

# Quantitative Analysis of Property Tax and Floor Area Ratio Regulation with respect to Efficiency and Income Distribution

Mayu HIROTA<sup>1</sup>, Tatsuhito KONO<sup>2</sup> and Nozomu TAKAMURA<sup>3</sup>

<sup>1</sup>E-mail:mayu.hirota.p4@dc.tohoku.ac.jp

<sup>2</sup>Member of JSCE, Professor, Graduate School of Information Sciences, Tohoku University  
(Aoba 6-6-06, Aramaki, Aobaku, Sendai, Miyagi 980-8579, Japan)

E-mail:kono@plan.civil.tohoku.ac.jp

<sup>3</sup>Bachelor Student, Dept. of Civil Eng., Tohoku University  
(Aoba 6-3-6, Sendai, Miyagi 980-8579, Japan)

E-mail:nozomu.takamura.s8@dc.tohoku.ac.jp

\*Alphabetical listing of authorship.

We explore the relation between optimal property taxation and optimal land use regulation in a spatial setting, represented by a monocentric urban model with homogenous people. As is well-known, both policies are equivalent in terms of efficiency if lump-sum tax is available. However, previous studies have not considered the financial constraint of the government supplying public services with tax revenue. As many optimal taxation papers show, taxation except for lump-sum tax generally yields welfare losses. Accordingly, the equivalency generally does not hold. Our study explores the efficiency and income distribution of property tax and land use regulation with explicitly considering a government budget constraint. For this purpose, we set four regimes: Regime 1 in which congestion pricing and head tax are available, Regime 2 in which land use regulations are imposed with head tax, Regime 3 where optimal spatially-variable property taxation is imposed without head tax, and Regime 4 in which FAR regulations are imposed with a constant rate of property tax. Our results can be summarized as follows. Regime 1 is the first best. Regimes 2 and 4 are the second best. Regime 3 is the least efficient because the financial constraint additionally generates deadweight losses in the floor market. But Regimes 1 and 2, which use an impractical head tax, are basically infeasible in the real world. Regime 3 is less efficient than Regime 4. But land rent is always positive in Regime 3, while land rent can be negative in Regime 4. When land rent is negative, landowners do not provide land. In this respect, Regime 3 has an advantage.

**Key words:** location-dependent property tax, FAR regulation, efficiency, distribution effects

## 1. Introduction

Most cities suffer from congestion. To cope with such congestion problems, cities often use FAR regulations, which can change the population density distribution. Property tax can be a substitution policy to FAR regulations.

Actually, a spatially-variable property tax can serve as a complete substitute to FAR regulations in terms of efficiency, as proved in Pines and Kono (2012). But this equivalency holds as long as lump-sum tax is available. In reality, lump-sum tax is not imposed. So, we need to take account of the government budget constraint.

Kono, Pines and Yokoi (2019) explore a spatially-variable property tax in such a case. This previous study shows that a spatially-variable property tax is not equal to FAR regulations in terms of efficiency, when we take account of the government budget constraint.

This study focuses on property tax and FAR

regulations with explicitly considering a government budget constraint.

## 2. Model

We use the setup of Kono, Pines and Yokoi (2019). The city's geography is represented by the location of a center at some point in space and the exogenous supply of land around it, which is potentially available for housing production. The latter is represented by a function  $\bar{\phi}[x]$ , which is the length of the arc for residential area at distance  $x$  from the center where,  $x \in [0, b, \bar{j}]$ ,  $\bar{\phi}[0] = 0$ ,  $b$  is the most distant location from the city center where land is used for residence, and  $\bar{j}$  is the jurisdiction's radius (by assumption,  $\bar{j} \geq b$ ).

The city accommodates  $\bar{N}$  identical households. The utility function of a household who lives  $x$  miles from the center is given as  $\tilde{u}[z[x], h[x]]$ , where  $z[x]$

is the consumption of a composite good, which is also the numeraire, and  $h[x]$  is the consumption of housing at  $x$ . The initial endowment of a representative household is  $I$  units of a composite good and an equal share in the housing-producing firm which entitles them to an equal share in the aggregate firms' profit.

The spatial distribution of the  $\bar{N}$  identical households is represented by a function  $N[x]$ , the population residing between  $x$  and the boundary where  $x \in [0, b]$ .

Let  $n[x] \equiv \bar{\phi}[x]\delta[x]$  measured by households/miles units. Then,

$$N[x] = \int_x^b n[x']dx',$$

A round trip between  $x$  and the city center, where all the workplaces are located, costs  $T[x]$  units of the composite good.  $T[x]$  is

$$T[x] = \int_0^x t[N[s]]ds \text{ at } \forall x \in [0, b],$$

where  $t[N[s]]$  is the marginal transportation cost with respect to distance from the city center as a function of the traffic volume at  $s$ . The city has multiple congested radial roads and infinite circumferential roads on which travelling is costless. A toll  $p[x]$  may be imposed on commuting where  $x \in [0, J]$ .

Housing is supplied by numerous price-taking firms. The available urban land is used to produce housing, along with composite goods. Let  $c[H[x]]$  units of the composite good be required per unit of land to produce  $H[x]$  units of housing per unit of land area. Then, if housing is supplied at a positive level, profit maximization requires that

$$\tau[x] = \frac{q[x] - c'[H[x]]}{q[x]} \text{ at } \forall x \in [0, b],$$

where  $\tau[x]$  is the spatially-variable ad valorem tax on housing.

The net profit  $\Pi[x]$  from housing production per unit of land is:

$$\Pi[x] = H[x]q[x](1 - \tau[x]) - (c[H[x]] + \bar{r}) \text{ at } \forall x \in [0, b].$$

A viable city requires a fixed quantity of local public goods (hereafter, LPG), which costs  $\bar{G}$ .  $\bar{F}$  is transferred from a higher-level of government. A planner finances  $\bar{G} - \bar{F}$  by the optimal set of tax instruments available. Although our main concern is financing the LPG only with an optimal structure of excise tax on housing (property tax, hereafter), we begin our discussion by considering alternative tax regimes, each representing a distinct combination of taxes/subsidies. Accordingly, we define four regimes:

- Regime 1. Congestion pricing and a head tax.
- Regime 2. FAR regulation and a head tax.
- Regime 3. Spatially variable rate of property tax.
- Regime 4. FAR regulation and the common rate of

property tax.

We define the budget constraint of the planner in general terms. Accordingly:

$$\begin{aligned} \bar{G} - \bar{F} - \bar{N}l - \int_0^b N[x]p[x]dx \\ - \int_0^b \bar{\phi}[x]H[x]q[x]\tau[x]dx = 0, \end{aligned}$$

where  $l$  is a head tax rate.

### 3. Numerical Simulations

We divide the city into four discrete rings with an equal width (2.5km). The rings are indexed by  $i \in \{1, \dots, 4\}$ , where  $i = 1$  at the zone adjacent to the CBD edge.

We consider a hypothetical city with  $\bar{N} = 500,000$  and  $700,000$ . The utility function is specified as  $u = z + 0.8 \ln(h)$ , where  $0.8$  is determined so that the housing expenditure comprises 16% of the household income. We set the household income as  $\bar{I} = 5$  (million yen). The floor production function is specified as  $c(H) = (\frac{H}{0.0028})^{1/0.75}$ , where the parameters are estimated from 112 samples of Japanese buildings. Transportation cost function has  $t(x) = \eta + \delta(\frac{N(x)}{capa})^{2.82}$ , where  $\eta$ ,  $\delta$ , and  $capa$  are set up for a hypothetical city in Japan.

Table 1 shows the social welfare by regime and income distribution in the case of 500,000 and 700,000 households. Regime 0 (market equilibrium) is included in Table 1. Comparing to Regime 0, we can show how much efficiency has been increased by property tax and FAR regulation. The social welfare in Regime 1 is, needless to say, the first best. We see that the level of social welfare in Regime 2 is equal to that in Regime 4. Then, we find that the social welfare in Regime 3 is the lowest. These results are consistent with theoretical results. The introduction of a head tax in Regimes 1 and 2 is equivalent to not taking account of the government budget constraint, Regimes 3 and 4 take into account the government budget constraint. Comparing Regimes 3 and 4, we can find the welfare improvement ratio against the first best are about 20% and 50% in Regime 3 and Regime 4, respectively. So, Regime 4 is better. But cell (a) in Table 1 shows a negative value of landowner's profit in Regime 4, so landowners do not provide land. The landowner's profit in Regime 3 cannot be negative, which can be theoretically proved. From these points, Regime 3 can be a promising policy for congestion externalities.

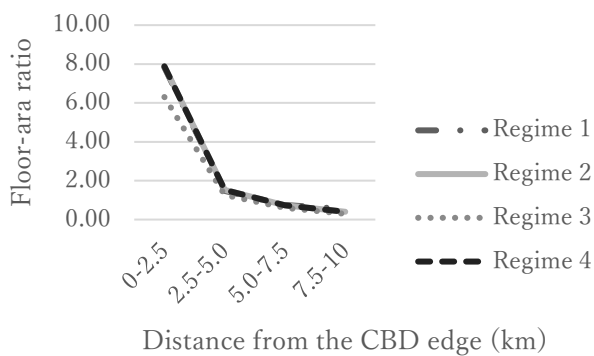
Figure 1 shows the floor area ratio at each location by regime. The FAR near the CBD in Regime 2 and Regime 4 is close to Regime 1. But that in Regime 3 is slightly lower.

Finally, the characteristics of each regime are summarized as follows. Regimes 1 and 2 have relatively high efficiencies, but use a lump-sum tax, which is not practically used worldwide. So, these two regimes are thought to be infeasible in the real world. Regime 3 is less efficient than Regime 4. But land rent is always positive in Regime 3, while land rent can be negative depending on the locations and the economic situations in Regime 4. When land rent is negative, the landowner does not provide land. In this sense, Regime 3 is feasible, while Regime 4 can be infeasible due to negative land rent.

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**Table1:** Social welfare and income distribution

households	Regime	landowners'	Residents	Social	Welfare
		profit (10 <sup>11</sup> yen)	utility (10 <sup>11</sup> yen)	welfare (10 <sup>11</sup> yen)	improvement ratio (%)
500,000	0	1.0	39.2	40.2	100.0
	1	1.0	39.6	40.6	
	2	1.0	39.4	40.4	52.9
	3	0.7	39.5	40.3	17.5
	4	0.002	40.4	40.4	52.9
700,000	0	1.4	52.7	54.1	100.0
	1	1.4	53.7	55.1	
	2	1.4	53.2	54.6	50.2
	3	1.0	53.3	54.4	23.7
	4 (a)	-0.1	54.7	54.6	50.2



**Figure 1:** FAR by location (500,000 households)

#### 4. Concluding Comments

This study explores the property tax and FAR regulations in consideration of fiscal constraints, and clarifies their respective characteristics through quantitative analysis.

#### Reference

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