

# Optimization of Spatially-Differentiated Heights of Embankments and Financing Considering Residential Location Change

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This paper explores the efficient embankments construction from two viewpoints: 1) how tall the embankments should be constructed depending on the region, and 2) how to finance the enormous cost of the construction. To analyze the above two viewpoints, we developed a model to examine efficient embankment heights and financing methods using the kinematic wave method, an urban economic model, and a cost-benefit analysis. We then perform numerical simulations with respect to two cases: the case in which residents do not migrate between regions due to embankments construction (the population fixed case) and the case in which they migrate between regions (the small open city case). The results of numerical simulations for the Kitakami River, reveal that the benefits per household are larger in the downstream region than in the upstream region in the population fixed case. In addition, in the small open city case, benefits that landowners receive are larger in the upstream region than in the downstream region.

**Key Words :** *Flood control planning, Cost-benefit analysis, Financing method*

## 1. INTRODUCTION

There have been many torrential rains that we have never experienced before. In order to protect cities from such flood disasters, construction of river embankments are urgently necessary.

It is important for planning the construction of embankments to consider two viewpoints: 1) how tall the embankments should be depending on the region, and 2) how to finance the enormous cost of the construction. For instance, the efficient height of embankments depends on river conditions and population distribution conditions. In addition, efficiency and fairness of the embankments construction depends on who pays for the cost and how much they pay.

In this study, we formulate inundation damage using a runoff model and develop the model of the residents' behavior. Then, considering the endogenous changes in residential location resulting from construction of embankments, we explore the efficient height of embankments and financing methods using cost-benefit analysis and discuss the fairness

depending on the condition with implementing the efficient construction.

## 2. THE MODEL

In this study, we use two models: 1) the model on inundation damage, and 2) the model on the city and residents' behavior.

First, the model on inundation damage is as follows. The river basin is divided into two geographical regions: the upstream region ( $x=1$ ) and the downstream region ( $x=2$ ). In each region, several tributaries flow into the main river. Simultaneous construction of embankments cannot be done at once in the upstream region and the downstream region. For simplicity, we do not consider embankment collapses.

The flow of the main river is expressed using the kinematic wave method. Using the equation of continuity, the depth of the main river at each time can be solved successively. This can be expressed as,

$$h_x^t = h_x^{t-\Delta t} + r_x^t \Delta t + q_{in_x}^t \frac{\Delta t}{\Delta x} - q_{out_x}^t \frac{\Delta t}{\Delta x}, \quad (1)$$

where  $h_x^t$  is the depth of the main river in region  $x$ ,

$r_x^t$  is the amount of rainfall per second ( $m/s$ ),  $q_{in_x}^t$  is the unit-width flow rate into region  $x$ ,  $q_{out_x}^t$  is the flow rate at region  $x$ . Here,  $q_{out_x}^t$ , is derived by using the Manning formula.

Flooding occurs when water depth  $h_x^t$  exceeds the height of the embankment. At each time, the volume that exceeds the embankments height floods the outside of the main river. The total flood water from the main river,  $M_{total_x}$ , can be obtained by adding these volumes during the time when flooding occurs. This volume can be expressed as

$$M_{total_x} = \sum_{t_s}^{t_l} B_x (h_x^t - b_x) \times \Delta x, \quad (2)$$

where  $M_{total_x}$  is the total flood water in each region,  $t_s$  is the time when flooding starts,  $t_l$  is the time when flooding finishes,  $B_x$  is the width of the main river,  $b_x$  is the height of the embankment, and  $\Delta x$  is the length of the main river.

The flood water from the main river  $M_{total_x}$  and rainfall  $\sum_{t=1}^{\tau} r_x^t \Delta t$  are pooled on the possible flooded area  $A_x$ . (the volume that the sewerage system can drain is subtracted.)

According to this pooled volume, inundation depth  $H_x$  is determined endogenously.

$$H_x = \frac{M_{total_x}}{A_x} + \sum_{t=1}^{\tau} r_x^t \Delta t - \frac{V_{drainage}}{A_x}, \quad (3)$$

where  $\tau$  is the duration time of rainfall,  $V_{drainage}$  is possible drainage volume per day.

Using Eq. (3), the damage cost by flood from the main river and rainfall on the possible flooded area can be defined. This damage cost can be expressed as

$$D_x(b_x, r_s) = A_{r_x} \times N \times d_x \times p(H_x), \quad (4)$$

where  $D_x$  is the damage cost, subscript  $s$  is recurrence interval,  $r_s$  is rainfall of each recurrence interval,  $A_{r_x}$  is building area per household,  $N$  is the total number of households,  $d_x$  is the ratio of household in each region to all households living along the main river,  $p(H_x)$  is the damage cost per square meter according to the inundation depth.

By calculating the expected value for the damage costs of each recurrence interval, the expected damage cost can be expressed as,

$$E_x = \sum_{s \in S} \frac{1}{S} \times D_{(x,s)}. \quad (5)$$

Next, the model on the city and residents' behavior is as follows. The city consists of three regions: the upstream region, the downstream region, and the rest of the region. The upstream region and downstream region have rivers, which suffer damage due to flooding. On the other hand, rest of the region is assumed to be an area that will not be damaged by this river flooding. Households residing in each region behave under the following budget constraints, and is expressed as

$$y_x = z_x + R_x f_x + E_x + G_x$$

where  $y_x$  is income,  $z_x$  is composite good,  $R_x$  is land rent,  $f_x$  is floor area, and  $G_x$  is amount of tax.

In addition, the utility function for households is expressed as

$$U_x = z_x + u_x(f_x) + \beta_x$$

Where  $U_x$  is utility function,  $u_x$  is utility depending on floor area, and  $\beta_x$  is amenity level.

Households then maximize utility under the above budget constraints and utility function. In this model, since the land rent is set to be a bid rent, the households' utility is maximized by maximizing the bid rent. The maximization of land rent can be expressed as

$$\max_{f,z} R_x \equiv \max_{f_x} \frac{1}{f_x} \{y_x + u_x(f_x) + \beta - U_x - E_x - G_x\}$$

Next, the government determines the embankment heights and the financing method with the aim of maximizing social welfare. This study analyzes two cases: the population fixed case in which households do not migrate among regions due to embankments construction, and the small open-city case in which they migrate.

First, the social welfare in the population fixed case consists of the increase in land rent and increase in household utility subtracting the cost of embankments construction, and is expressed as

$$\max_{f,z} SW \sum_{T=1}^{100} (R_1 L_1 + R_2 L_2 + U_1 N_1 + U_2 N_2) - (C_1 + C_2)$$

where  $i$  is discount rate,  $N_x$  is number of households,  $C_x$  is cost of embankments construction.

Second, in the small open city case, social welfare consists of the increase in land rent minus the cost of embankments construction, and is expressed as

$$\max_{f,z} SW \sum_{T=1}^{100} \left(\frac{1}{1+i}\right)^T (R_1 L_1 + R_2 L_2) - (C_1 + C_2).$$

We consider the four regimes: Regime 1) The government pays the entire cost of embankments construction, Regime 2) The government pays half costs of the embankments construction, Regime 3) the upstream region and the downstream region share the costs of embankments construction for the entire watershed, Regime 4) the upstream region and downstream region each pay for their own local embankments construction costs. Table 1 then shows the amount of tax per household  $G_x$  depending on the case.

**Table1:** amount of tax per households  $G_x$  depending on the regime

	$G_1$	$G_2$
Regime 1	0	0
Regime 2	$\frac{C_1 + C_2}{2N_{all} \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$	$\frac{C_1 + C_2}{2N_{all} \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$
Regime 3	$\frac{C_1 + C_2}{(N_1 + N_2) \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$	$\frac{C_1 + C_2}{(N_1 + N_2) \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$
Regime 4	$\frac{C_1}{N_1 \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$	$\frac{C_2}{N_2 \sum_{T=1}^{100} \frac{1}{(1+i)^T}}$

### 3. SIMULATIONS AND RESULTS

We set the parameters that match the Kitakami River to some extent and explore the efficient embankments construction and financing methods. In this section, we show the numerical results for Regime 1 and Regime 2 regarding the population fixed case and the small open city case, respectively.

First, Table 2 shows the results of the numerical simulations in the population fixed case. In the population fixed case, there are no benefits for the landowners, and all benefits are attributed to the households. This is because, in the population fixed case, land rents do not increase as a result of embankments construction. In addition, the benefits per household are larger in the downstream region than in the upstream region. This is because the expected damage cost before the embankment construction is large in the downstream region, and the expected damage cost is greatly reduced by the construction.

**Table2:** Numerical results of the population fixed case

	Population Fixed Case			
	Case 1		Case 2	
	upstream-region	downstream-region	upstream-region	downstream-region
Efficient embankments height [m]	3.7	3.8	3.7	3.8
Total benefit of households [10 <sup>6</sup> yen]	9,404	5,242	9,364	5,231
Benefit per household [10 <sup>6</sup> yen / person]	843	1,613	840	1,610
Total benefit of landowners [10 <sup>6</sup> yen]	0	0	0	0
Cost of river improvements [10 <sup>6</sup> yen]	727	783	727	783
Social welfare [10 <sup>6</sup> yen]	13,137		13,084	
<b>Market equilibrium</b>				
Floor are [ m ]	90	307	90	307
Number of household [ 10 <sup>4</sup> ]	11.15	3.25	11.15	3.25
Land rent [ yen ]	3,544	1,035	3,544	1,035
Amount of tax per person [ yen ]	0	0	491	491

Next, Table 3 shows the results of numerical simulations for the small open city case. In the small open city case, there are no benefits to the households, and all benefits are attributed to the landowners. This is because the level of household utility is

constant in the small open city case. In addition, benefits that the landowners receive are larger in the upstream region than in the downstream region. This is because the upstream region has a higher utility level and the land rent rises significantly due to embankments construction.

**Table3:** Numerical results of the small open city case

	Small Open Case			
	Case 1		Case 2	
	upstream-region	downstream-region	upstream-region	downstream-region
Efficient embankments height [m]	3.8	3.9	3.8	3.9
Total benefit of households [10 <sup>6</sup> yen]	0	0	0	0
Total benefit of landowners [10 <sup>6</sup> yen]	11,397	7,576	11,357	7,551
Cost of river improvements [10 <sup>6</sup> yen]	783	841	783	841
Social welfare [10 <sup>6</sup> yen]	17,349		17,284	
<b>Market equilibrium</b>				
Floor are [ m ]	63	155	63	155
Number of household [ 10 <sup>4</sup> ]	16.0	6.47	16.0	6.46
Land rent [ yen ]	5,084	2,058	5,075	2,055
Amount of tax per person [ yen ]	0	0	528	528

### 4. CONCLUSION

This paper clarifies, using a Kinematic wave method and cost-benefit analysis, that in the population fixed case the benefits per household are larger in the downstream region than in the upstream region. In addition, in the small open city model, we show that benefits that the landowners receive are larger in the upstream region than in the downstream region.

Additionally, we will perform numerical simulations for Regimes 3 and 4 and compare the results of those four regimes. We will then discuss how it is efficient and fair to finance the embankments construction.

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