SPATIAL EQUILIBRIUM OF CAR SHARING SERVICE MARKET

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In the car-sharing service market, which is currently becoming widespread, monopoly supplier will place cars at spots and users visit these spots to use the service. There exist thick market externalities in car sharing service markets: the more cars are supplied in a market, the more customers will visit the market, and vice versa. In this paper, an equilibrium model of multi spot markets is presented considering both agglomeration mechanism caused by thick market externalities and dispersion mechanism caused by access costs to spots. From the analysis, we can show that services are not always fully provided especially in areas with less population density because of the profitability, and that the equilibrium is not social optimum. Political implications are proposed which fill the gap between market equilibrium and social optimum.

***Key words**: car sharing service, market equilibrium, social optimal, scale economy*

1. INTRODUCTION

In the car-sharing service market, which is currently becoming widespread, suppliers will place cars at some spots and consumers visit these car sharing spots to use the service. It is very similar to the car rental market which is very hot in Japan. As of January 2019, the total number of registered car sharing rentals has exceeded 1 million. Car rental companies closely link production, sales and use. In developed countries, people have formed a relatively mature consumption concept, and their understanding of car rental is relatively comprehensive. The advantages of renting a car have been widely recognized [1)].

However, there are still some problems in the car sharing service nowadays.

At present, it can be seen from the map that in the Kyoto area, the spots of car rental services are mainly concentrated in stations and downtown areas with large populations, but not many in the rural areas. Since most of the rental car companies in Japan are provided by private enterprises, the main consideration for their location selection is maximizing profits. Hence, site selection with this goal often sacrifices the utility of some consumers in rural and remote areas for high cost of moving. This will undoubtedly reduce the overall social welfare and require the government to make reasonable efforts.

In response to the above problems, this study presents an equilibrium model of multi spot markets considering both agglomeration mechanism caused by thick market externalities and dispersion mechanism caused by access costs to spots. From the analysis, we can show that services are not always fully provided especially in areas with less population density because the profit might be negative there, and the result is not social optimum. Furthermore, the study proposes the political implications of bridging the gap between market equilibrium and social optimum.

In Section 2, previous studies and the basic concept of this study are described. In Section 3, the car-sharing service market is formulated through using the queuing theory, and the consumers'(passengers') behavior and the supplier's profit are analyzed and modeled. In Section 4, scale economy, characteristics of market equilibrium and the comparation between market equilibrium and social optimum in these cases are analyzed using the constructed model separately from the side of uniform distribution and urban-rural cases. Section 5 summarizes the study and presents future issues.

2. BASIC CONCEPT OF THE STUDY

(1) Overview of previous studies

With the spread of car sharing services, the number of studies on the impact of this service on society is increasing. Firstly, in the research on the willingness to use car-sharing services, Susan et al. [8] conducted a feasibility study of car-sharing services in Beijing, mainly through surveys and interviews to collect local people's views on car-sharing services and other modes of transportation. In the study of theoretical research, Matsushima et al. [2] simulated the behavior of suppliers and passengers separately and analyzed the market equilibrium both in car sharing markets. Also, they analyzed the impact of transaction cost on market equilibrium through theoretical research. In addition, Ishimura et al. [4] conducted the factor analysis focusing on the financial burden of car sharing services. Moreover, Taguchi et al. [5] analyzed the relationship between the permissible walking time to a car sharing spot and selection behavior of passengers. However, most of these studies discussed the relationship between a single factor and individual service selection behavior and did not analyze the structure of the car sharing service market. Furthermore, these researches were not considered from the space domain. Whether the position and quantity of the spot market has an effect on the equilibrium is not considered in the above research. However, related papers on the taxi market mentioned the impact from spatial perspective on market equilibrium, Matsushima et al. [3),6] performed a series studies that due to changes in transaction costs in the taxi service market, frequent visits by customers and taxis to the spot market give each other external benefits. In these studies, the matching between customers and taxis is expressed using a double queuing model. In addition to proposing a market equilibrium model that considered regionality, they also considered the mechanism of the endogenous formation of the taxi markets. However, there are certain differences between the taxi markets and the car-sharing markets, which will be explained in the following sections.

In the related topics of practical application, Murakami [9] analyzed the related processes of car sharing services in detail and proposed that the automation of car rental services can be realized through Internet technology. Matthew Barth and Michael Todd [10] studied how to predict the vehicle travel path and arrival time in the multi-service point car-sharing service and proposes a heuristic algorithm to solve this problem. Narayanan [11] et al. found that when the number of users in car-sharing service system is small, the data-driven demand model proposed in this research confirmed that the demand is affected by supply, seasonal variations and days of a week. Costain et al. [12] investigate the users' behavior of a car-sharing service and state that the service provides a segment of the population with enhanced accessibility. Habib et al. [13] present an econometric model for the behavior of carsharing users. The model shows a high degree of fit to the observed dataset and provides many behavioral details of car-sharing users. Most of the above research are considered from the side of application but this research will focus more on the theoretical side and how the scale economy happens in car sharing service market.

(2) Scale economy impact

Economies of scale in microeconomics refers to the phenomenon that the expansion of production scale leads to the increase of economic benefits, and it is the characteristic that the long-term average total cost decreases with the increase of output. The theory of external economies of scale was first proposed by the famous. economist Marshall in 1890, and then developed by Krugman and other scholars.

The theory of external economies of scale believes that under the same conditions, regions with larger industry scales are more efficient than regions with smaller industry scales. The expansion of industry scale can cause the increase in returns of scale of manufacturers in the region, which will lead to certain industries and their auxiliary departments are highly concentrated in the same or several locations on a large scale, forming external economies of scale.

Matsushima et al. [3] confirmed the existence of economies of scale in the taxi markets: the more taxies gather at a market, the more customers will visit the market, and vice versa. In Fernandez L.'s research [14], a microeconomic model is developed to study the main characteristics of production costs in urban bus corridors. A multiproduct formulation is used, considering trips during peak and off-peak periods as different products. It was found that in the short run the existence of fixed costs produces ray scale economies, but in the long run, economies of scale are produced only by administration fix expenditures, which also can increase more than proportionally than production after some production level. In this paper, he examines the issues of economies of scale in the provision of fixed-route bus transit service in the USA, using level of contracting as a variable to classify agencies into three different size groups. The structure of his paper is similar to this research, but the analysis is from different sides.

Using a significantly larger pooled data set constructed from the National Transit Database of the US Federal Transit Administration from 1992 to 2000, Iseki [15] found that agencies with different levels of contracting exhibit very different relationships between cost per vehicle hour and agency size. Applying the observed range of agency size (number of cars), he also found diseconomies of scale for all agency sizes with all levels of contracting, even when he utilized a quadratic function for the regression equation.

(3) Comparison between social optimal and equilibrium

Market distortion means that the optimal allocation of resources cannot be achieved through the allocation of resources through the market. It is generally believed that the causes of market distortion include monopoly, externalities, public goods and incomplete information.

Due to the existence of externalities or external influences, the market mechanism cannot effectively allocate resources. For producers who generate external economy, because their private profit is smaller than social welfare (because social welfare is equal to the sum of private benefits and external benefits, and external benefits cannot be obtained by producers through market prices), they lack production enthusiasm, so their output level will be lower than the social optimal output level.

As for the car sharing market, in order to maximize the profit, the supplier will find the value of each variables from the equilibrium of the model. However, outcomes from equilibrium often result in lower social welfare for not taking consumers' utility into account. When the optimization objective becomes to maximize social welfare, the difference between the two outcomes is distortion.

3. FORMULATION OF THE MODEL

(1) Preconditions

Let us consider the case where there are multiple spot markets in a certain area and model the spatial equilibrium of spot markets from the behavior by consumers and supplier. Assume a linear market

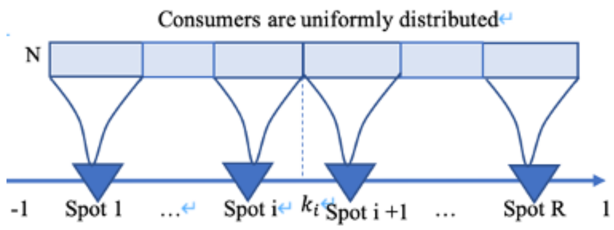

Fig.1 Linear market with multiple spots in assumption.

with the length of 2 which consider a district along a street from -1 to 1 as shown in Figure 1. Homogeneous consumers are uniformly distributed over the linear market. Car- sharing services are only provided at the spot markets. Consumers only consider using car-sharing services as a mode of travel and can only trade in spots. Moreover, consumers will not leave the market until the end of the service and will choose the spot market that maximizes their utility. From supplier's point of view, the spots are set at x_i, and i is the serial number of spots $(1 \leq i \leq R)$. Supplier has to consider how to set the number of cars in each spot and the price.

Consumers who want to use car-sharing services can use the service at once if there is a car available at the spot. When all the cars are in use, the consumer needs to wait until the service to another consumer ends and a car becomes available again. When there are R spots (Figure 1) in this area, the consumer in this area will compare his utility in each spot and go to the highest one to use the service when it's over 0. When the utility of all of the spots is less than 0, he will give up using the car-sharing service. That is, the consumer needs to compare his utility acquired from using the service, the price of the car-sharing service, the sum of transaction costs arising from waiting in the queue and moving to spots to determine which spot to choose.

(2) Behavior of a consumer.

Assume that the consumer uses the service according to a Poisson distribution with an average arrival rate of λ_i ($i = 1,2,3,\dots,R$) and the consumers arrival rate λ_i and the average use time of the car sharing service $1/\mu$ are already known(exogenous) in the short run.

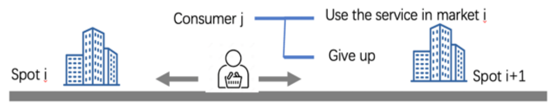

Fig.2 Behavior model of a consumer.

Utility of a consumer j using car sharing services at spot i can be expressed as (1):

$$U_i^j = u - \frac{c}{\mu} - d_i^j(k) - AT_i(s_i, n_i) \tag{1}$$

The first term u is the utility consumer can acquire from car sharing service. The second term means the cost of using the service per person which is supposed to be fixed in each spot. The third term is the cost of moving to market i based on the distance from the position of consumer j. The cost of a consumer j(located at x) going to spot i (x_i) and spot i+1(x_{i+1}) (-1<x_i<x_{i+1}<1) can be expressed as (2a) and (2b):

$$d_i^j(k) = \alpha|x - x_i| \tag{2a}$$

$$d_{i+1}^j(k) = \alpha|x_{i+1} - x| \tag{2b}$$

In (2a) and (2b), α is the parameter of moving cost related to per unit distance. The last term is the waiting cost in the queue of spot i, in this term, s_i is the number of cars served in spot i.

The expected utility of the representative consumer who purchases services at the market is given by:

$$EU_i^j = u - \frac{c}{\mu} - d_i^j(k) - aT_i(\lambda_i, s_i) \tag{3}$$

As shown in Figure 2, the consumer will choose the market which can get more utility, so we can get the following expression

$$i^* = \arg\max\{U_1, \dots, U_i, U_{i+1}, \dots, U_R, U_0\} \tag{4}$$

$$set\ U_0 = 0$$

to guarantee the utility of consumer above 0, otherwise he will give up using the service. Here i^* is the number of markets which can maximize consumer's utility.

(3) Derivation of expected waiting time:

Through the M/M/S type queue model, the number of consumers who are waiting to use car sharing service (L_q^i) and the number of consumers who are using or waiting to use it (L_i) can be obtained as:

$$L_i(\lambda_i, s_i) = \frac{{s_i}^{s_i}{\rho_i}^{s_i+1}}{s_i!(1-\rho_i)^2}P_0 + \rho_i s_i \qquad (5)$$

$$L_q^i(\lambda_i, \mu, s_i) = \frac{{s_i}^{s_i}{\rho_i}^{s_i+1}}{s_i!(1-\rho_i)^2}\frac{1}{\sum_{m=0}^{s_i-1}\left(\frac{{s_i}^m{\rho_i}^m}{m!}\right)+\frac{{s_i}^{s_i}{\rho_i}^{s_i}}{s_i!(1-\rho_i)}} \qquad (6)$$

So, the expected waiting time can be calculated as follow based on Little's formula:

$$T_i(\lambda_i, s_i) = \frac{L_q^i(\lambda_i,\mu,s_i)}{n_i\lambda_i} = \frac{{s_i}^{s_i}{\rho_i}^{s_i+1}}{s_i!(1-\rho_i)^2}\frac{1}{s_i\mu}P_0 \qquad (7)$$

$$P_0 = \frac{1}{\sum_{m=0}^{s_i-1}\left(\frac{{s_i}^m{\rho_i}^m}{m!}\right)+\frac{{s_i}^{s_i}{\rho_i}^{s_i}}{s_i!(1-\rho_i)}} \qquad (8)$$

$$T_i(n_i, s_i) = \frac{{s_i}^{s_i}{\rho_i}^{s_i+1}}{s_i!\,(1-\rho_i)^2}\frac{1}{s_i\mu}\frac{1}{\sum_{m=0}^{s_i-1}\left(\frac{{s_i}^m{\rho_i}^m}{m!}\right)+\frac{{s_i}^{s_i}{\rho_i}^{s_i}}{s_i!\,(1-\rho_i)}}$$

Where in this function, $\rho_i = \frac{n_i\lambda_i}{s_i\mu}$ (9), from the nature of queuing theory,

$$0 < \rho_i < 1 \quad (10)$$

should be satisfied otherwise the waiting time extends to infinity.

(4) Number of consumers in each spot.

If total number of consumers in the linear area is N. Assume that the number of consumers in spot market i is n_i, passenger density in this area is ζ which is exogenous. k_i is the boundary of market i and $i+1(-1 < x_i < k_i < x_{i+1} < 1)$,that is, consumer j at spot k_i can acquire the same utility to both markets, then there will be 2 possible cases when R markets(R≥2) exist.

Case 1:all the consumers in the area use the service. Utility at k_i is above 0 and between the section:

$$U_i(c, s_i, k_i, n_i) = U_{i+1}(c, s_{i+1}, k_i, n_{i+1}) \geq 0$$

Also, the utility of the consumer at the boundary of area:

$$U_i(c, s_i, k_{i-1}, n_i) \geq 0$$
$$U_{i+1}(c, s_{i+1}, k_{i+1}, n_{i+1}) \geq 0$$

In this case, the passenger arrival rate in spot i and i+1 is:

$$\lambda_i = \frac{(k_i - k_{i-1})\zeta}{2}; \qquad (11a)$$

$$\lambda_{i+1} = (k_{i+1} - k_i)\zeta/2 \qquad (11b)$$

The number of users in spot i and i+1 is:

$$n_i = N * \frac{(k_i - k_{i-1})}{2}; \qquad (12a)$$

$$n_2 = N * \frac{(k_{i+1} - k_i)}{2} \qquad (12b)$$

Case 2: part of the consumers use the service.
When the price per unit time of car-sharing service is high or the number of cars in the market is low, the utility of using the service may be negative in some part of this linear area, so some consumers may forgive using the service.
That is when:

$$U_i(c, s_i, k_{i-1}, n_i) < 0$$
$$U_{i+1}(c, s_{i+1}, k_{i+1}, n_{i+1}) < 0$$

then

$$n_i \leq N * \frac{(k_i - k_{i-1})}{2};\ n_2 \leq N * \frac{(k_{i+1} - k_i)}{2} \qquad (13)$$

(5) Behavior of supplier

The supplier will set the number of cars and the price of using the service per unit time to maximize the profit. Determining the number of markets is also one of important behaviors of suppliers which will be explained in the next part. So, the profit function can be expressed as follow (14):

$$\Pi(c, s_i, n_i) = c\sum_{i=1}^{R}\frac{n_i \cdot \lambda_i}{\mu} - F\sum_{i=1}^{R} s_i \qquad (14)$$

where $n_i\frac{\lambda_i}{\mu}$ is the usage charge per user shown in Eq. (1). Therefore, the first term on the right-hand side is the sum of usage fees born by $\sum n_i$ users, and represents the income earned when $\sum s_i$ vehicles are parked. The second term on the right side represents the fixed cost of installing $\sum s_i$ cars. Assuming that s_i represents a continuous value, the condition for profit maximization with regard to s_i is expressed by the following equation:

$$\frac{\partial\Pi(c, s_1, s_2 \ldots, s_R, n_1, n_2, \ldots, n_R)}{\partial s_i} = \sum_{m=1}^{R}\frac{\partial n_m}{\partial s_i}\frac{\lambda_m}{\mu}c - F = 0 \qquad (15)$$

The condition for profit maximization with regard to c is expressed by the following equation:

$$\frac{\partial\Pi(c,s_1,s_2\ldots,s_R,n_1,n_2,\ldots,n_R)}{\partial c} = \sum_{m=1}^{R}\frac{\partial n_m}{\partial c}\frac{\lambda_m}{\mu}c + \sum_{i=1}^{R} n_i\frac{\lambda_i}{\mu} = 0 \qquad (16)$$

The equilibrium of this model can be obtained by Eqs (11), (12), (15) and (16), endogenous variables are s_i, n_i and c.

(6) Location of spots

Determining the number of markets is also one of the important behaviors of suppliers as stated in the last subsection. When the passenger density ζ is already known and the total number of markets is R.

Starting from one spot situation, as shown in Figure 3, the spot can be gradually moved from the area boundary -1 to 1, so as to find the market position that makes the maximum profit in one market situation, that is, satisfying the constraint $-1 < x_1 < 1$. The maximum profit of 1 spot is assumed to be π_1. When there are R spots exist, the principle is the same, that is, move each spot in turn from the edge to find the moment of maximum profit, but it should meet the following constraint below:

$$-1 < x_1 < x_2 < \cdots < x_R < 1$$

The maximal profit when R spots exist is assumed to be π_R. Then compare the maximum profit under different spot quantities to find the most suitable number R^*, that is:

$$R^* = \arg\max\{\pi_1, \ldots, \pi_R\}$$

(7) Social Welfare

Social welfare is governmental provision of economic assistance to persons in need. Here maximizing societal welfare could be expressed by using the consumer's utility function and supplier's profit function.

$$\Pi(c, s_i, n_i) = c\sum n_i\frac{\lambda_i}{\mu} - F\sum s_i \qquad (17)$$

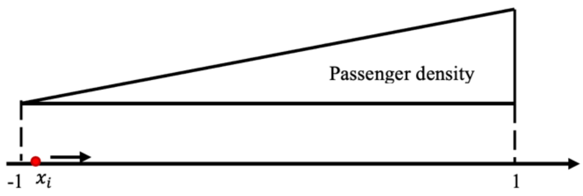

Fig.3 Location of spot x_i

Consumer surplus(cs) of the consumer at position k is:

$$cs(k) = max\{U_i(x_i, k), i = 1, \ldots, R, U_0\} \qquad (18)$$

So, the total consumer surplus (CS) for consumers in this area is shown as the formula (19) below:

$$CS = \zeta\int_{-1}^{1} cs(k)\,dk \qquad (19)$$

So, the social welfare is:

$$\text{SW}=\Pi + CS \qquad (20)$$

Assuming that s_i represents a continuous value, the condition for social optimum (SO) with regard to s_i is expressed by the following equation:

$$\frac{\partial \text{SW}(c, s_1, s_2\ldots, s_R, n_1, n_2, \ldots, n_R)}{\partial s_i} = \sum_{m=1}^{R}\frac{\partial n_m}{\partial s_i}\frac{\lambda_m}{\mu}c - F + \frac{\partial CS}{\partial s_i} = 0 \qquad (21)$$

The condition for social optimum with regard to c is expressed by the following equation:

$$\frac{\partial \text{SW}(c, s_1, s_2\ldots, s_R, n_1, n_2, \ldots, n_R)}{\partial c} = \sum_{i=1}^{R}\frac{\partial n_i}{\partial c}\frac{\lambda_i}{\mu}c + \sum_{i=1}^{R} n_i\frac{\lambda_i}{\mu} + \frac{\partial CS}{\partial c} = 0 \qquad (22)$$

The result of this model can be obtained by Eqs (11), (12), (21) and (22), endogenous variables are s_i, n_i and c.

4. ANALYSIS OF MARKET EQUILIBRIUM

(1) Scale economy in car-sharing model

The average length of consumers' queue can be expressed as $L_i(\lambda_i, \mu, s_i)$,For $\mu > \lambda_i > 0$ and $\theta > 0$, it holds that

$$L_q^i(\lambda_i, \mu, s_i) = \frac{s_i^{s_i} \rho_i^{s_i+1}}{s_i!(1-\rho_i)^2} P_0 = L_q^i(\theta\lambda_i, \theta\mu, s_i)$$

(23)

Which means that the average length of queue remains unchanged when both arrival rates increased with the same rate θ. On the other hand, the average waiting time will decrease when $\mu > \lambda_i > 0$ and $\theta > 1$.

$$T_i(\lambda_i, \mu, s_i) = \frac{L_q^i(\lambda_i,\mu,s_i)}{n_i\lambda_i} =$$

$$\frac{s_i^{s_i} \rho_i^{s_i+1}}{s_i!(1-\rho_i)^2} \frac{1}{s_i\mu} \frac{1}{\sum_{m=0}^{s_i-1}\left(\frac{s_i^m \rho_i^m}{m!}\right) + \frac{s_i^{s_i} \rho_i^{s_i}}{s_i!(1-\rho_i)}} = \theta T_i(\theta\lambda_i, \theta\mu, s_i)$$

(24)

Because $\theta > 1$, we can derive that:

$$T_i(\theta\lambda_i, \theta\mu, s_i) < T_i(\lambda_i, \mu, s_i) \qquad (25)$$

In other words, if the average arrival rate of passengers and cars in market i increases, then the average waiting time of passengers and the number of taxis in that market will decrease. That is, the more customers and taxis that use the market, the more efficient the market will be which shows the influence of scale economy.

(2) Numerical analysis

The profit of supplier can be expreesed as follow:

$$\Pi(c, s_i, n_i) = c \sum_{i=1}^{R} \frac{n_i \cdot \lambda_i}{\mu} - F \sum_{i=1}^{R} s_i$$

When the passenger density in the model is uniform, the passenger arrival rate will also be even under the above assumptions. Firstly, optimize the model from supplier's perspective. The supplier's objective is undoubtedly to maximize the profit, However, as the passenger density in the region increases, there will undoubtedly be a need to increase the number of markets to attract more consumers to use car-sharing services to increase the profit.

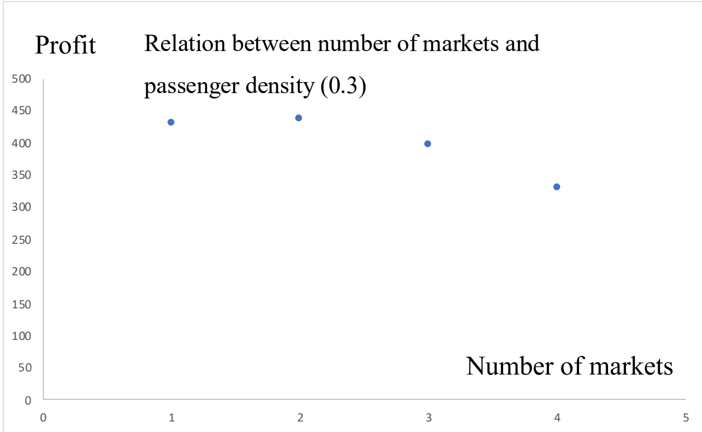

Fig.4 Relation between number of markets and passenger density (0.3).

Table.1 Number of markets when passenger density changes

Numbers of markets	ζ=0.1	ζ=0.3	ζ=0.6	ζ=0.9	ζ=1.2	ζ=1.5
1	105.54*	430.61	911.6	1391.78	1871.96	2352.05
2	44.83	436.54*	981.22*	1522.48	2063.2	2603.65
3	/	396.62	968.5	1531.83*	2093.36*	2654.35*
4	/	328.78	913.08	1479.29	2042.44	2604.15

Take the case when the passenger density is 0.3 as an example for analysis. In this case, two markets can provide the maximum profit for the supplier as Figure 3 shows. It can be seen that profit first increases and then decreases with the increase of the market number. Since the two markets are completely symmetrical when the consumer density is uniformly distributed, the location of k_1 is 0.

Table 1 shows the relation between number of markets and profit for several value of passenger density, which shows that the equilibrium is unstable. The number of markets with stable spatial spots increases as passenger density increases. The value with * means the maximum profit when **ζ** changes.

Table 1 shows how the optimum number of markets change when passenger density changes. When the number of spots increases, the area covered by each spot in the region decreases, so the cost of moving also decreases. Based on the numerical calculations, the desirable number of spots through this calculation example is the effect of centralization due to the external economics of market thickness and the decentralization due to the increase in costs.

(3) Comparation between SW and Profit

In order to maximize the profit, the supplier will find the value of each variables from the equilibrium of the model. However, outcomes from equilibrium often result in lower social welfare by not taking into account consumers' utility. When the optimization objective becomes to social optimum, the difference between the two outcomes where the distortion occurs. Here in the uniform distribution cases, taking the example when passenger density here is 0.5, we can know that the market number is 2 from the last part. In the following Table 2, by comparing the results of the two optimization objectives, it can be seen that in the case of social optimum (SO), the supplier's profit will be negative, which is undoubtedly more harmful to the supplier. From the supplier's point of view, in order to maximize profit, the value of price would be high, but this will lead to lower passenger utility. Therefore, it is necessary to formulate relevant regulations to balance these two.

(4) Implications

For the problems in Table 2, this part proposes a restriction, that is, to ensure that the supplier's profit is above 0. From Table 3, we can find that the maximum social welfare will not drop a lot but the profit of supplier will increase. That is, this implication will effectively protect the profits of suppliers without unduly reducing consumer utility. To a certain extent, the impact of distortion on the interests of supply and demand sides is mitigated through the non-negative constraint.

(5) Analysis in uneven passenger density cases

However, there are often urban-rural population differences in many areas. In this case, the uneven distribution of population density will have a great impact on the market equilibrium. From the supplier's point of view, due to the smaller population in rural areas, setting up spots in the same way as in urban areas will not bring the same return in, suppliers will therefore tend to open markets in areas with higher population densities. However, from the perspective of government, when suppliers only consider their own interests, consumers in some rural areas will give up using the service due to factors such as moving costs. Therefore, a case study is carried out for the above situation. Here, the local passenger density is simulated by a proportional function for simplification, and the slope is adjusted to represent the population difference.

In the uneven passenger density cases, the passenger density function is set as 1/2x+1/2 to ensure that the mean passenger density is consistent with the uniform distribution cases for comparation. Figure 5 shows the comparation between consumers' utility function in market equilibrium case and social optimum case, it can be found that in the market equilibrium case, not all the consumers will use the service because the utility of some consumers will be negative and only one market exists at position 0.41 where the consumer can get highest utility in this case. Table 4 shows the results of equilibrium and social optimum separately in uniform and urban-rural cases, we can find that when the objective is to social optimum, the number of markets is 2, however, under the same settings, the result shows that 1 market will maximize the profit of supplier.

Table.2 Comparation between SO and Equilibrium

	SO	Eqilibrium	Implication(Profit>0)
Number of markets	2		
Number of users at spot 1	100		
Number of users at spot 2	100		
Number of provided cars at spot 1	9	6	9
Number of provided cars at spot 2	9	6	9
Price	0	8.6685	1.006
location of spot 1	-0.5	-0.5	-0.5
location of spot 2	0.5	0.5	0.5
Profit of single market	-47.435	886.85	801.845
Profit	-94.87	1673.7*	106.41
Sum U	1905.133	100	1603.69
SW	1810.263*	1773.7	1710.1

Table.3 Implication in uniform distribution case

	SO	Eqilibrium
Number of markets	2	
Number of users at spot 1	100	
Number of users at spot 2	100	
Number of provided cars at spot 1	9	6
Number of provided cars at spot 2	9	6
Price	0	8.6685
location of spot 1	-0.5	-0.5
location of spot 2	0.5	0.5
Profit of single market	-47.435	886.85
Profit	-94.87	1673.7*
Sum U	1905.133	100
SW	1810.263*	1773.7

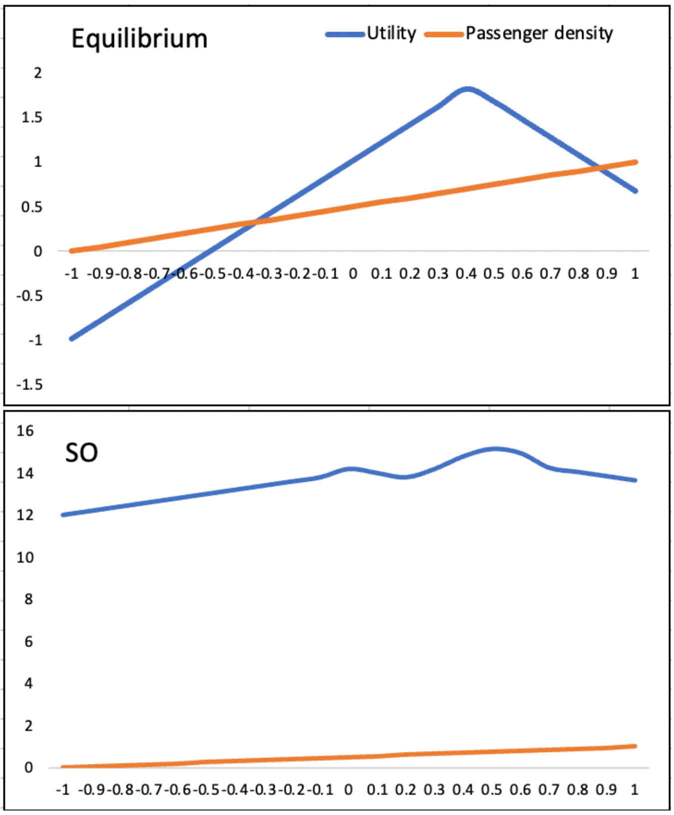

Fig.5 Comparation between market equilibrium and social optimum

Table.4 Comparation between uniform and urban-rural case

	Urban-rural cases		
	Eqilibrium	SO	Constraint (c=c*)
Passenger density	1/2x+1/2	1/2x+1/2	1/2x+1/2
Number of markets	1	2	2
Number of users at spot 1	175	68.6	95.7
Number of users at spot 2	/	131.4	104.3
Number of provided cars at spot 1	14	8	7
Number of provided cars at spot 2	/	10	7
Price of spot	8.8	2.7	2.7
Location of spot 1	/	-0.064	0.207
Location of spot 2	0.414	0.686	0.99
Profit of spot 1	1657.6	-167.1	233.1
Profit of spot 2	/	-364.5	287.6
Profit	1657.6*	-531.6	520.7
Sum U	1020	3228.5	2444.1
SW	2677.6	3174.9*	2964.9

Table.5 Price regulation(c=c*) in urban-rural case

	Urban-rural cases		Uniform distribution cases	
	Eqilibrium	SO	Eqilibrium	SO
Passenger density	1/2x+1/2	1/2x+1/2	0.5	0.5
Number of markets	1	2	2	2
Number of users at spot 1	175	68.6	100	100
Number of users at spot 2	/	131.4	100	100
Number of provided cars at spot 1	14	8	6	9
Number of provided cars at spot 2	/	10	6	9
Price	8.8	2.7	8.7	0
Location of spot 1	/	-0.064	-0.5	-0.5
Location of spot 2	0.414	0.686	0.5	0.5
Profit of spot1	1657.6	-167.1	886.85	-47.4
Profit of spot2	/	-364.5	886.85	-47.4
Profit	1657.6*	-531.6	1673.7	-94.9
Sum U	1020	3228.5	100	1905.2
SW	2677.6	3174.9*	1773.7	1810.3

Since the supplier's profit in the SO table is less than zero, the government will find ways to formulate certain regulations to increase the supplier's profit, which is different from the positive method of restricting the profit in Table 3. It is shown that the price obtained from SO case is used as the actual price in the market equilibrium. The results in the last column of Table 5 show that the profit of the supplier is no longer negative, and the overall social welfare only decreases slightly, which is undoubtedly an effective regulation.

5. Summary and Future work

In the process of car-sharing services, passengers use the service by going to the market where they can benefit the most. In this paper, an equilibrium model of multi spot markets is presented considering both agglomeration mechanism caused by thick market externalities and dispersion mechanism caused by access costs to spots. In other words, the equilibrium of the car-sharing service market depends on the difference between the increase in benefits from scale economy and the increase in the cost of using the service for passengers who are far from the market when the population increases. This study analyzes the model from both a uniformly distributed market and a market with uneven population density, and finds the relationship between the number of markets， market equilibrium and social optimum. However, there are still some problems remained to be solved in the future.

The problem of spatial equilibrium in the spot market of car sharing market is complex, and many research issues remain. First of all, this research is a theoretical research mainly based on assumptions, not the actual situation. In real life, the situation often deviates from the assumed conditions, and some assumptions cannot be satisfied. Therefore, in future research, we should consider how to adjust the assumptions in the process. to make the results more realistic. Secondly, from the data perspective, by looking for data that is closer to the model for analysis, we can analyze the impact of scale economy on the model and the impact of discrete mechanisms on passengers in actual situations.

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