

# Two-dimensional Geographical Position as a Factor in Determining the Growth and Decline of Retail Agglomeration

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We investigate where retail stores agglomerate in a road network with radial roads and a ring road in a two-dimensional space. Per-distance travel cost on the radial roads can be different from that on the ring road. The transition of the two-dimensional agglomeration patterns of retail stores is investigated with decreases in the travel costs. Results show 1) a difference in improvement sequences in the radial and ring roads generates a difference in the agglomeration patterns with different welfare levels and 2) how the two-dimensional geographical position of shopping agglomerations ensuring the highest welfare level differs from that in equilibrium.

**Key Words:** *Agglomeration, Bifurcation, Monopolistic competition, Two-dimensional road network*

## 1. Introduction

Shopping is an indispensable daily activity in our lives. The hollowing-out of urban commercial centers has been an economic geographical progressing problem over the past several decades. One of the factors driving the hollowing-out is a decrease in travel costs caused by automobility and road improvements. Road improvements, however, provide social benefits to consumers. We explore how road improvements in a two-dimensional road network affect the agglomeration pattern and social welfare.

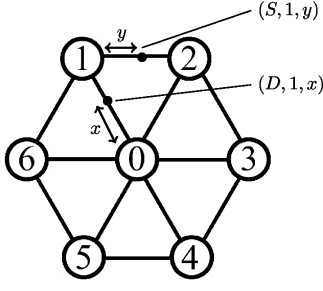
The location of retail stores has been studied for almost a century since Hotelling<sup>1)</sup>. One feature of Hotelling's framework is a simplified urban space: a line segment where consumers are distributed uniformly. Although several studies extend this feature to capture some unique economic mechanisms,<sup>2),3),4),5)</sup> urban spaces in the real world are more complex than the spaces employed by those studies. One realistic factor increasing complexity is a road network embedded in a two-dimensional space. The road network generates geographical heterogeneity, such as a center and suburbs. In order to explore recent urban problems (e.g., the hollowing-out of the center), it is

essential to differentiate the center from the suburbs.

We focus on heterogeneity in road networks observed in the real world. Actually, per-distance travel cost near a center is different from those near suburbs in a real network. Moreover, roads in a city are not simultaneously improved by a local government. Focusing on mainly how an improvement sequence on a road network affects the agglomeration patterns of retail stores and social welfare, we investigate where retail stores are located in such a heterogeneous road network.

We build on the spatial price competition model proposed by Tabuchi<sup>6)</sup>. This model comprises a homogeneous space, monopolistic competition among retail stores, and a dynamical system that describes changes in the sizes of marketplaces where the retail stores are located. Our paper differs from Tabuchi<sup>6)</sup> in a space where retail stores can be located. We employ a regular-hexagonal shape with one center and six suburbs (Fig.1), which are potential marketplaces for retail stores. Moreover, we relax the uniform per-distance travel cost assumption employed in many spatial competition models.

The contribution of our paper is twofold. First, we show that a difference in improvement sequences in the



**Fig.1** City shape. Black lines: the road network in the city; node 0: the center in the city; nodes 1, . . . , 6: the suburbs.

road network generates a difference in agglomeration patterns in equilibrium even for the same travel costs parameters. Second, we show that the scale of agglomeration of retail stores in each marketplace as well as the two-dimensional location pattern of marketplaces in which stores operate at a market equilibrium differ from those at the first-best situation particularly when the travel costs are low. This implies that policymakers should guide stores to form an appropriate location pattern with policies such as land-use regulations.

The rest of our paper is organized as follows. A spatial price competition model is introduced in Section 2. Agglomeration patterns of retail stores are introduced in Section 3. Our theoretical results are verified with numerical comparative statics analysis of the distribution of retail stores in Section 4. Section 5 concludes our paper.

## 2. Model

We consider a city composed of seven potential marketplaces labeled 0, 1, . . . , 6. Marketplace 0 and marketplaces 1, . . . , 6 are in the center and suburbs, respectively. The center is connected to the suburbs by radial roads, whereas the suburbs are located on a ring road.

We consider two types of goods: horizontally differentiated goods and an outside good. The differentiated goods are supplied by a large number of profit-maximizing retail stores in the marketplaces. The outside good is supplied by perfectly competitive firms and chosen as a numéraire good.

### (1) Consumers and retail stores

Consumers in the city are uniformly distributed over the road network with the density normalized to 1. Let  $\mathcal{L}$  denote all the positions on the road network. The utility of consumers residing at  $\ell \in \mathcal{L}$  and visiting marketplace  $j$  is given by  $U(\ell, j) = \ln M_j(\ell) + A(\ell)$ , where  $M_j(\ell) = \left( \int_0^{n_j} q(\ell, k)^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}}$ .  $q(\ell, k)$  is the consumption of the  $k$ th variety,  $n_j$  is the mass of varieties supplied in marketplace  $j$ ,  $\sigma (> 1)$  is the elasticity of substitution between any two varieties, and  $A(\ell)$  is the consumption of the outside good.

If consumers choose to visit marketplace  $j$ , then the budget constraint is given by  $\int_0^{n_j} p_j(k)q(\ell, k) dk + t(\ell, j) + A(\ell) = W$ , where  $p_j(k)$  is the price of the  $k$ th variety in marketplace  $j$ ,  $W$  is the income, and  $t(\ell, j)$  is the travel cost paid by the consumers.<sup>1</sup>

Solving the utility maximization problem, we obtain demand functions:

$$q(\ell, k) = p_j(k)^{-\sigma} R_j^{-1}, \quad (1)$$

$$A(\ell) = W - t(\ell, j) - 1, \quad (2)$$

where  $R_j = \int_0^{n_j} p_j(k)^{1-\sigma} dk$ . We assume that income  $W$  is high so that  $A(\ell)$  is positive in equilibria.

Retail stores are located in marketplaces. These stores share the same marginal production cost  $c$  and the same fixed cost  $f$ . We assume that retail stores in the same marketplace are under monopolistic competition. The total number of retail stores at each marketplace is determined by free entry.

Let  $\pi_i(k)$  be the profit of the retail store producing the  $k$ th variety at marketplace  $i$ .  $\pi_i(k)$  is given by

$$\pi_i(k) = (p_i(k) - c)Q_i(k) - f, \quad (3)$$

where  $Q_i(k)$  is the total demand for the  $k$ th variety at marketplace  $i$ . Using (1), we obtain the profit-maximizing prices, which are the same across all the varieties and marketplaces:  $p_i(k) = p^*$  ( $\forall i, k$ ), where  $p^* = c\sigma/(\sigma - 1)$ . We regard  $\pi_i(k)$  as  $\pi_i$  because each firm at the same marketplace can be treated symmetrically.

### (2) Market area

Substituting  $p^*$  into (1), we obtain demand for the  $k$ th variety (i.e.,  $q(\ell, k)$ ) for consumers at  $\ell (\in \mathcal{L})$  visiting marketplace  $j$ :  $q(\ell, k) = 1/(p^*n_j)$ . Substituting

<sup>1</sup> We ignore commuting in order to focus on how decreases in the travel cost (i.e., road improvements) affect the equilibrium of shopping stores.

this function and (2) into the utility function, we obtain the indirect utility of the consumers:

$$V(\ell, j) = \sigma_{-1} \ln n_j - t(\ell, j) + V_D, \quad (4)$$

where  $\sigma_{-1} = (\sigma - 1)^{-1}$ ,  $V_D = -\ln p^* + W - 1$ . We define the set of the indirect utilities that the consumers can obtain by visiting a marketplace:

$$\mathcal{V}(\ell) = \{V(\ell, 0), V(\ell, 1), \dots, V(\ell, 6)\}, \quad \ell \in \mathcal{L}.$$

Using  $\mathcal{V}(\ell)$ , we mathematically define the market area.

**Definition 1.** The market area of marketplace  $i$  ( $i = 0, 1, \dots, 6$ ) is the following set:

$$\mathcal{M}_i = \{\ell \in \mathcal{L} \mid \max \mathcal{V}(\ell) = V(\ell, i)\}. \quad (5)$$

### (3) Market equilibrium

Let  $\mathbf{n} = (n_0, \dots, n_6)^\top$  denote the distribution of the retail stores across the marketplaces in the city. The market equilibrium condition for  $\mathbf{n}$  is the following condition:

$$\begin{cases} \pi_i = 0 & \text{if } n_i > 0, \\ \pi_i \leq 0 & \text{if } n_i = 0, \end{cases} \quad i = 0, 1, \dots, 6. \quad (6)$$

We employ a dynamical system to investigate the stability of equilibria:

$$\frac{d\mathbf{n}}{dt} = \mathbf{F}(\mathbf{n}), \quad (7)$$

where  $\mathbf{F}(\mathbf{n}) = (F_0(\mathbf{n}), F_1(\mathbf{n}), \dots, F_6(\mathbf{n}))^\top$  and  $F_i(\mathbf{n}) = n_i \pi_i$  ( $i = 0, 1, \dots, 6$ ). The market equilibrium is equal to the stationary point of dynamics (7). We investigate the market equilibria by finding stationary points of dynamics (7). A stationary point is linearly-stable if every eigenvalue of Jacobian matrix  $\partial \mathbf{F} / \partial \mathbf{n}$  has a negative real part.

### (4) Travel cost

Consumers have several route choices to the marketplaces. The consumers choose the route with the lowest travel cost.

We mathematically define  $\mathcal{L}$  to express travel costs, which are determined by the distance between consumers and a marketplace. Let  $D$  and  $S$  denote the radial roads and the ring road in the city, respectively. Since we assume that the length of each road between the marketplaces is 1, we can represent  $\mathcal{L}$  by  $\mathcal{L} = \mathcal{A} \times \mathcal{P} \times X$ , where  $\mathcal{A} = \{D, S\}$ ,  $\mathcal{P} = \{1, 2, \dots, 6\}$ , and  $X = (0, 1)$ .  $(D, i, x) \in \{D\} \times \mathcal{P} \times X$  is equal to position  $x$  distant from the center on the radial road between the center and suburb  $i$  (e.g., see  $(D, 1, x)$  in Fig.1). Similarly,  $(S, i, y) \in \{S\} \times \mathcal{P} \times X$  is equal to

position  $y$  distant from suburb  $i$  on the ring road between suburb  $i$  and  $j$  ( $\equiv i+1 \pmod{6}$ ) (e.g., see  $(S, 1, y)$  in Fig.1).

For consumers residing at  $\ell = (D, i, x) \in \{D\} \times \mathcal{P} \times X$  (i.e., consumers residing along the radial roads), the travel cost is given by

$$t(\ell, j) = \begin{cases} \phi x & (j = 0), \\ \min \{\phi(1+x), \phi(1-x) + \tau L_{ij}\} & (j \in \mathcal{P}), \end{cases} \quad (8)$$

where  $\phi$  is the per-distance travel cost on the radial roads,  $\tau$  is the per-distance travel cost on the ring road, and  $L_{ij} = \min \{|i-j|, 6-|i-j|\}$ .  $\phi x$  is the travel cost when the consumers visit the center;  $\phi(1+x)$  is when the consumers visit suburb  $j$  ( $\in \mathcal{P}$ ) via the center;  $\phi(1-x) + \tau L_{ij}$  is when via the suburbs. On the other hand, for consumers residing at  $\ell = (S, i, x) \in \{S\} \times \mathcal{P} \times X$  (i.e., consumers residing along the ring road), the travel cost is given by

$$t(\ell, j) = \begin{cases} \phi + \tau(1/2 - |x - 1/2|) & (j = 0), \\ \tau \times \min \{|i+x-j|, 6-|i+x-j|\} & (j \in \mathcal{P}). \end{cases} \quad (9)$$

$\phi + \tau(\dots)$  is the travel cost when the consumers visit the center via the nearest suburb;  $\tau \times \min \{\dots\}$  is the travel cost when the consumers visit marketplace  $j$  via the shortest route along the ring road.

### (5) Welfare

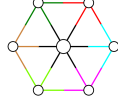
We measure the efficiency of the distribution of retail stores. Since the retail stores' profits are zero in the equilibria by condition (6), social welfare  $SW$  is total consumer utility (in monetary terms).

## 3. Agglomeration patterns of retail stores

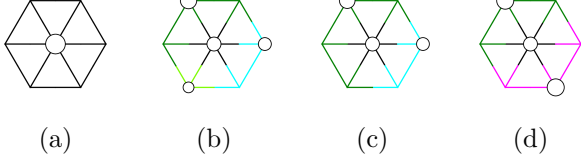
We focus on some agglomeration patterns of retail stores. These patterns are possible market equilibria, which are investigated in Section 4. We focus on the agglomeration pattern of retail stores in which every marketplace has a market area (i.e.,  $\mathcal{M}_i \neq \emptyset$  ( $i = 0, 1, \dots, 6$ )).

We define this market area pattern as market pattern (D), and the equilibria that forms market pattern (D) as the dispersion (Fig.2).

We also focus on the equilibria in which some marketplaces have no market area. We call these equilibria corner equilibria. We investigate four corner equi-



**Fig.2** The dispersion. Black area:  $\mathcal{M}_0$ ; green:  $\mathcal{M}_1$ ; red:  $\mathcal{M}_2$ ; sky blue:  $\mathcal{M}_3$ ; pink:  $\mathcal{M}_4$ ; yellowgreen:  $\mathcal{M}_5$ ; brown:  $\mathcal{M}_6$ ; the size of  $\bigcirc$ : the number of retail stores.



**Fig.3** Corner equilibria under investigation. (a): the full agglomeration; (b) the period-doubling pattern; (c) the asymmetric pattern; (d) the linear pattern.  $\bigcirc$ : the number of retail stores. Black area:  $\mathcal{M}_0$ ; green:  $\mathcal{M}_1$ ; sky blue:  $\mathcal{M}_3$ ; pink:  $\mathcal{M}_4$ ; yellowgreen:  $\mathcal{M}_5$ .

libria (Fig.3): the full agglomeration ( $\mathcal{M}_0 \neq \emptyset$ ), the period-doubling pattern ( $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_3, \mathcal{M}_5 \neq \emptyset$ ), the asymmetric pattern ( $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_3 \neq \emptyset$ ), and the linear pattern ( $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_4 \neq \emptyset$ ). These equilibria are possible agglomeration patterns into which the dispersion changes with decreases in  $\phi$  and  $\tau$ .

#### 4. Dependency of agglomeration patterns on improvement sequences in the road network

Conducting the numerical comparative statics analysis of equilibria, we verify that road improvement sequences generate differences in the agglomeration patterns and the social welfare in equilibrium. We set  $(\sigma, f) = (6.0, 20)$ . We focus on two simple improvement sequence patterns: (1) the radial roads are improved first and (2) the ring road is improved first. We investigate the following two cases of changes in the travel costs:

- The radial-roads first case:

$$(\phi, \tau) = \underbrace{(1.0, 1.0)}_{\text{Stage } \alpha} \xrightarrow{\text{Transition 1}} \underbrace{(0.35, 1.0)}_{\text{Stage } \beta_1} \xrightarrow{\text{Transition 2}} \underbrace{(0.35, 0.17)}_{\text{Stage } \gamma}.$$

- The ring-road first case:

$$(\phi, \tau) = \underbrace{(1.0, 1.0)}_{\text{Stage } \alpha} \xrightarrow{\text{Transition 1}} \underbrace{(1.0, 0.17)}_{\text{Stage } \beta_2}$$

$$\xrightarrow{\text{Transition 2}} \underbrace{(0.35, 0.17)}_{\text{Stage } \gamma}.$$

The radial-roads first case is that the radial roads are improved first, and the ring road is improved next. On the other hand, the ring-road first case is that the ring road is improved first.

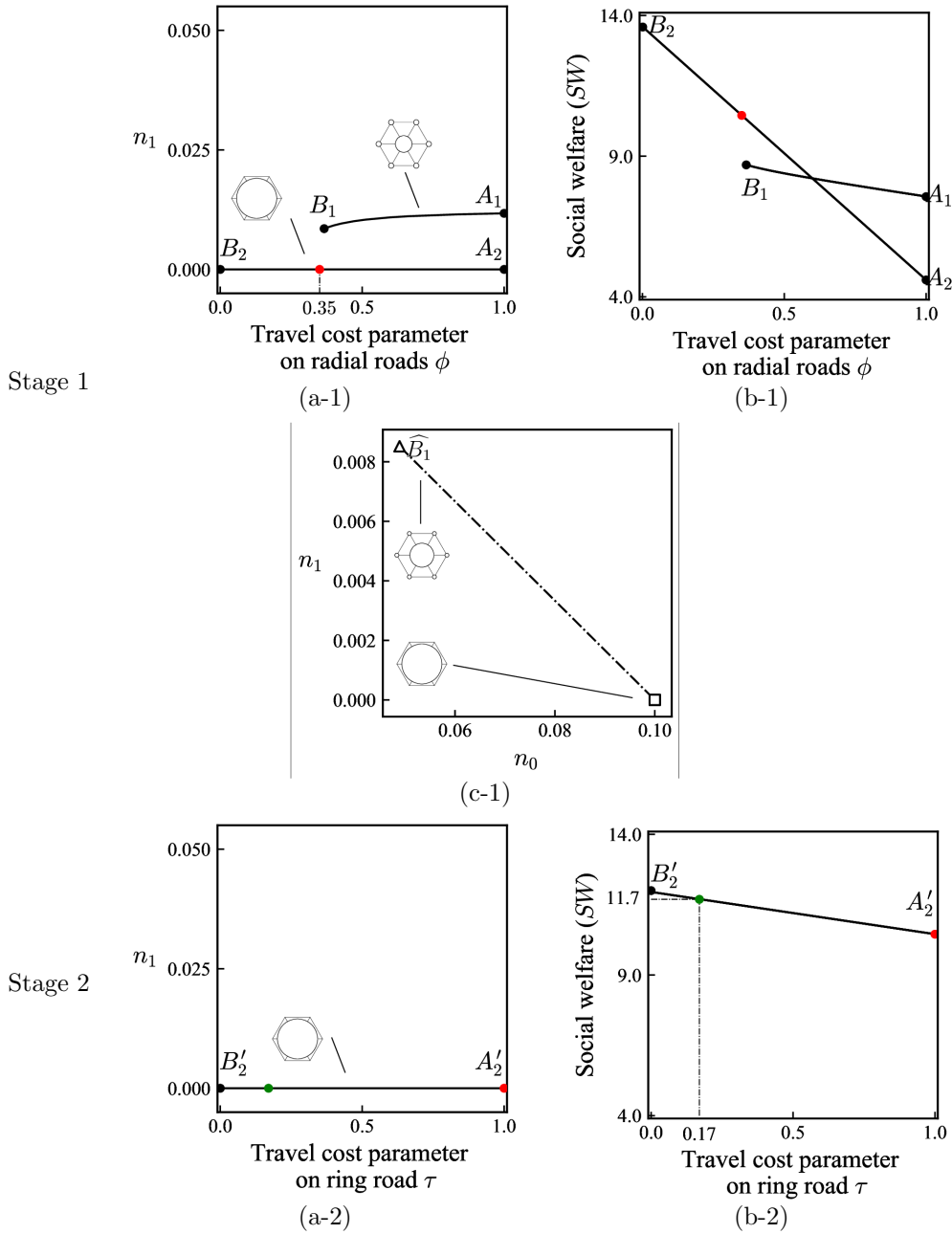
First, we focus on the result of the radial-roads first case shown in Fig.4. Fig.4(a-1) is the comparative statics analysis with a decrease in  $\phi$ , which is equal to Stage 1. Solid lines  $A_1B_1$  and  $A_2B_2$  are the stable dispersion and the stable full agglomeration, respectively. Both the dispersion and the full agglomeration exist for large  $\phi$  ( $> 0.37$ ). In the dispersion, number of retail stores in the center  $n_0$  increases and market area of the center  $\mathcal{M}_0$  expands with a decrease in  $\phi$ . For small  $\phi$  ( $= 0.37$ ),  $\mathcal{M}_0$  entirely covers the radial roads.

We investigate how a point in a neighborhood of  $B_1$  changes under dynamics (7). Let  $\widehat{B}_1$  denote the point. The solution starting at  $\widehat{B}_1$  under dynamics (7) is shown in Fig.4(c-1). This solution converges at the square marker ( $\square$ ). The point marked by the square marker shows the full agglomeration. In summary, the dispersion changes into the full agglomeration when the radial roads are improved.

The full agglomeration is always stable (Fig.4(a-1)). The red point in Figure 4(a-1) is equal to the agglomeration pattern at stage  $\beta_1$  (i.e., the full agglomeration).

We focus on stage  $\gamma$  of the radial-roads first case. Fig.4(a-2) is the comparative statics analysis with a decrease in  $\tau$ , which is equal to Transition 2. The green point in Fig.4(a-2) shows the agglomeration pattern of the final stage (i.e., the full agglomeration). Hence, the radial-roads first case results in the full agglomeration emerging from the dispersion.

Next, we focus on the result of the ring-road first case shown in Fig.5. Figure 5(a-1) is the comparative statics analysis with a decrease in  $\tau$ , which is equal to Stage 1. Solid line  $A_1B_1$  is the stable dispersion;  $A_3B_3$  is the period-doubling pattern;  $A_4B_4$  is the linear pattern;  $A_5B_5$  is the asymmetric pattern. The dispersion becomes unstable at point  $B_1$ , which is a bifurcation point. Three unstable equilibria emerge at this point. When the dispersion is unstable, a small perturbation to this state generates an agglomeration pattern where large agglomerations and small agglomerations alternately emerge on the ring road

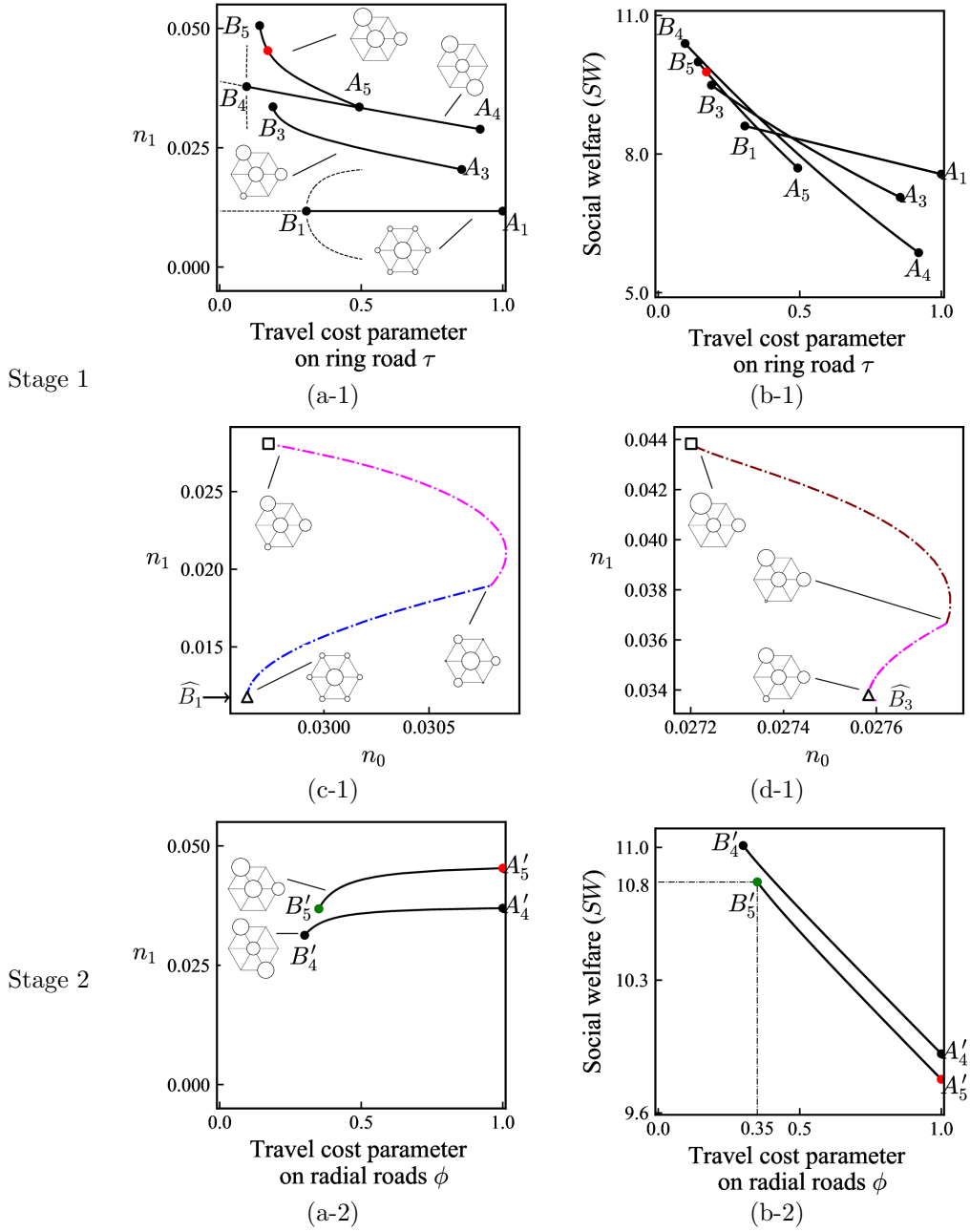


**Fig.4** The radial-roads first case. (a) The market equilibria with decreases in the travel costs. Solid line: stable equilibria. (b) Social welfare of stable equilibria in (a). (c-1) The solution starting at a point in the neighborhood of point  $B_1$ . Dashed-dotted line: the solution under market pattern (F).

To investigate the change from the unstable equilibrium, we investigate how a point in a neighborhood of bifurcation point  $B_1$  changes under dynamics (7). The solution starting at this point under dynamics (7) is shown in Fig.5(c-1). The solutions shown with the blue line and the pink line in Fig.5(c-1) are obtained under market areas (D) and (P), respectively. The solution converges at the point marked by the square marker ( $\square$ ). This result shows that the dispersion changes into the period-doubling pattern. Moreover, in the period-doubling pattern,  $n_1$  increases with a

decrease in  $\tau$ . This pattern disappears at point  $B_3$  (Figure 5(a-1)).

To elucidate the change from point  $B_3$ , we investigate how a point in the neighborhood of point  $B_3$  changes under dynamics (7). The solution starting at this point is shown in Fig.5(d-1). Near  $\widehat{B}_3$ , we obtained this solution with dynamics (7) in market pattern (P). On the other hand, for small  $n_5$ , the solution was obtained under that in market pattern (A). This result shows that the period-doubling pattern changes into the asymmetric pattern. Hence, the asymmetric



**Fig.5** The ring-road first case. (a) The market equilibria with decreases in the travel costs. Solid line: stable equilibria; dashed line: unstable equilibria (b) Social welfare of stable equilibria in (a). (c-1) The solution of the dynamics starting at a point in a neighborhood of  $B_1$ . Blue dashed-dotted line: the solution under market pattern (D); pink: market pattern (P). (d-1) The solution starting at a point in a neighborhood of  $B_3$ . Pink: market pattern (P); brown: market pattern (A).

pattern emerges at stage  $\beta_2$  is the red point in Fig.5(a-1).

We focus on stage  $\gamma$ . Fig.5(a-2) is the comparative statics analysis with a decrease in  $\phi$ , which is equal to Stage 2. In both the asymmetric pattern and the linear pattern,  $n_1$  decreases with a decrease in  $\phi$ . The green point in Figure 5(a-2) is equal to the agglomeration pattern of stage  $\gamma$  (i.e., the asymmetric pattern). This pattern is not the agglomeration pattern that

emerges in the radial-roads first case. In summary, the improvement sequences in the road network finally generate the difference in the agglomeration pattern. This observation is main finding 1.

We focus on the welfare of the linear pattern shown in the ring-road first case. In the ranges of travel costs ( $0.01 < \tau < 0.34$  in Fig.5(b-1) and  $0.35 < \phi \leq 1.00$  in Fig.5(b-2)), the welfare in the linear pattern is higher than that in the other patterns in the ring-road first

case. However, not the linear pattern but the asymmetric pattern emerges from the dispersion in the market system. That is, the two-dimensional shape of the location in the market system is not that of the first-best location. This result is main finding 2, which indicates that policies that change the locations of marketplaces are needed (e.g., land-use regulations).

## 5. Conclusion

We have investigated how improvement sequences on a two-dimensional road network affect the agglomeration patterns of retail stores and social welfare. We have two main findings: (1) the improvement sequence in the road network finally generates the difference in agglomeration patterns and (2) the two-dimensional shape of the locations in the market system differs from that in the first-best location. In particular, the asymmetric pattern emerges if the ring road is improved first.

## Acknowledgement

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