton's numerical method. The solution is the optimum number of stops $(s-1)_c^{**}$. The corresponding optimum headway h_c^{**} for call-on service is determined from equation (13).

(4) Number of Stops and Headway Model (Min. Travel Cost)

The objectives of this model is to determine the optimum combinations of the number of stops, headway, and fleet size that minimizes the total travel cost for a given vehicle dynamic characteristics, congestion levels and transit line for local and call-on service. The model is formulated as:

Minimize total travel cost, i.e., Min. $C_t = \{C_u + C_o\}$ ---(30)

Where, $C_u = P\psi_c.T_u\ \{(s-1),h\}$ ----------(31)

And, $C_o = F + V_c N\ \{(s-1),h\}$ ----------(32)

Given Parameters: P, p, L, l, a, b, V, V_a, ψ_c, V_c, μ, F, F_b, F_r, m.

Optimized parameters: number of stops $(s-1)$, headway h, and fleet size N (number of stops, headway).

(a) Optimization

After, substituting the value of C_u and C_o from equation (18) and (19) into the objective function equation (30), differentiate it with respect to $(s-1)$and h, and set each differential to zero for the simultaneous optimization of the number of stops and headway that minimize the total travel cost. The optimum combination of the number of stops and headway can be determined by simultaneously solving the following two equations for local service.

$$P\psi_c\left[l/L.\xi - L(3-2x)/\left\{6V_a(s-1)^2\right\}\right] + 2V_c\,\xi/h = 0 \text{ -----(33)}$$

$$P\psi_c\{2p\mu l/L + (1+2q)/2\} - [n\xi + L/V + T_t]2V_c/h^2 = 0 \text{ --(34)}$$

Similarly, we derived the optimum combination between the number of stops and headway that minimize the total travel cost for call-on service as:

$$\xi\left\{1-e^{\frac{-2ph}{s-1}}\left(1+\frac{2ph}{s-1}\right)\right\}\left(P\psi_c.\frac{l}{L}+\frac{2V_c}{h}\right) - P\psi_c.\frac{L(3-2x)}{6V_a(s-1)^2}=0 \text{ ----------(35)}$$

$$P\psi_c\left[\frac{l}{L}\left\{2pe^{\frac{-2ph}{(s-1)}}.\xi+2p\mu\right\}+\frac{1}{2}(1+2q)\right] + \frac{2V_c}{h}\left\{2pe^{\frac{-2ph}{(s-1)}}\ \xi+2p\mu\right\} - \frac{2V_c}{h^2}\left\{n^c\xi + L/V + 2ph\mu + T_t\right\}=0 \text{ ------(36)}$$

Now, simultaneously solving the equation (33) and (34), and (35) and (36) by Newton's numerical method for $(s-1)$ and h will give the optimum combination of the number of stops and headway that minimizes the total cost for local and call-on service respectively. The optimum pairs of values of $(s-1)$ and h is obtained for a given fleet size and passenger generation rate. By substituting the values of the optimum headway and number of stops into the equation (12) and (13), we determined the corresponding optimum fleet sizes for local and call-on service respectively.

(5) Number of Stops and Headway Model (Min. Users Travel Time)

The objectives of this model is to determine the optimum combinations of number of stop and headway for a given fleet size, vehicle dynamic characteristics, congestion levels that minimize the users travel time for local and call-on service. The model can be formulated as:

Minimize user travel time: $Min\{T_u(s-1,h)\}$ ----------(37)

Subject to $N\{s-1,h\}=C$

Optimized parameter: $(s-1)$ and h.

Given Parameters: p, L, l, a, b, V, V_a, μ, m.

Local service: After substituting the value of h from equation (12) into (9) and T_u from equation (9) into (37), differentiate equation (37) with respect to $(s-1)$ and h, and set each to zero to derive the optimum combination of the number of stops and headway for a given fleet size and vehicle dynamic characteristics.

$$(s-1)_l^{***} = \left\{L(3-2x)\Big/ 6V_a\xi\left[\frac{l}{L}\left\{1+\frac{4p\mu}{N-4p\mu}\right\}+\frac{1+2q}{N-4p\mu}\right]\right\}^{1/2}$$

----------(38)

$$h_l^{***} = \frac{2\left[\xi\left\{L(3-2x)\Big/ 6V_a\xi\left[\frac{l}{L}\left\{1+\frac{4p\mu}{N-4p\mu}\right\}+\frac{1+2q}{N-4p\mu}\right]\right\}^{1/2}+\frac{L}{V}+T_t\right]}{N-4p\mu}$$

----------(39)

Solving the equation (38) and (39) simultaneously for $(s-1)$ and h will give the optimum pair of values of $(s-1)_l^{***}$ and h_l^{***} for a given fleet size and passenger generation rate that minimizes the users travel time for local service.

Call-on Service: The number of stops and headway in the users travel time equation (10) and fleet-size constraint equation (13) for call-on services are related in a complicated manner. The substitution method in above would not be practical. Hence, we used Lagrange multiplier method and defined the problem as:

$$M\{(s-1),h,u\} = T_u\{(s-1),h\} + u\{C - N(s-1,h)\} \text{ ------(40)}$$

Where, u is the Lagrange multiplier. Differentiate the equation (40) with respect to $(s-1)$, h and u respectively and set each differential to zero for optimization as:

$$\frac{\partial M}{\partial(s-1)} = \frac{l}{L}\xi\left\{1-\left(1+\frac{2ph}{s-1}\right)e^{\frac{-2ph}{(s-1)}}\right\} - \frac{L(3-2x)}{6V_a(s-1)^2}$$

$$-\frac{2u\xi}{h}\left\{1-\left(1+\frac{2ph}{s-1}\right)e^{\frac{-2ph}{(s-1)}}\right\} = 0 \text{ ----------(41)}$$

$$\frac{\partial M}{\partial h} = \frac{l}{L}\left\{2pe^{\frac{-2ph}{(s-1)}}.\xi + 2p\mu\right\} + \frac{1}{2}(1+2q) +$$

$$\frac{2u}{h^2}\left\{\left(n^c\xi + L/V + 2ph\mu + T_t\right) - h\left(2pe^{\frac{-2ph}{(s-1)}}\xi + 2p\mu\right)\right\} = 0\text{-(42)}$$

$$\frac{\partial M}{\partial u} = C - \frac{2}{h}\left(n^c\xi + L/V + 2ph\mu + T_t\right) = 0\text{----------(43)}$$

Similarly, by solving the equation (41), (42) and (43) by Newton's numerical method we determined the optimum combinations of the number of stops and headway for given fleet size and vehicle dynamic characteristics that minimize the average users travel time.

There are many non-liner equations in this paper. Some of the equations are very complicated. We could not prove the unique solutions for all these equation. Therefore, we analyzed the validity of those equations in section 6. The simulation results reflected the correct interrelation between the variables.

6. Results and Discussions

The numerical results obtained here represented graphically to illustrate the optimum parametric interrelationships and interdependencies in different model situations at different congestion levels for local and call-on service for the minimum users travel time and travel cost. We performed the sensitivity test and determined the effect of small changes of basic parameters to the transit performance and their mutual influences based on field data and assumed data. In the transit system some variables like passenger generation rate and traffic congestion often change with respect of time and day of operation. Therefore, we examined the effect of small changes of passenger generation rate, fleet size and the number of stops and traffic congestion on transit performance and their parametric interdependencies. In order to determine the scale of sensitivity of the basic parameters to the transit performance we used a range of values of passenger generation rates from 0 to 1000 per/hr, the number of stops 14 and 25 and the fleet size 15 and 20 instead of using a single average value obtained from field survey. The followings are the average parametric values obtained from the field survey of bus Route No. 1 of DMA that are used to perform the numerical calculations.

$L = 13.7$km; $l = 7.9$ km; $V = 12.5$ m/see; $V_a = 1.25$ m/sec; $a = 0.745$ m/sec^2; $(s-1) = 14$; $b = 0.815$ m/sec^2; $x = 0.7$; $p = 7$ persons/min; $h = 7.48$ min, $\mu = 2.57$ sec/per; $q = 0.2$, and $T_t = 7.19$min, ψ_c =250taka/hr and V_c =20taka/hr.

(1) Headway Model (Min. Travel Cost)

The relationships between the optimum value (headway, fleet size, number of stoppings and the users travel time) for a given number of stops $(s-1) = 14$ and $(s-1) = 25$, and the passenger demand that minimizes total travel cost for (a) local and (b) call-on service are shown in Fig.1 and Fig.2 respectively. From Fig.1 and Fig.2 it is found that the optimum headway that minimizes the total travel cost for a given number of stops is continually decreasing with the increasing of passenger demand because the optimum headway is inversely proportional to the square root of demand. In this model the access/egress time component of the users travel time is not changing since the number of stops is given. But the waiting time and stopping time for boarding and alighting passengers change with the decreasing headway and increasing demand. Therefore, the corresponding optimum users travel time decreases with the increasing of passenger demand even though in usual case the user travel time should be increased. In the low passenger demand the headway is found more sensitive to small change in passenger demand in comparison with the change in the large demand. The optimum fleet size increases with the demand in all services. The users travel time, fleet size and headway increase with en route congestion and vehicle interactions. From Fig.1 (b) and 2(b), we found that the optimum number of stoppings *nc* for call-on service depends on the number of stops, passenger demand and headway and en route congestion. It is found that the *nc* for call-on service increase with passenger demand and approaches to the number of stops i.e., to local service.

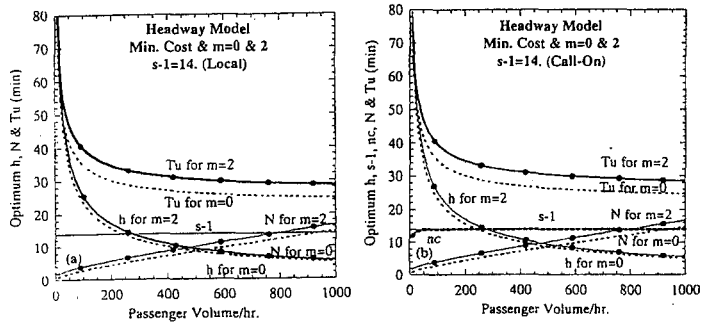

Fig.1: The relationships between the optimum value (headway, fleet size, number of stoppings and users travel time) for a given number of stops $(s-1) = 14$ and the passenger demand that minimize the total travel cost for (a) local and (b) call-on service.

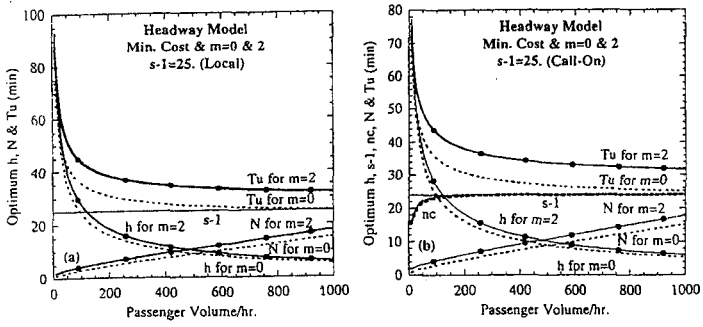

Fig.2: The relationships between the optimum headway, fleet size, number of stoppings and users travel time for a given number of stops $(s-1) = 25$ with the passenger the passenger demand that minimize the total travel cost for (a) local and (b) call-on service.

(2) Number of Stops Model (Min. Travel Cost)

The relations between the optimum value (the number of stops, fleet size and the users travel time) for a given headway ($h = 7$ min) and the passenger demand that minimize the total travel cost are shown in Fig 3 (a) local and 3 (b) call-on service. Fig.3 (a) showed that the optimum number of stops that minimizes the total travel cost for given headway is increasing with the increase of passenger demand for local service. The optimum number of stop is found highly sensitive to small change in passenger demand in low demand region in comparison to change in large demand region. As the headway is given, waiting time is not changing and optimum number of stops makes tradeoff between access/egress time and vehicle stopping time for boarding and alighting passenger. Therefore, the optimum users travel times firstly decrease with the increase of optimum number of stops toward the minimum and further increases with the number of stops. The optimum users travel time and fleet size increase and the number of stop decreases with the increase of traffic congestion. Usually, the number of stoppings for call-on service increases with the passenger demand and approaches to the number of stops for very large passenger demand. However, in Fig. 3(b) we could see that for a very small demand the optimum number of stops and stoppings approach toward infinity and decrease with the increasing of passenger demand to a minimum point and afterwards remain constant. But the operation with infinite number of stops is request-stop service where transit stops anywhere along the route for single boarding or alighting. These situations explained that for very large demand call-on service approaches to the local and for very small demand to the request-stop service. The optimum number of stops and stoppings also decrease with the increasing of congestion for call-on service, in Fig 3(b). The optimum fleet size increases with congestion for both the local and call-on service.

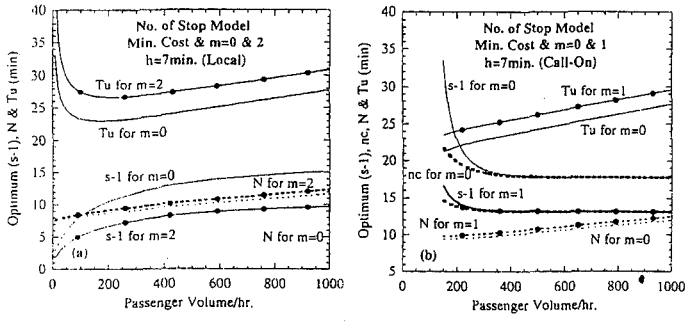

Fig.3: The relationships between the optimum value (the number of stops, fleet size, number of stoppings and users travel time) for a given headway ($h = 7$ min) and the passenger demand that minimize the total travel cost for (a) local and (b) call-on service.

(3) Headway and Number of Stop Model (Min. Travel Cost)

The relations between the optimum combination (the number of stops, headway, users travel time and fleet size) and the passenger demand that minimize the total travel cost for congestion level (a) $m = 0$ and (b) $m = 2$ for local service are shown in Fig. 4. From this model a set of optimum parameters (headway, number of stops and fleet size) can be determined simultaneously for the minimum travel cost. It is observed that in optimum combination the optimum headway continuously decreases and optimum number of stop increases for the minimization of total travel cost with the increasing of demand. The optimum headway is more sensitive to passenger demand at small demand in comparison with the large demand. However, the optimum headway is too sensitive in comparison with the optimum number of stops. The optimum number of stops is also more sensitive at the low demand and its sensitiveness reduces with the increasing of demand and becomes insensible for very large demand. Since the optimum number of stops becomes insensitive for very large demand, the optimum headway continuously decreases and optimum fleet size increases with the increasing of demand. In comparison with Fig. 4(a) and 4 (b) it is observed that the optimum headway and fleet size increase but the number of stops remarkably decreases with the increasing of en route congestion and vehicles interactions. Conversely, when the optimum number of stops is specified for a fixed route transit system, the optimum headway and vehicle capacity must increase with the increasing of congestion to meet-up the increasing demand.

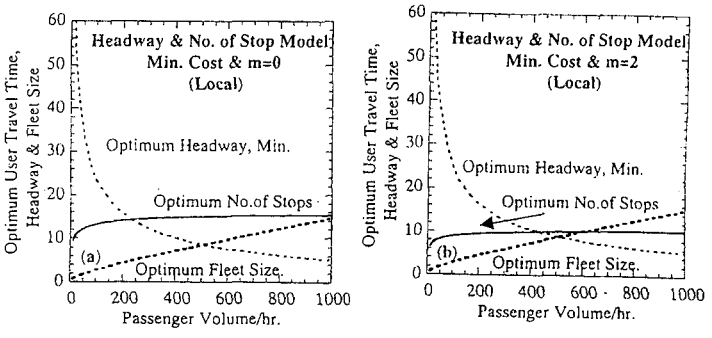

Fig.4: The relationships between the optimum combination (the number of stops and headway and the fleet size) and the passenger demand that minimize the total travel cost for (a) $m = 0$ and (b) $m = 2$ for local service.

(4) Headway and Number of Stop Model (Min. Users Travel Time)

The relationships between the optimum combination (the number of stops and headway) for a given fleet size (a) $N = 15$ and (b) $N = 20$, and the passenger demand for the minimum users travel time are shown in Fig. 5(a) and (b). When the fleet size is limited, the selection of right combination of the optimum number of stops and headway should be made from the feasible combinations of two parameters for a given fleet size. From this model, we could determine the different sets of feasible combinations of the optimum number of stops and headway that minimize the users travel time. It is observed that in optimum combination that minimizes the users travel time, the optimum number of stops decreases and the optimum headway increases with the increasing of demand for local service for given fleet size. With the increasing of the number of passenger generation the bus standing time and the passengers boarding and alighting time increase. In order to maintain the minimum users travel time, the number of stops should be reduced to make up for the

extra time spent for boarding and alighting at stops. Therefore, the optimum number of stops reduces with the increasing of passenger demand. Furthermore, the optimum headway increases with the increasing of passenger demand and also with the increasing of traffic congestion and vehicle interactions. In comparison with Fig. 5 (a) and 5 (b), the optimum headway and number of stops are reversibly sensitive to fleet size changes. For small fleet $N = 15$, the optimum headway is quite sensitive and gradually reduces its sensitivity with the increasing of fleet size $N = 20$; and the optimum number of stops is less sensible to small fleet $N = 15$ and become more sensible with the increasing of fleet size $N = 20$.

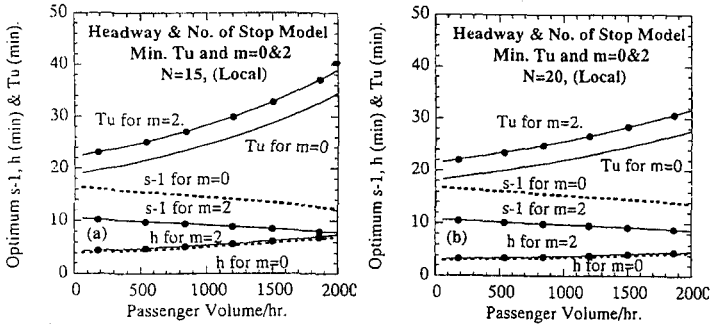

Fig.5: The relationships between the optimum combination (the number of stops and headway) for given fleets size (a) $N = 15$ and (b) $N = 20$ and the passenger demand that minimize the users travel time for local service.

7. Conclusions

This study developed a planning principles to determine the optimum number and locations of stops, headway, fleet size and the types of operations for a given transit route, vehicle dynamic characteristics that minimizes the total travel cost and users travel time for large fleet size and limited fleet size. This study also analyzed the sensitivities to the optimal variables for the different optimal headway and number of stops model conditions and different services, and determined the effects of small changes of the basic variables on transit performance and mutual influences. One of the most significant findings resulting from these sensitivity analyses was that the optimum number of stops, optimum headway, optimum fleet size that minimizes the total travel cost and users travel time are quite sensitive to the vehicle dynamic characteristics, passengers demand, and traffic congestion.

The optimum headway that minimizes the total travel cost was continually decreasing with the increasing of passenger demand because the optimum headway was inversely proportional to the square root of passenger demand and user's time unit value for local service. The corresponding optimum users travel time also decreased continuously as the waiting time and passenger boarding and alighting time changed with the decreasing of headway. The optimum number of stoppings for call-on service is found less in comparison with the number of stops at low passenger demand and increased with the increasing of demand and approached to the number of stops.

The optimum number of stops that minimize the total travel cost for a given headway for local service increased with the increasing of demand. This optimal number of stops was highly sensitive to small change in passenger demand in low demand region and gradually reducing its sensitivity with the increasing of demand. For very small demand the optimum number of stops and stoppings for minimum travel cost for call-on service approached toward infinity and decreased with the increasing of passenger demand to a minimum point and afterward remained constant even continuing the increasing of demand. It was concluded that transit service should be operated as request-stop service for small passenger demand; and with the increasing of demand the call-on service and eventually the local service. It was observed that the optimum number of stops that is derived from the minimum total cost objective converges to that the number of stops derived from the minimum users travel time objective with the increasing of passenger demand.

In the optimum combination, the optimum headway continuously decreased and the optimum number of stops increased that minimize the total travel cost with the increasing of demand. The optimum headway was too sensible in comparison with the optimum number of stops to the change in passenger demand. However, both the optimum headway and number of stops were more sensitive in low demand and its sensitivity gradually reduced with the increasing of demand, but the number of stops became insensible for very large demand because the optimum headway continuously decreased and optimum fleet size increased with increasing demand.

For limited fleet size, it was revealed that in optimum combination that minimizes the users travel time, the optimum number of stops decreased and the optimum headway increased with the increasing of passenger demand. The optimum number of stops and headway was reversibly sensible to the fleet size changes. For small fleet size the number of stop was less sensible but headway was quite sensible to the fleet size changes. For large fleet size the headway was relatively insensible and number of stops was sensible to the fleet size changes.

The optimum users travel time, fleet size and headway increased and the number of stop decreased with the increasing of traffic congestion or vehicles interactions. Traffic congestion/vehicle interactions reduced the average running speed of vehicles and made delay arrivals of bus at stops. The more number of passengers were accumulated at stops in delay period, and hence the bus standing time for boarding and alighting passengers increased and consequently proportionally increased the users travel time. Therefore, to maintain the minimum users travel time the optimum number of stops should be reduced to make up for the extra time spent for longer stoppings.

Although the optimum models have been developed under some limitations of assumptions, these optimization procedures conceptually represented the accurate algorithms and simulation results reflected the correct interrelation between the variables. Therefore, in practice it could be very useful and effective for planning tool to alleviate the existing problems of DMA's transit systems, and to offer an effective, reliable, convenient and scheduled bus transit system. The improved service could

be benefited to the users by reducing the users travel time and travel cost and to the operators by maximizing revenue and reducing system operating cost.

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総コスト最小によるダッカ大都市圏の公共交通サービスの最適化

ウディン. MD. ザヒール・松井　寛・藤田素弘

ダッカ大都市圏には local, call-on, request stop, express service 等様々な特徴を備えた公共交通サービスが私的セクターによって競合的に提供されているが、必ずしも利用者にとって好ましいサービスシステムになっていない. 本研究では利用者の待ち時間を含む交通コストと運営コストの総和を最小とするような公共交通サービス、たとえば、運行間隔、停留場数、車両サイズを決定するモデルについて提案し、その感度分析について検討している.

Optimization of Dhaka's Mass Transit Services for Minimum Travel Time and Cost.

By Uddin Md. Zahir, Hiroshi Matsui and Motohiro Fujita.

Abstract

This study developed the methodology to determined the optimum headway, number of stops, fleet size that minimized the users travel time and total travel cost for given vehicle dynamic characteristics, congestion levels and fixed route of Dhaka Metropolitan Area (DMA) for local, call-on and request stop service. This study also examined the sensitivities of the optimal basic variables to the changes in system parameters like passenger demand, fleet size, and traffic congestion and identified the factors influenced on the optimum conditions significantly. It was seen that optimum number of stops and stoppings are a function of passenger demand. The transit service should be operated as request-stop service for small demand; with the increasing demand the call-on service and eventually the local service. For large demand the optimum number of stops derived from the minimum total travel cost objective and minimum users travel time objective was satisfied. The optimum users travel time, fleet size and headway increased and the number of stops decreased with the increasing of en route congestion and vehicle interactions.
