Branch-and-Price-and-Benders-Cut Algorithm for the Many-to-Many Hub Location Routing Problem

Yuehui Wu[1], Ali Gul Qureshi[1] and Tadashi Yamada[1,2]

[1] Department of Urban Management, Graduate School of Engineering, Kyoto University
(Katsura Campus, Nishikyo-ku, Kyoto-shi, Kyoto, 615-8246, Japan)
E-mail: wu.yuehui.34w@st.kyoto-u.ac.jp (Yuehui Wu)
E-mail: qureshi.aligul.4c@kyoto-u.ac.jp (Ali Gul Qureshi)
[2] Graduate School of Management, Kyoto University
(Yoshida Honmachi, Sakyo-ku, Kyoto-shi, Kyoto, 606-8501, Japan)
E-mail: yamada.tadashi.2x@kyoto-u.ac.jp (Tadashi Yamada)

The paper focuses on a variant of hub location routing problem arising in the design of intra-city express service networks, named as many-to-many hub location routing problem (MMHLRP), in which each non-hub node should be served by exactly one hub, and a maximum duration limit is imposed on each tour. A set partitioning formulation of the problem is provided, and a solution algorithm is developed based on the benders decomposition scheme and branch-and-price scheme to exactly solve the problem. The problem is decomposed into a master problem (MP) and a subproblem (SP) via benders decomposition first, and the MP and SP are solved iteratively to solve the primal problem. In each iteration, the MP is solved by a branch-and-price algorithm first. Then the dual problem of the SP is solved to generate benders cuts based on the solution of the MP. Numerical experiments are conducted on the instances generated from the Australian Post dataset to test the performance of the model and the developed algorithm. Computational results prove that the algorithm outperforms the CPLEX and is able to provide optimal solutions for the generated instances within reasonable computational time, which indicates the feasibility and efficiency of the model and algorithm proposed. Furthermore, the results can provide insights into the MMHLRP, making it available for decision-making.

Key Words: *intra-city express, hub location routing problem, exact solution, benders decomposition, branch-and-price*

1. INTRODUCTION

Freight transportation plays a significant role in the economy of any country, which consists of basically two types of service segments: full truckload (TL) and less-than-truckload (LTL). TL is more suitable for large freight flows that need to be directly transported between two points, for example, between suppliers and clients or factories and warehouses; while LTL is more appropriate for customers that cannot be individually serviced by a truck and have then to share the carrier's transportation resources with other customers.

One important application in the LTL segment is the transportation of mails and parcels in intra-city express service systems. In this study, we focus on the operational characteristics of cargo companies that offer "delivery within the same day in the city service", "next day delivery service", or "delivery within 24 hours service", e.g., Yamato Transport, Japan Post and so on. Based on the characteristics, we introduce a network planning problem for the design of their service systems and propose an exact solution algorithm to solve the problem.

Generally, in these systems, flows of mails and parcels are exchanged between post offices via local tours and hubs, instead of linking each pair of post offices directly, which is usually very expensive. The flows are collected at origin post offices, routed via one or two hubs, and distributed at destination post offices. Local tours departing from hubs are designed to collect and distribute flows at post offices, through which collection and distribution processes are handled simultaneously. Both the vehicles and hubs are assumed to be uncapacitated, since the referred services are usually only available for documents and small packages. However, a maximum duration limit is imposed on each local tour to ensure service quality. With these considerations, the problem (many-to-many hub location routing problem, MMHLRP) includes locating one or several hubs, allocating each post office to exactly one hub, and designing local tours under practical constraints. The structure of the target network is illustrated by **Fig.1**, which shows a solution with 3 hubs, 11 non-hub nodes, and 4 vehicles in total.

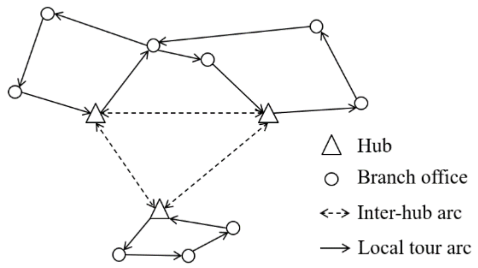


Fig.1 Example of MAHLRP solution

Camargo et al.[1] first introduced the MMHLRP based on the hub location routing problem (HLRP). HLRP was proposed by Nagy and Salhi[2] under the name of many-to-many location routing problem, which incorporates the routing aspects into the hub location problem (HLP). Real-world applications of the HLRP can be found in many network planning problems for intra-city express systems[3], postal service systems[4], telecommunication systems [5], and ship cargo systems[6].

The main contribution of this study can be summarized in two aspects: i) The MMHLRP is formulated as a set partitioning formulation first, and a branch-and-price-and-benders cut algorithm is proposed to solve it to optimality. ii) Series of numerical experiments are conducted to test the proposed algorithm and to provide insights into the MMHLRP, which will be made available for future references and decision-making.

The remainder of the paper is structured as follows: Section 2 presents the problem descriptions and the model formulation of the MMHLRP, while Section 3 introduces the proposed solution algorithm. Section 4 conducts numerical experiments to prove the algorithm's efficiency, followed by Section 5 as a conclusion of the study.

2. MATHEMATICAl FORMULATION

Given a set H of potential hubs and a set C of clients, we define the problem on a directed complete graph $G = (N, A)$ with node set $N = H \cup C$ and arc set A, where arc set A is the set of all arcs (i, j): $i, j \in N$. For each pair of clients $i \in C$ and $j \in C$, it is associated with a given amount of freight flow d_{ij} to be shipped from i to j. The total pickup demand and delivery demand of each client $i \in C$ is denoted by $O_i = \sum_{j\in C} d_{ij}$ and $D_i = \sum_{j\in C} d_{ji}$, respectively. Each potential hub $m \in H$ is addressed with a fixed cost F_m of setting up a hub at site m. A fleet of homogeneous vehicles are employed to complete local tours, and each vehicle has a fixed cost f. Both the hubs and vehicles are assumed to be uncapacitated. However, a maximum duration limit is imposed on each local tour to ensure service quality. Furthermore, each client should be served by exactly one hub.

In the network, each arc $(i, j) \in A$ has a nonnegative travel cost c_{ij} and a nonnegative travel time t_{ij}, which satisfy the triangle inequality. Like many other studies related to the MMHLRP, the costs of local tours are calculated as the sum of the travel cost of the used arcs, while the inter-hub transportation cost is determined by both travel distance and transferred flow[7)8)]. A unit inter-hub transportation cost, denoted as α, which also works as the discount factor[1)8)], is used to make inter-hub transportation cost and local tour cost comparable (as in Yang et al.[7)]). Moreover, an allocation cost needs to be paid when assigning each client $i \in C$ to each hub $m \in H$.

We use the following decision variables to formulate the MMHLRP as a set partitioning formulation, called SPF. Let R denote the set of all feasible single-vehicle routes. The cost of route $r \in R$ is donated as c_r. Furthermore, let a_{ir} equal 1 if route r serves client $i \in C$ and w_{mr} equal 1 if it departs from hub $m \in H$. For each route $r \in R$, we define a binary variable θ_r which is equal to 1 if and only if route r is selected in the solution of the MMHLRP. For each client $i \in C$ and each hub $m \in H$, z_{im} is a binary variable which is equal to 1 if and only if client i is served by hub m. b_m is a binary variable which is equal to 1 if and only if potential hub $m \in H$ is opened. Moreover, y_{ijmn} denotes the fraction of flow from client i to client j passing via hubs m and n.

$$\min \sum_{m\in H} F_m b_m + \sum_{r\in R} c_r\theta_r + \sum_{i\in C}\sum_{m\in H} c_{im}(O_i + D_i) z_{im} + \sum_{i\in C}\sum_{j\in C}\sum_{m\in H}\sum_{n\in H} \alpha d_{ij} c_{mn} y_{ijmn} \tag{1}$$

$$s.t. \sum_{r\in R} a_{ir}\theta_r \geq 1 \;\forall i \in C \tag{2}$$

$$\sum_{r\in R} w_{rm} d_r \theta_r \leq b_m \;\forall m \in H \tag{3}$$

$$\sum_{m\in H} z_{im} = 1 \;\forall i \in C \tag{4}$$

$$z_{im} \leq \sum_{r \in R} w_{rm} a_{ir} \theta_r \ \forall i \in C,\ m \in H \tag{5}$$

$$\sum_{n \in H} y_{ijmn} = z_{im} \forall i \in C, j \in C, m \in H \tag{6}$$

$$\sum_{m \in H} y_{ijmn} = z_{jn} \forall i \in C, j \in C, n \in H \tag{7}$$

$$\theta_r \in \{0,1\} \ \forall r \in R \tag{8}$$

$$b_m \in \{0,1\} \ \forall m \in H \tag{9}$$

$$z_{im} \in \{0,1\} \ \forall i \in C, m \in H \tag{10}$$

$$0 \leq y_{ijmn} \leq 1 \ \forall i \in C, j \in C, m \in H, n \in H \tag{11}$$

The objective function (1) minimizes the total operating cost, including the fixed cost of hubs, local tour cost, allocation cost, and inter-hub transportation cost. Note that the fixed cost of vehicles is included in c_r for each $r \in R$. Constraint (2) ensures that each client is served. Constraint (3) guarantees that clients can only be served by open hubs. Single allocation is guaranteed by constraint (4). Constraint (5) links allocation variables and routing variables. Constraints (6)-(7) ensure that the flow between each pair of clients can only be routed via the hubs they are assigned to. Constraints (8)-(11) are variable domains.

3. SOLUTION ALGORITHM

The SPF cannot be used to solve the MMHLRP directly, mainly due to the exponential number of feasible routes and the huge number of continuous variables. We present a branch-and-price-and-benders cut algorithm to deal with these two problems, as shown in **Fig.2**. In the algorithm, we decompose the SPF into a master problem (MP) and a subproblem (SP) via benders decomposition first, and the MP and SP are solved iteratively to solve the primal problem. In each iteration, the MP is solved by a branch-and-price algorithm first. Then the dual problem of the SP is solved to generate benders cuts based on the solution of the MP. Moreover, the upper bound (UB) and lower bound (LB) are updated if needed. The algorithm stops when the LB equals the UB. The specific details of the algorithm will be presented in the following subsections.

(1) Benders decomposition

Benders decomposition is a method for solving mixed-integer programming (MIP) problems that have the constraint set with a special structure: when fixing the complication variables (integer variables), the mathematical program reduces to a simple, easy to solve linear problem.

Benders decomposition iteratively solves two levels of coordination. At a higher level, a relaxed version of the original problem with the set of the integer variables and its respective constraints is responsible for fixing these integer variables' values and providing an LB for the problem. This level is known as the MP. At a lower level, the dual of the original problem with the values of the integer variables temporarily fixed is used to produce benders cuts and provides a UB for the problem. This stage is known as the SP.

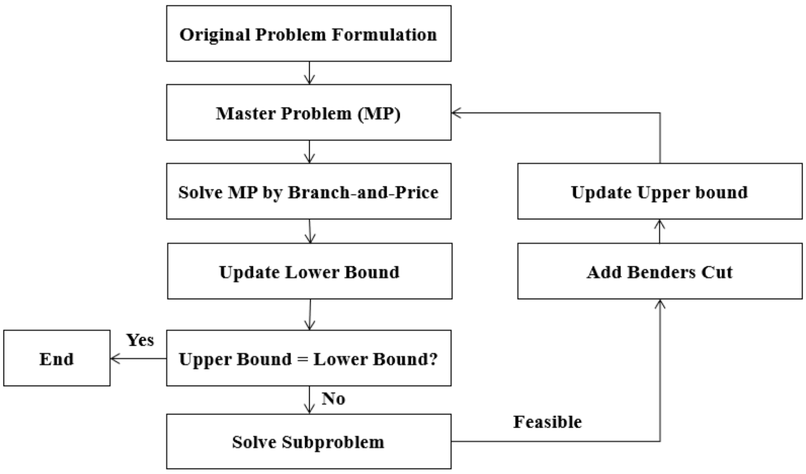


Fig.2 Flowchart of the proposed algorithm

The algorithm alternates between the solution of MP and the SP until the UB and the LB converge towards an optimal solution, if one exists.

a) Benders subproblem for the MMHLRP

Let $\Omega = \{(\theta,\ z): (2) - (5) \& (8) - (10) \text{ hold}\}$ be the set of feasible values for the variables θ and z. Then for fixed values $\theta = \bar{\theta}$ and $z = \bar{z}$ such that $(\bar{\theta},\ \bar{z}) \in \Omega$, a primal linear SP can be defined for each pair of clients $i \in C$ and $j \in C$:

$$min \sum_{i \in C} \sum_{j \in C} \sum_{m \in H} \sum_{n \in H} \alpha d_{ij} c_{mn} y_{ijmn} \tag{12}$$

$$s.t. \sum_{n \in H} y_{ijmn} = \bar{z}_{im} \forall i \in C, j \in C, m \in H \tag{13}$$

$$\sum_{m\in H} y_{ijmn} = \bar{z}_{jn} \forall i \in C, j \in C, n \in H \qquad (14)$$

$$0 \le y_{ijmn} \le 1 \ \forall i \in C, j \in C, m \in H, n \in H \qquad (15)$$

Furthermore, let $\delta_{ijm} \in \mathbb{R}$ and $\gamma_{ijn} \in \mathbb{R}$ be the dual variables associated with constraint (13) and constraint (14), respectively. Then for each pair of clients $i \in C$ and $j \in C$, the dual problem of the SP is stated as below:

$$max \sum_{m\in H} \bar{z}_{im}\gamma_{ijm} + \sum_{m\in H} \bar{z}_{jn}\varphi_{ijn} \qquad (16)$$

$$s.t. \gamma_{ijm} + \varphi_{ijn} \le \alpha d_{ij} c_{mn} \forall m \in H, n \in H \qquad (17)$$

$$\delta_{ijm} \in \mathbb{R} \ \forall m \in H \qquad (18)$$

$$\gamma_{ijn} \in \mathbb{R} \ \forall n \in H \qquad (19)$$

Note that for any vector $z \in \mathcal{Z}$ such that $\mathcal{Z} = \{z: (4)-(5), 0 \le z_{im} \le 1, \forall i \in C, m \in H\}$, the primal (12)-(14) and dual (16)-(19) SPs are feasible and bounded (Elisangela et al., 2013). Thus, whenever a dual SP is solved, the following benders optimality cut is generated for the corresponding pair of clients $i \in C$ and $j \in C$, in which η_{ij} is the auxiliary benders variables, representing an inter-hub transportation cost estimator for this pair of clients. $(\bar{\gamma}_{ijm}, \bar{\varphi}_{ijn}) \in \mathcal{D}$ is the obtained dual solution of the SP, in which $\mathcal{D}$ represents the dual solution place of the SP.

$$\eta_{ij} \ge \sum_{m\in H} \bar{\gamma}_{ijm} z_{im} + \sum_{n\in H} \bar{\varphi}_{ijn} z_{jm} \qquad (20)$$

Moreover, we use the methodology applied in Papadakos[9] and Elisangela et al.[10] to generate Pareto-optimal cuts to accelerate the algorithm.

b) Benders masterproblem for the MMHLRP

Using the generated cuts, the MP is formulated as below:

$$\min \sum_{m\in H} F_m b_m + \sum_{r\in R} c_r \theta_r + \sum_{i\in C}\sum_{m\in H} c_{im}(O_i + D_i) z_{im} + \sum_{i\in C}\sum_{j\in C} \eta_{ij} \qquad (20)$$

$$s.t. (2)-(5), (8)-(10) \qquad (21)$$

$$\eta_{ij} \ge \sum_{m\in H} \bar{\gamma}_{ijm} z_{im} + \sum_{n\in H} \bar{\varphi}_{ijn} z_{jm} \ \forall (\bar{\gamma}_{ijm}, \bar{\varphi}_{ijn}) \in \mathcal{D}, i \in C, j \in C \qquad (22)$$

$$\eta_{ij} \ge 0 \ \forall \ i \in C, j \in C \qquad (23)$$

(2) Branch-and-price

The MP is solved via a classical branch-and-price approach embedded with a bounded bidirectional labeling algorithm as the solution approach of the pricing subproblem[11], which is shown in **Fig. 3**.

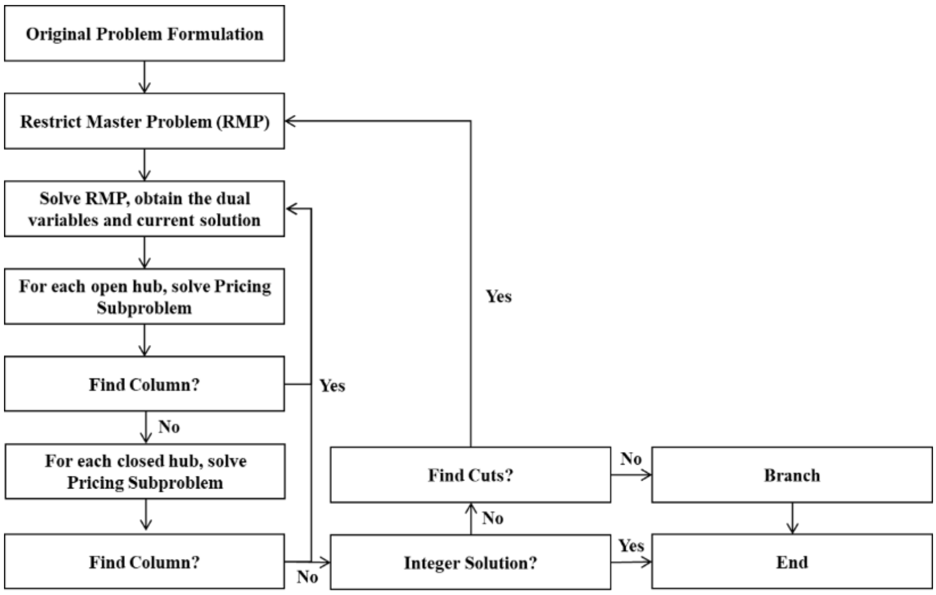


Fig.3 Flowchart of the branc-and-price algorithm (for each branch node)

4. NUMERICAL EXPERIMENT

In this section, numerical experiments are conducted to test the proposed model and algorithm. The instances used in the experiments are introduced first, followed by a comparison between the proposed algorithm and the CPLEX to show its efficiency.

(1) Instance generation

To test the proposed algorithm, we follow the fashion of Camargo et al.[1] and generate instances with different α based on Australia Post (AP) dataset. AP dataset, available at OR-Library (http://people.brunel.ac.uk/~mastjjb/jeb/orlib/), is derived from a real-world hub location problem for Australia Post, which describes postcode districts, the direction of their coordinates, and flow volumes (mail flow). We generated 20 instances from the AP dataset with up to 30 clients. The instances are named as AP$n.\alpha$ where n is the number of clients and $\alpha \in \{2,4,6,8\}$ represents the applied discount factors 0.2, 0.4, 0.6, and 0.8, respectively.

The initial instances generated by the generator include the coordinates of the clients and the flow among the clients. We used the coordinates of the clients in the initial instances directly. The potential hub set is assumed to be the same as the client set. For the sake of simplicity, the travel cost and the travel time between each pair of nodes are set as the Euclidean distance between them. The fixed cost of vehicles is set as 1000, and the maximum duration of each tour is set as 50000. The fixed cost of hubs is generated randomly, following the fashion of Ebery et al.[12].

(2) Experiment results

Both the proposed algorithm and the CPLEX are applied to the generated instances, and a computational time limit of 10800 seconds is imposed on each method. The obtained results are shown in **Table 1**, in which UB, LB, Gap, and T represent the upperbound, the lowerbound, the gap between UB and LB, and the calculational time in seconds of the corresponding method, respectively. Ite denotes the number of iterations used to reach the convergence of the benders decomposition. Hub shows the selected hubs in the obtained solution. $Speedup$ denotes the speed-up between the two methods.

a) Solution quality

As shown in **Table 1**, the computational times for the CPLEX solver reached 10800 seconds for most of the instances. Actually, CPLEX only found optimal solutions for the four smallest instances with 10 clients, which indicates the difficulty of the problem. For the instances that CPLEX cannot solve optimally, the optimality gaps remained very large (35.97% on average and 59.93% in the worst case). Meanwhile, the CPLEX solver did not find feasible integer solutions within the time limit for all the instances with more than 20 clients. However, the proposed branch-and-price-and-benders cut algorithm was able to find optimal solutions for all the instances in reasonable computational time. These results prove that the proposed algorithm outperforms the CPLEX in the solution quality and can provide optimal solutions for this variant of problem.

b) Computational time

For the instances solved optimally by both the CPLEX solver and the proposed algorithm, the computational times were significantly reduced by the proposed algorithm, and the average computational time for these instances decreased from 1307.59 seconds to 1.20 seconds. Our algorithm solved the instances with 10 clients and 15 clients very fast, in which all instances were solved within 15s of CPU time. When the problem size increased, the computational time increased critically as the number of feasible routes increased exponentially in this case. The average CPU times for solving the instances with 20, 25, and 30 clients are 21.94 seconds, 260.10 seconds, and 709.03 seconds, respectively. It is also revealed in **Table 1** that the instances with larger unit inter-hub transportation cost take more computational time than those with smaller unit inter-hub transportation

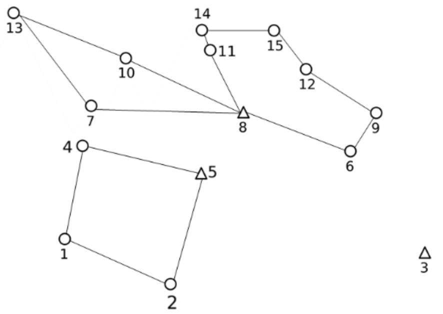
Table 1 Comparision between the proposed algorithm and the CPLEX

	CPLEX				Branch-and-Price-and-Benders Cut Algorithm					
Instance	UB	LB	Gap (%)	T	LB	Gap (%)	T	Ite	Hub	$Speedup$
AP10.2	294022.52	294022.52	0.00	3561.89	294022.52	0.00	1.38	3	2,5,7,9	2584.82
AP10.4	324923.22	324923.22	0.00	195.58	324923.22	0.00	1.63	2	5	119.91
AP10.6	324923.22	324923.22	0.00	810.66	324923.22	0.00	1.19	1	5	682.37
AP10.8	324923.22	324923.22	0.00	662.22	324923.22	0.00	0.61	1	5	1083.83
AP15.2	348050.32	294263.36	15.45	10800	348050.32	0.00	7.01	1	3,5,8	-
AP15.4	381769.79	267977.91	36.55	10800	381769.79	0.00	4.82	1	3,5,8	-
AP15.6	408657.31	287249.09	34.81	10800	408657.31	0.00	6.92	1	5,8	-
AP15.8	427552.82	291017.62	32.49	10800	427552.82	0.00	10.68	1	8	-
AP20.2	434094.18	245631.7	43.42	10800	434094.18	0.00	29.82	3	2,3,10	-
AP20.4	466301.22	279169.1	40.13	10800	466301.22	0.00	20.91	2	3,10	-
AP20.6	497567.98	259091.7	47.93	10800	497567.98	0.00	18.97	2	3,10	-
AP20.8	528834.73	316389.5	40.17	10800	528834.73	0.00	18.07	2	3,10	-
AP25.2	493213.14	197634.2	59.93	10800	493213.14	0.00	180.63	1	5,7,23	-
AP25.4	552702.78	257625.5	53.39	10800	552702.78	0.00	384.84	2	5,7,23	-
AP25.6	590406.58	271309.3	54.05	10800	590406.58	0.00	214.42	1	5,7,13,23	-
AP25.8	617427.16	285722.9	53.72	10800	617427.16	0.00	260.49	2	5,8,17	-
AP30.2	588526.29	263179.9	55.28	10800	588526.29	0.00	561.29	3	3,6,10,20,25	-
AP30.4	639155.07	318959.9	50.10	10800	639155.07	0.00	597.33	2	2,5,22,25	-
AP30.6	683912.55	335243.7	50.98	10800	683912.55	0.00	1105.41	2	3,6,16,25	-
AP30.8	718633.67	351591.9	51.07	10800	718633.67	0.00	575.69	1	2,6,16,25	-

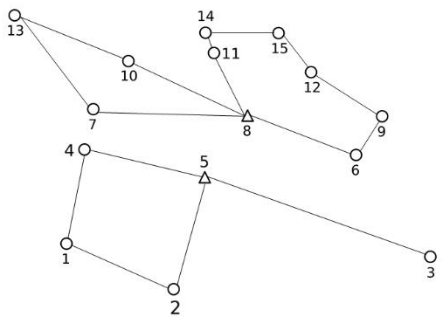
cost, which is because that larger branch trees are needed in those instances.

c) Networks

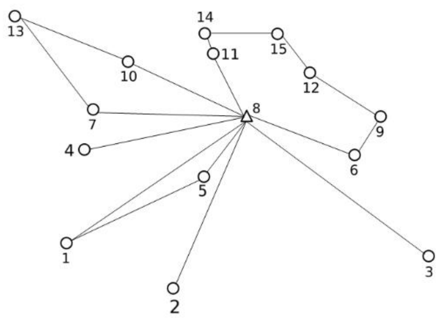
The optimal solutions for instances with 15 clients are illustrated in **Fig. 4**, where the figure notations are the same as those used in **Fig. 1**. It is observed in the figure that the allocation between clients and hubs, as well as the number and location of open hubs, changes as the unit inter-hub transportation cost varies. Furthermore, some hubs are always selected, even though the unit inter-hub transportation cost varies, e.g., hub 8 in instances with 15 clients. Also, as expected, the results show that generally, the number of installed hubs decreases with the increase of unit inter-hub transportation cost.



(a) AP15.2 and AP15.4



(b) AP15.6



(c) AP15.8

Fig.4 Optimal solutions of instances with 15 clients

5. CONCLUSION

In this paper, we focused on a many-to-many hub location routing problem (MMHLRP), arising in the design of intra-city express systems, e.g., those for Yamato Transport and Japan Post. Each post office should be served by exactly one hub, and a maximum duration limit is imposed on each tour. We formulated the problem as a set partitioning formulation first, and then introduced an exact solution algorithm based on the benders decomposition framework and bracg-and-price framework to exactly solve this problem. The problem is decomposed into a master problem (MP) and a subproblem (SP) via benders decomposition first, and the MP and SP are solved iteratively to solve the primal problem. In each iteration, the MP is solved by a branch-and-price algorithm first. Then the dual problem of the SP is solved to generate benders cuts based on the solution of the MP. To assess the performance of the model and algorithm proposed, numerical experiments were undertaken with the instances generated from the Australain Post (AP) dataset. Outcomes of the computational studies reveal that, for all the generated instances, the proposed algorithm outperformed CPLEX, not only in solution quality but also in computational time. In fact, the algorithm yielded the optimal solutions for instances with up to 30 non-hub nodes, while the CPLEX solver could merely solve instances with 10 non-hub nodes exactly. Further works could be devoted to include heuristic pricings into the framework, propose efficient valid inequalities, and apply the algorithm to variants of this problem, such as those with time windows or stochastics factors.

REFERENCES

1) de Camargo, Ricardo Saraiva, Gilberto de Miranda, and Arne Løkketangen: A new formulation and an exact approach for the many-to-many hub location-routing problem, *Applied Mathematical Modelling*, Vol. 37.(12-13), pp. 7465-7480, 2013.
2) Nagy, Gábor, and Said Salhi: The many-to-many location-routing problem, *Top*, Vol. 6(2), pp. 261-275, 1998.
3) Bostel, Nathalie, Pierre Dejax, and Mi Zhang: A model and a metaheuristic method for the hub location routing problem and application to postal services, *2015 International Conference on Industrial Engineering and Systems Management (IESM)*, IEEE, 2015.
4) Çetiner, Selim, Canan Sepil, and Haldun Süral: Hubbing and routing in postal delivery systems, *Annals of Operations research*, Vol. 181(1), pp. 109-124, 2010.
5) Catanzaro, Daniele, et al: A branch-and-cut algorithm for the partitioning-hub location-routing problem, *Computers & operations research*, Vol. 38(2), pp. 539-549, 2011.
6) Fontes, Fábio Francisco da Costa, and Gilles Goncalves: A

variable neighbourhood decomposition search approach applied to a global liner shipping network using a hub-and-spoke with sub-hub structure, *International Journal of Production Research*, Vol. 59(1), pp. 30-46, 2021.
7) Yang, Xiao, Nathalie Bostel, and Pierre Dejax: A MILP model and memetic algorithm for the hub location and routing problem with distinct collection and delivery tours, *Computers & Industrial Engineering*, Vol. 135, pp. 105-119, 2019.
8) Karimi: The capacitated hub covering location-routing problem for simultaneous pickup and delivery systems, *Computers & Industrial Engineering*, Vol. 116, pp. 47-58, 2018.
9) Papadakos: Practical enhancements to the Magnanti–Wong method, *Operations Research Letters*, Vol. 36(4), pp. 444-449, 2008.
10) de Sá, Elisangela Martins, Ricardo Saraiva de Camargo, and Gilberto de Miranda: An improved Benders decomposition algorithm for the tree of hubs location problem, *European Journal of Operational Research*, Vol. 226(2), pp. 185-202, 2013.
11) Righini and Salani: New dynamic programming algorithms for the resource constrained elementary shortest path problem, *Networks: An International Journal*, Vol. 51(3), pp. 155-170, 2008.
12) Ebery, Jamie, et al: The capacitated multiple allocation hub location problem: Formulations and algorithms, *European journal of operational research*, Vol. 120(3), pp. 614-631, 2000.