SPATIAL PREDICTION OF APARTMENT RENT PRICE USING MACHINE LEARNING-BASED APPROACHES

Takahiro YOSHIDA¹, Daisuke MURAKAMI² and Hajime SEYA³

¹Member of JSCE, Project Assistant Professor, Dept. of Urban Eng., The University of Tokyo (3-1, Hongo 7, Bunkyo-ku, Tokyo 113-8656, Japan) E-mail: yoshida.takahiro@up.t.u-tokyo.ac.jp (Corresponding Author)
²Member of JSCE, Assistant Professor, Dept. of Data Sci., The Institute of Statistical Mathematics (10-3, Midori-cho, Tachikawa, Tokyo 190-8562, Japan) E-mail: dmuraka@ism.ac.jp
³Member of JSCE, Associate Professor, Dept. of Civil Eng., Kobe University (1-1, Rokkodai-cho, Nada-ku, Kobe, Hyogo 657-8501, Japan) E-mail: hseya@people.kobe-u.ac.jp

Employing a large dataset (at most, the order of $n = 10^6$), this study attempts enhance the literature on the comparison between regression and machine learning (ML)-based rent price prediction models by adding new empirical evidence and considering the spatial dependence of the observations. The regressionbased approach incorporates the nearest neighbor Gaussian processes (NNGP) model, enabling the application of kriging to large datasets. In contrast, the ML-based approach utilizes typical models: extreme gradient boosting (XGBoost), random forest (RF), and deep neural network (DNN). The out-of-sample prediction accuracy of these models was compared using Japanese apartment rent data, with a varying order of sample sizes (i.e., $n = 10^4$, 10^5 , 10^6). The results showed that, as the sample size increased, XGBoost and RF outperformed NNGP with higher out-of-sample prediction accuracy. XGBoost achieved the highest prediction accuracy for all sample sizes and error measures in both logarithmic and real scales and for all price bands (when $n = 10^5$ and 10^6). A comparison of several methods to account for the spatial dependence in RF showed that simply adding spatial coordinates to the explanatory variables may be sufficient.

Key Words : Apartment rent price prediction, large data, Nearest neighbor Gaussian processes (NNGP), Deep neural network (DNN), Extreme gradient boosting (XGBoost), Random forest (RF)

1. INTRODUCTION

Online automatic real estate price estimation services, such as Zestimate (a service of the Zillow Group in the United States), are increasing in popularity. Seya and Shiroi (2021)¹⁾ reported that accurate price assessments and predictions are crucial for real estate agents, as well as end users. Considering the perspective of the agency, reducing appraisal costs and improving transparency are advantageous. Meanwhile, the perspective of end users involves the improvement of information asymmetry between real estate agencies and end users to a certain extent. Massive property data and statistics- or machine learning (ML)-based real estate sales and rent price prediction methods are the means of support for agents and users.

Traditionally, regression approaches are employed

to prices of real estate and rent, although automated assessment of real estate sales and rent prices using techniques involving massive data and ML-based has garnered attention^{2,3}). Efron (2020)⁴) reported regression-based approaches being typically used for prediction as well as attribution, that is, the individual predictors being assigned significance (i.e., significance testing). However, combining weak learners in ML-based approaches (e.g., random forest (RF) or extreme gradient boosting [XGBoost]) is not effective in the case of attribution. Thus, regressionbased approaches can offer advantages. However, considering pure prediction, simple functional forms, such as linear, logarithmic, and Box-Cox, commonly employed in regression-based approaches, may be inadequate for capturing the nonlinearity of the data. Therefore, examining the extent of the difference in prediction accuracy from ML-based methods is crucial.

Constructing prediction models of real estate sales or rent prices results in certain challenges in accommodating factors such as neighborhood quality as explanatory variables (covariates)⁵⁾. Hence, considering the spatial dependence inherent in the data is important^{6,7)}. In geo-(spatial) statistics, regression-based kriging was established to incorporate spatial dependence among error terms, typically applying a Gaussian process (GP) to the $errors^{8)}$. Certain studies, such as James et al. $(2005)^{9)}$, Bourassa et al. $(2010)^{10}$, and Seya et al. $(2011)^{11}$, reported that high predictive accuracy was offered by the kriging approach compared to multiple regression models (ordinary least squares [OLS]) in the property-related literature. Because the OLS model structure is straightforward, it enables parameter determination using relatively small samples. However, with kriging, as the price information of neighboring properties is reflected in the predicted results through spatial dependence, it results in a situation different from that of OLS. That is, several prediction benefits of increasing the sample size exist¹⁾. In contrast, in the case of the ML approach, research that attempts to introduce spatial dependency into the model remains in its early stages, although some interesting studies have recently been conducted^{12,13)}.

This study aims to supplement the literature with the addition of new empirical evidence via comparison of regression- and ML-based rent price prediction models used on a large dataset (at most, in the order of $n = 10^6$), and considering the spatial dependence among observations as an expansion of that reported by Seya and Shiroi $(2021)^{1}$. The former approach required kriging. However, OLS was also employed to set a general benchmark. Moreover, with increasing sample size (e.g., when $n = 10^5$), application of kriging directly becomes difficult, requiring $O(n^3)$ computational cost to invert the variance-covariance matrix. Therefore, we considered the nearest neighbor Gaussian processes (NNGP) model, allowing the application of kriging to massive data via sparse approximation^{14,15,16}. Although exist several methods exist to conduct spatial statistical modeling with big data^{17,18}, NNGP is reliable owing to it consistent competitive results in comparative studies¹⁹⁾. Furthermore, for the latter, certain representative models, namely, RF, XGBoost, and deep neural network (DNN) were employed.

Several trials have compared and investigated the predictive accuracy of real estate sales and rent prices between regression- and ML-based approaches. However, certain limitations should be overcome, including [1] small sample sizes, [2] disregard for spatial dependence, and [3] tailored and ad hoc

hyperparameter settings. Hence, in this study, we [1] examined different and relatively large sample sizes $(n = 10^4, 10^5, \text{ and } 10^6)$, [2] considered spatial dependence, and [3] finely calibrated the hyperparameters via cross-validation.

Further, this study employed the LIFULL HOME's dataset to evaluate monthly residential apartment rental prices in Japan for empirical evidence. This dataset was also used by Seya and Shiroi (2021)¹⁾ and comprises rental property cross-sectional data and image data up to September 2015. The rental property cross-sectional data include rent, lot size, location (i.e., municipality, zip code, nearest station, and time consumed to walk to the nearest station), year it was built, layout of the room, building structure, and equipment for 5.33 million properties throughout Japan. Meanwhile, the image data comprise 83 million pictures that outline the floor plans and details about the interiors for every property. This study employed only the former data.

Among the 5.33 million properties, 4,588,632 properties were retained after the missing data was excluded. Thereafter, $n = 10^4$, 10^5 , and 10^6 properties were randomly sampled from the cleaned data. Subsequently, the regression-based (OLS and NNGP) and ML-based approaches (RF, XGBoost, and DNN) were compared via a validation process while considering the difference in sample size and out-of-sample predictive accuracy of rent prices.

The remainder of this paper proceeds as follows. Section 2 presents a short review of existing literature. Section 3 explains the models used in this comparison study. Further, Section 4 details the results obtained from the comparative analysis using the LIFULL HOME dataset. Finally, the concluding remarks, along with the scope for future research, are presented in Section 5.

2. LITERATURE REVIEW

This section presents a review of the literature regarding the prediction of real estate sales and rent prices. Studies have postulated that spatial regression models exhibit a high predictive accuracy compared to the OLS model⁹. Seya et al. (2011)¹¹ examined the performance of various spatial prediction models that considered spatial dependence by employing a dataset comprising apartment rents of 23 wards in Tokyo for empirical comparison. They showed the benefit of considering spatial dependence in the error term (e.g., kriging, geoadditive model, and spatial error model) or regression coefficients (e.g., geographically weighted regression (GWR) model). However, a limitation was the small size (i.e., 529 for parameter estimation and 150 for validation). Geostatistical models (kriging) and spatial econometric models are extensively used to consider spatial dependence among errors. Many studies have applied both methods to hedonic price modeling. However, for the purpose of spatial (i.e., out-of-sample) prediction, the former, which requires no spatial weight matrix, is more natural and flexible²⁰⁾. However, a comparison by Seya et al. (2011)¹¹⁾ revealed that the differences in the predictive accuracy between the kriging and spatial econometric models are negligible, compared to the differences between OLS and kriging. For kriging, application to massive data, on the order of a million, can be achieved by considering various approximations¹⁹⁾.

Various methods have been developed to model spatial dependence among regression coefficients in different fields, including geography, statistics, and $ML^{21,22,23,24}$. Because the housing market is often segmented, the local model (i.e., spatially varying coefficient (SVC) model) can be applied. Hence, SVC models have been employed for hedonic price modeling in several studies. However, their application to massive amounts of data remains in its nascent stages^{25,26}). For instance, the scalable GWR model, proposed by Murakami et al. (2020)²⁶⁾, was applied to our dataset. However, the parameter estimation procedure (i.e., the bandwidth selection procedure) was not completed within 24 hours when $n = 10^5$. Thus, application of the model to a dataset with a sample size of $n = 10^6$ and above is difficult.

Several attempts have been made to achieve the results based on similar motivation. Seya and Shiroi $(2021)^{1}$ reviewed studies that employed neural network (NN) methods. Valier (2020)²⁷⁾ reported that 57 cases are available wherein ML-based models, including NN, were more accurate in predicting the values than the 13 cases wherein regression performed better. Zurada et al. $(2011)^{28}$ suggested that, although many recent studies have compared regression with artificial intelligence (AI)-based methods in the context of mass appraisal, useful comparison of the published results is a challenge because the models in many studies were built by considering relatively small samples. Therefore, for a more comprehensive comparative study, a dataset containing over 16,000 sales transactions was used. They found that non-traditional regression-based methods performed better in all simulation scenarios, specifically with homogeneous datasets. However, AI-based methods performed well with less homogeneous datasets under certain simulation scenarios. Seya and Shiroi $(2021)^{1}$, upon which the present study was built, compared the performances of OLS, NNGP, and DNN. They found that, with an increase in sample size from $n = 10^4$ to 10^6 , the DNN's out-of-sample predictive accuracy approaches that of NNGP and is

nearly equal in the order of $n = 10^6$. However, in terms of both higher- and lower-end predictive accuracy for which rent prices deviate from the median, DNN may have better results than NNGP. Seya and Shiroi $(2021)^{11}$ have a clear limitation in that they only used the DNN method to represent the ML approach.

Several studies have employed tree-based techniques to realize the ML approach. Pace and Hayunga (2020)²⁹⁾ examined tree-based techniques, including classification and regression trees (CART)³⁰⁾, boost ing^{31} , and $bagging^{32}$. Further, they compared these techniques to the spatio-temporal linear model³³, considering over 80,000 real estate prices in the United States. Bagging was found to work well and could yield lower out-of-sample residuals than global spatiotemporal methods; however, its performance was poorer than local spatiotemporal methods. Mayer et al. $(2019)^{34}$ used a large dataset consisting of over 123,000 single-family houses sold in Switzerland between 2005 and 2017. They reported that the gradient boosting (GB) approach performed far better than the other methods. It was followed by mixed effects regression, the NN method, and the RF approach in terms of performance. Based on the online housing platform, Ming et al. (2020)³⁵⁾ used 33,224 pieces of data reflecting Chengdu housing rentals in China. They empirically compared the predictive performance of RF, LightGBM, and XGBoost and found that XGBoost performed the best. Ho et al. $(2021)^{36}$ used three ML algorithms, namely, support vector machine (SVM), RF, and GB, to appraise property prices. They applied these methods to examine a data sample of approximately 40,000 housing transactions over a period of over 18 years in Hong Kong and then compared the results of these algorithms. They found that RF and GBM outperformed the SVM in terms of predictive power.

3. MODELS

This section introduces regression- and ML-based approaches to the spatial prediction.

(1) Regression-based approaches

a) NNGP

Consider *D* as the spatial domain and *s* the coordinate position (X, Y). The spatial regression model, also referred to as the spatial process model^{17,37)}, can be expressed as:

$$y(s) = m(s) + w(s) + \varepsilon(s),$$

$$\varepsilon(s) \sim N(0, \tau^2),$$
(1)

where y(s) is the spatial process for real estate rental prices, which is decomposed to m(s), w(s), and $\varepsilon(s)$. Further, τ^2 represents a variance parameter

termed as a nugget representing the micro-scale variation and measurement error³⁸⁾. Typically, it is assumed that $m(s) = x(s)'\beta$, where x is an explanatory variable vector at point s, and β is the corresponding regression coefficient vector. Further, w(s)is assumed to follow the GP: $w(s) \sim GP(0, C(\cdot, \cdot | \theta))$, with the mean being zero and the covariance function being $C(\cdot, \cdot | \theta)$ (where θ is a parameter vector that typically includes the parameter ϕ [where $1/\phi$ is called the range]; it controls the range of the influence of spatial dependence, and the parameter σ^2 represents the variance of the spatial process and referred to as the partial sill). Finally, $\varepsilon(s)$ is an uncorrelated pure error term.

For sample obtained at points $s_1, ..., s_n$, with $y(s_i)$ and $x(s_i)$ denoting the dependent variable and explanatory variables observed at location s_i , we obtain $w = (w(s_1), w(s_2), ..., w(s_n))'$ from the multivariate Gaussian distribution: $w \sim N(\theta, C(\theta))$. Here, θ is an $n \times 1$ vector of zeros, and the element of the $n \times n$ matrix $C(\theta)$ is expressed as $C(s_i, s_j | \theta)$ (i = 1, ..., n; j = 1, ..., n). Further, $y \sim N(X\beta, \Lambda(\tau^2, \theta))$ can be expressed as the spatial process model, where $\Lambda(\tau^2, \theta) = C(\theta) + \tau^2 I$, where I is an $n \times n$ identity matrix.

The prediction of the response $y(s_0)$ at a particular point s_0 is called kriging. For the kriging predictor, the inverse of the $n \times n$ variance–covariance matrix Λ is required. Thus, a cost of $O(n^3)$ is accrued for the computation, which on the order of $n = 10^5$ with a standard personal computer environment poses a challenge. Hence, various approaches are available for approximating the spatial process $w(s)^{18,19}$. This study employs the NNGP model¹⁴, which was originally proposed by Vecchia (1988)³⁹⁾. The joint density of the spatial process w (the full GP) is expressed as the product of conditional densities p(w) = $p(w(s_1)) \prod_{i=2}^{n} p(w(s_i)|w(s_1), ..., w(s_{i-1}))$. There after, Datta et al. (2016)¹⁴⁾ assumed the following approximation for this joint density:

$$\tilde{p}(\boldsymbol{w}) = p(\boldsymbol{w}(\boldsymbol{s}_1)) \prod_{i=2}^{n} p(\boldsymbol{w}(\boldsymbol{s}_i) | \boldsymbol{w}(N(\boldsymbol{s}_i))), \quad (2)$$

where $N(s_i)$ is a neighbor set of s_i and serving as the *k*-nearest neighbors of s_i in NNGP. Thus, the complete GP is approximated by the NNGP and can be expressed as a joint density using the nearest neighbors. Further, Datta et al. (2016)¹⁴⁾ demonstrated that the approximation of Eq. (2) leads to an approximation of the precision matrix C^{-1} to \tilde{C}^{-1} , as expressed as follows:

$$\widetilde{\boldsymbol{C}}^{-1} = \left(\boldsymbol{I} - \widetilde{\boldsymbol{A}}\right)' \boldsymbol{D}^{-1} (\boldsymbol{I} - \widetilde{\boldsymbol{A}}), \tag{3}$$

where \tilde{A} is a sparse and strictly lower triangular matrix, with its diagonal elements represented by zeros,

with non-zero entries at most *k*-entries in each row. Further, $D = \text{diag}(d_{ii})$ is a diagonal matrix whose elements are conditional variances based on the full GP model. Further, as \tilde{A} can be provided as a $k \times k$ (k << n) matrix, and \tilde{C}^{-1} is sparse, significant reduction in the computational load can be achieved. The spatial process model provided through NNGP can be expressed as follows:

$$\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \widetilde{\boldsymbol{\Lambda}}(\tau^2, \boldsymbol{\theta})), \qquad (4)$$

where $\widetilde{\boldsymbol{\Lambda}}(\tau^2, \boldsymbol{\theta}) = \widetilde{\boldsymbol{C}}(\boldsymbol{\theta}) + \tau^2 \boldsymbol{I}.$

The Bayesian Markov chain Monte Carlo (MCMC)¹⁴⁾, Hamiltonian Monte Carlo⁴⁰⁾, and maximum likelihood methods⁴¹⁾ can be employed to estimate the parameters of the NNGP model. This study employed the MCMC. and as the NNGP parameters are $\boldsymbol{\beta}$ and $\boldsymbol{\phi} = (\tau^2, \sigma^2, \phi)' = (\tau^2, \boldsymbol{\theta})'$, a prior distribution must be set for each parameter and subsequently multiplied by the likelihood function to obtain the conditional posterior distributions (the full Bayesian NNGP). However, because this study employs massive data to a maximum order of $n = 10^6$, implementing the full Bayesian NNGP within a practical computational time is a challenge. Therefore, this study employs the conjugate NNGP, which was proposed by Finley et al. $(2017)^{15}$. Assume $\tilde{P}(\phi)$ is the approximate nearest neighbor of a spatial correlation matrix corresponding to an approximate nearest neighbor of $\tilde{\boldsymbol{C}}(\boldsymbol{\theta})$. Then, the conjugate NNGP can then be expressed as:

$$\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \widetilde{\boldsymbol{M}}), \tag{5}$$

where $\tilde{\boldsymbol{M}} = \tilde{\boldsymbol{P}}(\phi) + \alpha \boldsymbol{I}$ and $\alpha = \tau^2/\sigma^2$. The reason for employing the conjugate NNGP because, assuming that α and ϕ are known, the conjugate normal-inverse Gamma posterior distribution for $\boldsymbol{\beta}$ and σ^2 can be used. Further, it enables obtaining the predictive distribution for $y(\boldsymbol{s}_0)$ as a *t*-distribution. Thus, performing MCMC sampling is simple. Section 4 explains the setting of the values of α and ϕ .

(2) ML-based approaches a) RF

RF is a bagging-type ensemble of decision trees that trains several trees in parallel. RF was proposed by Breiman (2001)⁴²⁾ by combining CART and bagging. In the RF algorithm, decision trees constructed from bootstrap samples are combined to conduct a prediction, where each decision tree is trained *independently*. The training procedure can be described as follows: (1) Bootstrap samples are drawn as a randomized subset from the training data, and (2) a decision tree is constructed for every sample, using a randomized subset of predictor variables. This variable selection step helps balance low tree correlation with reasonable predictive strength. (3) Aggregation

(i.e., averaging) was performed for each predicted result. Note that there are certain hyperparameters in RF that need to be calibrated, which will be explained in Subsection 4.3.

b) XGBoost

XGBoost is an efficient and scalable approach based on GB developed by Friedman et al. $(2000)^{43}$ and Friedman (2001)⁴⁴⁾. To generate the final prediction results, GB uses decision trees as weak learners in a sequential learning process, in the form of an ensemble of weak predictions such as decision trees. It has three main components: (1) a loss function to be optimized, (2) a weak learner to predict, and (3) an additive model to add weak learners to optimize the loss function. Chen et al. (2016)⁴⁵⁾ improved the algorithm by adding a regularization term to reduce overtraining (overfitting) and called it XGBoost. This improved algorithm significantly reduces processing time; however, compared to RF, XGBoost has more hyperparameters that need to be calibrated. Again, this is explained in Subsection 4.3.

c) DNN

DNN is a mathematical model with a network structure wherein layered units are connected to neighboring layers. Each element that comprises a network is referred to as a unit or node. The first layer is the input layer, while the last is the output layer. The remaining layers referred to as hidden layers. Further, the indices for layers are expressed as l = 1, ..., L, with the first layer being the input layer and the Lth the output layer. In a DNN, the previous layer transmits the results of the non-linear transformations on the received inputs to the next layer, which enables the outputs at the output layer to be derived as an estimation result. Thus, an observation was conducted in each layer, via linear transformations using a weight matrix W_{l+1} ($m_l \times m_{(l+1)}$) and non-linear transformations using an activation function f(.). The transformation from the *l*th layer output z_l ($m_l \times 1$) to the (l + 1)th layer output z_{l+1} $(m_{(l+1)} \times 1)$ can be performed using the following equations:

$$\boldsymbol{u}_{l+1} = \boldsymbol{W}_{l+1} \, \boldsymbol{z}_l + \boldsymbol{b}_{l+1}, \tag{6}$$

$$z_{l+1} = f(u_{l+1}),$$
 (7)

where \boldsymbol{b}_{l+1} is the $m_l \times 1$ bias vector and $\boldsymbol{f}(\boldsymbol{u}_{l+1})$ is the activation function vector. The final output is denoted by $(z_L \equiv \hat{y})$. When determining \boldsymbol{W}_{l+1} and \boldsymbol{b}_{l+1} for a regression (where y is continuous), the mean squared error (MSE) of the actual value y and the predictive value \hat{y} are often used as the loss function g, expressed as:

$$g = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2.$$
 (8)

The process of determining W_{l+1} and b_{l+1} to mini-

mize g is referred to as DNN learning, which is performed using the gradient algorithm, whereas backpropagation is used to calculate the gradient⁴⁶⁾. However, several hyperparameters must be calibrated in the DNN, including the number of layers and units in the hidden layers, learning rate, and batch size. DNN parameter tuning is commonly performed using grid and random searches⁴⁷⁾.

d) ML approaches and spatial dependence

Hengl et al. (2018)⁴⁸⁾ argued that RF is essentially a non-spatial approach to spatial prediction because the sampling locations and the general sampling pattern are ignored during the estimation. In this study, we examined and discussed appropriate methods for introducing or considering spatial dependence in the ML framework. In this experiment, we focused on RF because it has fewer hyperparameters than DNN and XGBoost, which may make it easier to understand the impact of differences in model structure on prediction accuracy. Certain possible approaches are as follows: geographical covariates, spatial autoregressive term, eigenvectors of a distance marix, and other approaches.

Regarding on geographical covariates, as mentioned in Sekulić et al. $(2021)^{49}$, one approach to include a geographic context into RF is to introduce the X and Y coordinates as covariates. We refer to this method as the RF_coordinates. Hengl et al. $(2018)^{38}$ proposed the use of buffer distance maps from observation points as covariates. This relative distance method is similar to RF_coordinates; the difference is that the latter has a small number of covariates, while the former has a large number of covariates. In this study, we focused on the RF_coordinates approach.

Regarding on spatial autoregressive term, certain studies have attempted to introduce spatially dependent RF by employing spatial econometrics⁵⁰⁾. Credit (2021)⁵¹⁾ proposed a method for constructing spatially explicit RF models by including spatially lagged (spatial autoregressive) variables to mirror various spatial econometric specifications. The approach entails the introduction of a spatial autoregressive term for the dependent variables (\mathbf{v}) and explanatory variables (X). Sekulić et al. $(2021)^{49}$ adopted a similar approach, wherein they directly introduced observations at the k nearest locations and the distances from these locations to the prediction location, which they termed random forest spatial interpolation (RF si). Note that Sekulić et al. $(2021)^{49}$ did not use the weighted average; rather, they directly introduced the actual observed values. In this study, we employ Credit's (2021)⁵¹⁾ approach, that is, the introduction of Wy with and without X-Y coordinates (RF_sar and RF_sar_coordinates, respectively), and the RF si approach by Sekulić et al. $(2021)^{49}$.

Regarding on eigenvectors of a distance matrix, considering the results by Murakami and Griffith (2019)⁵²⁾, Moran eigenvectors from *MCM*, where M = I - 11'/n is a centering operator, 1 is a vector of ones, and C is an $n \times n$ spatial weight matrix whose (i, j)th element equals $\exp(-d(i, j)/h)$, can be employed to consider spatial dependence. Here, d(i,j) represents the Euclidean distance between the sample sites i and j, and h is the maximum length of the minimum spanning tree connecting sample sites⁵³⁾. However, the calculated when n exceeds 1,000 the calculated eigenvalues cannot be guaranteed as accurate¹⁷⁾. Eigen decomposition is possible only when n < 10,000 in a standard computing environment. There exist several approximation methods for eigen-decomposition in the ML literature, with a popular approach, called the Nyström extension, being suitable for the Moran eigenvector approximation. Using the results of Murakami and Griffith $(2019)^{52}$, the first *h* approximate eigenpairs can be formulated as follows:

$$E = \begin{bmatrix} \mathbf{C}_{nh} - \mathbf{1} \otimes \left(\frac{\mathbf{1}'_h(\mathbf{C}_h + \mathbf{I}_h)}{h}\right) \end{bmatrix} E_h(\mathbf{A}_h + \mathbf{I}_h), \quad (9)$$
$$\widetilde{\mathbf{A}}_h = \frac{n}{h}(\mathbf{A}_h + \mathbf{I}_h) = \mathbf{I}_h \quad (10)$$

$$\widetilde{\boldsymbol{\Lambda}}_{h} = \frac{n}{h} (\boldsymbol{\Lambda}_{h} + \boldsymbol{I}_{h}) - \boldsymbol{I}_{h}, \qquad (10)$$

where C_h is an $h \times h$ matrix of a spatial weight matrix among h anchor points, defined by k-means centers (geometric centers of the clusters defined using the k-means method). A greater h yields a better approximation, but results in slower computation; that is, the approximation is influenced by the setting of the number of h. Murakami and Griffith $(2019)^{52}$ suggested that h = 200 be set to balance accuracy and computational efficiency. Although the values should be set only after examining the balance between the sample size and number of hours, this study applies their recommendations. In fact, the cases of h = 500and h = 1,000 have also been tested in our empirical comparison later. We found h = 200 to be the optimal setting in terms of predictive accuracy. This method is called as the RF esf and RF esf app with and without approximation, respectively. However, the former is applicable only when $n = 10^4$ or less for *C* constructed is sparse and *MCM* is not sparse.

Finally, regarding on other approaches, Georganos et al. $(2019)^{54}$ proposed a method for the remaining algorithms, referred to as geographical random forests, where, for each location *i*, a local RF is computed, but only *k* number of nearby observations are included. Thus, this results in the calculation of an RF at each training data point, with its own performance, predictive power, and feature importance. Saha et al. $(2020)^{12}$ proposed RF-GLS, an extension of RF for dependent error processes similar to the manner in

which generalized least squares (GLS) fundamentally extends OLS for linear models under dependence. This extension is based on the equivalent representation of local decision making in a regression tree as a global OLS optimization, which is subsequently replaced by a GLS loss, resulting in a GLSstyle regression tree. For spatial settings, RF-GLS coupled with Gaussian process-correlated errors can generate kriging predictions at new locations. However, based on our investigations, these two methods although having potential are not readily applicable to a massive dataset on the order of $n = 10^5$ or higher.

4. EMPIRICAL COMPARISON

(1) Dataset

The LIFULL HOME data was used on this study to obtain rent price predictions. The dataset used is the same as that used by Seya and Shiroi $(2021)^{1}$; however, we repeated the description to maintain consistency. Of 5.33 million properties, 4,588,632 properties (after excluding missing data) were employed as the original data. Further, the original data did not explicitly contain exact property positional coordinates owing to privacy concerns; however, the zip codes were available. Hence, to overcome this issue, the barycentric coordinates for zip codes (X and Y coordinates projected to the UTM54N WGS84 reference system) were employed instead. For cases involving multiple properties sharing the same location (e.g., a different room in the same apartment), small perturbations (random noise) were provided to each positional (X, Y) coordinate within the zip code. The natural logarithm of the monthly rent price (including maintenance fees) in yen is considered the dependent variable, and the explanatory variables used are listed in Tables 1-3. Typical variables were chosen to include descriptors of the location of the condominium (location variables) and the condominium itself (structural variables). "Walk time to nearest (train) station" (m), "Floor-area ratio" (%), and "Use district" (dummies) were employed as the location variables. Meanwhile, "Years built" (month), "Number of rooms" (#), "Direction" (dummies), "Building structure" (dummies), and "Room layout" (dummies) were employed as the structural variables Further, the number of explanatory variables (K) was 43. The descriptive statistics are presented in Tables 1-3.

(2) Experimental design

For the prediction of the 4,588,632 properties, random sampling was conducted at various sizes ($n = 10^4$, 10^5 , and 10^6), and 80 % of these data were utilized as training data for the learning models. The remaining 20% were employed as the test (validation)

Table 1 Descriptive statistics (continuous variables)

Variable	Min	Mov	Madian	Moon	SD	
variable	IVIIII	Iviax	Median	Mean	3D	
Rent price	5250	1250000000	63000	72850	1381803	
(yen)	5250	1250000000	03000	72830	1501075	
Years built	5	1010	220	226	125 6	
(month)	3	1812	228	250	155.0	
Walk time to						
nearest train	1	88000	640	781.5	661.3	
station (m)						
Number of	1	50	1	1 40	0.71	
rooms	1	30	1	1.48	0.71	
Floor-area	50	1000	200	234.1	130.6	
ratio (%)	50					
X (km)	-841	783.1	352.2	181.5	273.3	
Y (km)	2958	5029	3931	3942	195.3	

The "rent price" includes maintenance fees.

 Table 2 List of explanatory variables (discrete variables)

variable	Category			
Direction	North, Northeast, East, Southeast, South, South-			
	west, West, Northwest, Other			
Building	W, B, S, RC, SRC, PC, HPC,			
structure	LS, ALC, RCB, Others			
Room	R, K, SK, DK, SDK, LK, SLK, LDK, SLDK			
layout				
Use	Category I exclusively low residential zone (1			
district	Exc Low), Category II exclusively low residen-			
	tial zone (2 Exc Low), Category I exclusively			
	high-medium residential zone (1 Exc Med), Cate-			
	gory II exclusively high-medium residential zone			
	(2 Exc Med), Category I residential zone (1 Res),			
	Category II residential zone (2 Res), Quasi-resi-			
	dential zone (Quasi-Res), Neighborhood commer-			
	cial zone (Neighborhood Comm), Commercial			
	zone (Commercial), Quasi-Industrial zone (Quasi-			
	Ind), Industrial zone (Industrial), Exclusive indus-			
	trial zone (Exc Ind), Others (Others)			

For building structure: W: Wooden; B: Concrete block; S: Steel frame; RC: Reinforced concrete; SRC: Steel frame reinforced concrete; PC: precast concrete; HPC: Hard precast concrete; LS: Light steel, RCB: Reinforced concrete block

For room layout: The R refers to a room where there is only one room and there is no wall to separate the bedroom from the kitchen. For the others, K: includes a kitchen; D: includes a dining room: L: includes a living room; S: additional storage room. For example, LDK is a Living, Dining, and Kitchen area.

For use district: Category I exclusively low residential zone, Category II exclusively low residential zone, Category I exclusively medium-high residential zone, Category II exclusively mediumhigh residential zone, Category I residential zone, Category II residential zone, Quasi-residential zone, Neighborhood commercial zone, Commercial zone, Quasi-industrial zone, Industrial zone, Exclusively industrial zone

data to assess the predictive accuracy. Thus, the balance of sample sizes for the training and testing data followed three patterns: 8,000 vs. 2,000, 80,000 vs.

Table 3 Descriptive statistics (discrete variables)

Di	rection			Structure	
Category	Count	Share	Category	Count	Share
North	156843	0.0342	W	1024081	0.2232
Northeast	81173	0.0177	В	570	0.0001
East	595252	0.1297	S	844184	0.1840
Southeast	473041	0.1031	RC	1892428	0.4124
South	1749315	0.3812	SRC	190048	0.0414
Southwest	458125	0.0998	PC	11924	0.0026
West	404994	0.0883	HPS	802	0.0002
Northwest	78836	0.0172	LS	559974	0.1220
Others	591053	0.1288	ALC	58373	0.0127
			RCB	597	0.0001
			Others	5651	0.0012
Use district			Room layout		
Category	Count	Share	Category	Count	Share
1 Exc Low	780638	0.1701	R	423815	0.0924
2 Exc Low	25793	0.0056	Κ	1729903	0.3770
1 Exc Med	689879	0.1503	SK	6919	0.0015
2 Exc Med	321441	0.0701	DK	890584	0.1941
1 Res	1030319	0.2245	SDK	5123	0.0011
2 Res	211076	0.0460	LK	516	0.0001
Quasi-Res	59863	0.0130	SLK	138	0.0000
Neighbor-	386531	0.0842	LDK	1505821	0.3282
hood Comm	500551	0.0012	LDI	1000021	0.5202
Commercial	615630	0.1342	SLDK	25813	0.0056
Quasi-Ind	371672	0.0810			
Industrial	83826	0.0183			
Exc Ind	11949	0.0026			
Others	15	0.0000			

20,000, and 800,000 vs. 200,000. Due to the completely random sampling, no containment relations were possible where, for instance, 10^4 samples were contained in 10^5 samples. However, as the data size was sufficiently large, a quite low probability exists for the sample bias to conceal any trends. Thus, this study design (based on complete random sampling instead of conditionalization) would not significantly affect the results. Further, OLS, NNGP, RF, and XGBoost were estimated using R language, whereas the DNN was estimated using Python. However, to use the same random numbers for R and Python, a reticulate package that provides an R interface for Python modules, classes, and functions was used.

Further, for assessing the predictive accuracy, the following error measures were used: the mean absolute error (MAE), root mean squared error (RMSE), and mean absolute percentage error (MAPE). Here, \hat{y}_m and y_m represent the out-of-sample predictive and observed values, respectively, for the *m*th data. However, the first two measures may be affected by outliers for skewed distributions because it is unlikely that the noise will be Gaussian with constant variance. Moreover, calculating the RMSE on a skewed response variable will cause the resulting statistic to

be driven primarily by the observations of the highest magnitude (see descriptive statistics). Thus, all error measures were calculated while keeping y_m log-transformed. However, MAPE for log-transformed variables cannot be interpreted as percentages⁵⁵⁾. Hence, we also calculated the MAPE for the real scale.

$$MAE = \frac{1}{M} \sum_{m=1}^{M} |y_m - \hat{y}_m|, \qquad (11)$$

$$RMSE = \sqrt{\frac{1}{M} \sum_{m=1}^{M} (y_m - \hat{y}_m)^2}, \qquad (12)$$

$$MAPE = \frac{100}{M} \sum_{m=1}^{M} \left| \frac{y_m - \hat{y}_m}{y_m} \right|.$$
(13)

(3) Model setting

In this section, we describe the settings of each model, that is, for OLS, NNGP, RF, XGBoost, and DNN. The descriptions for OLS, NNGP, and DNN are similar to those in Seya and Shiroi (2021)¹⁾, but we repeat them to maintain the consistency of this article.

a) OLS

The variables are presented in Table 1. The rent price was used as the dependent variable. The other variables, except the X and Y coordinates, were used as explanatory variables. Table 4 presents the OLS results for $n = 10^6$. The adjusted R^2 value was found to be 0.5165.

b) NNGP

The full Bayesian NNGP is theoretically sound for estimation and prediction. However, as this study employs massive data, the conjugate NNGP proposed by Finley et al. (2017)¹⁵⁾ was used to aid in the reduction of the computational cost (see Subsection 3.1). The conjugate NNGP allows the acceleration of drawing samples by assuming α and ϕ to be known. Finley et al. (2017)¹⁵⁾ proposed assigning values to α and ϕ using a grid point search algorithm, which is based on the cross-validation (CV) score. However, the while performing a grid-point search for $n = 10^6$ the computational load it quite high. Therefore, an ad-hoc strategy was adopted in this study while assigning values to α and ϕ , as detailed by Seva and Shiroi $(2021)^{1}$. This was realized by using the spConjNNGP function in the spNNGP package of R. The determination of the number of nearest neighbors for consideration is required when employing the NNGP; thus, based on the CV, the number of nearest neighbors was set to 30^{11} .

c) RF

In this subsection, we describe the model setup for RF, estimated using the R package ranger⁵⁶⁾, which allows for the fast implementation of RF on high-

Table 4 Regression analysis results using OLS (in the case of $n = 10^6$)

Variables	Estimate	t-values		
Constant term	$1.08 imes 10^1$	$4.50 imes 10^3$	***	
Years built	-1.15×10^{-3}	-4.42×10^2	***	
Walk time to nearest	4.00 10-5	0.07 101	ماد ماد ماد	
station	-4.88×10^{-5}	-9.87×10^{4}	ጥጥጥ	
Floor-area ratio	$1.30 imes 10^{-3}$	$2.30 imes 10^2$	***	
Number of rooms	$1.49 imes10^{-1}$	$2.57 imes 10^2$	***	
Direction Northeast	$8.09 imes10^{-2}$	$2.80 imes 10^1$	***	
Direction East	-4.45×10^{-3}	-2.32×10^{0}	*	
Direction Southeast	5.40×10^{-3}	2.72×10^{0}	**	
Direction South	-2.33×10^{-2}	-1.29×10^{1}	***	
Direction Southwest	2.46×10^{-3}	1.23×10^{0}		
Direction West	1.94×10^{-3}	9.67×10^{0}	***	
Direction Northwest	7.39×10^{-2}	2.53×10^{1}	***	
Direction Others	-6.85×10^{-2}	-3.53×10^{1}	***	
Structure B	1.88×10^{-1}	6.39×10^{0}	***	
Structure S	9.41×10^{-2}	9.39×10^{1}	***	
Structure_BC	2.40×10^{-1}	2.37×10^{2}	***	
Structure_RC	2.40×10^{-1}	2.71×10^{2}	***	
Structure_BC	3.07×10^{-1}	2.00×10^{-10}	***	
Structure_FC	2.14×10^{-2}	3.48×10^{-100}	***	
Structure_HFC	9.13×10^{-2}	4.19×10^{1}	***	
Structure_LS	3.34×10^{-2}	4.73×10^{10}	***	
Structure_ALC	9.17×10 1.20 × 10 ⁻¹	5.19×10^{0}	***	
Structure_RCB	1.20×10^{-1}	$4.55 \times 10^{\circ}$	***	
Structure_Others	1.61×10^{-2}	1.81×10^{-1}	***	
Room layout_K	4.22×10^{-2}	3.62×10^{11}	***	
Room layout_SK	1.10×10^{-1}	1.39×10^{4}	***	
Room layout_DK	1.37×10^{-1}	1.00×10^{2}	***	
Room layout_SDK	3.65×10^{-1}	3.95×10^{11}	***	
Room layout_LK	2.79×10^{-1}	1.01×10^{1}	***	
Room layout_SLK	3.04×10^{-1}	$5.75 \times 10^{\circ}$	***	
Room layout_LDK	2.76×10^{-1}	2.12×10^{2}	***	
Room layout_SLDK	6.06×10^{-1}	1.39×10^{2}	***	
Use district_2 Exc	$-1.15 imes 10^{-1}$	$-2.72 imes 10^1$	***	
Low				
Use district_1 Exc	-1.52×10^{-1}	-1.22×10^{2}	***	
Med				
Use district_2 Exc	-2.77×10^{-1}	-1.83×10^{2}	***	
Med				
Use district_1 Res	-2.36×10^{-1}	-1.97×10^{2}	***	
Use district_2 Res	-2.48×10^{-1}	-1.38×10^{2}	***	
Use district_ Quasi-	-2.92×10^{-1}	-9.98×10^{1}	***	
Res	2.92 × 10).)0×10		
Use district_ Neigh-	-2.63×10^{-1}	-1.56×10^{2}	***	
borhood Comm	2.03 × 10	1.50 × 10		
Use district_ Com-	4.63×10^{-1}	1.83×10^{2}	***	
mercial	-4.03 × 10	-1.03×10		
Use district_ Quasi-	1.01×10^{-1}	1.26×10^{2}	***	
Ind	-1.91×10	-1.20×10		
Use district_ Indus-	2.46×10^{-1}	0.76×10^{1}	***	
trial	-2.40×10^{-1}	$-9.70 \times 10^{\circ}$		
Use district_Exc Ind	$-3.17 imes10^{-1}$	$-5.13 imes10^1$	***	
Use district_Others	$5.00 imes10^{-1}$	3.57×10^{0}	***	
Adjusted R^2	0.5165			

* significant at 5%; ** significant at 1%; *** significant at 0.1%.

dimensional data. According to Probst et al. $(2019)^{57}$, RF has several hyperparameters that must be set by the user. The number of trees (number of trees in the forest) must be set sufficiently high, and we set it to 500, which is a typical default value. The node size (minimum number of observations in a terminal node) was set to five. This is because it is generally considered to produce good results^{58,59)}, and a small preliminary experiment showed that the prediction results are fairly robust to these settings. Further, we confirmed that for the hyperparameter *mtry* (number of drawn candidate variables in each split), the default setting of K/3, where K is the number of explanatory variables, results in poor performance for some models (RF_si). Therefore, we attempted to optimize the value of mtry using the caret package in R to perform a grid search with fivefold cross-validation in the range [3 to K]. The calibration results of the *mtry* shown in Table 5. For RF sar are and RF_sar_coordinates, we employed the R code (rfsi function) provided by the developer, and the number of nearest neighbors k was also cross-validated in the range [3 to 35].

d) XGBoost

This subsection describes the model setting for XGBoost, which has a wider range of hyperparameters that need to be calibrated compared to RF. We observed that XGBoost performed worse than (welltuned) RF when the hyperparameters were set to the default values of xgboost and better than RF when the hyperparameters were calibrated using CV. The hyperparameters in xgboost include nround, which controls the maximum number of iterations; *max_depth*, which controls the depth of the tree; eta, which controls learning rate; gamma, which controls regularization; *colsample bytree*, which controls the number of features (variables) supplied to a tree: min child weight, which denotes the minimum number of instances required in a child node; and subsam*ple*, which is the number of samples (observations) supplied to a tree. Here, nround must be set to a sufficiently large value, and we set it to 100, considering the computation time. In addition, we set gamma to 0, indicating no regulation. The others, based on preliminary experiments, were selected from the following ranges via a five-fold cross-validated grid search

 Table 5 Calibration results of mtry

mtry	$n = 10^4$	$n = 10^5$	$n = 10^{6}$
RF_esf	7	-	-
RF_esf_app	7	9	13
RF_sar_coordinates	3	3	5
RF_sar	3	3	5
RF_si	29	51	58
RF_coordinates	5	5	7
RF_non_spatial	5	5	7

using the caret package in R: $max_depth = [9, 11, 13]$, eta = [0.1, 0.2], $colsample_bytree = [0.8, 1]$, $min_child_weight = [0.8, 1]$, and subsample = [0.8, 1]. Although these ranges are commonly used, they are suboptimal because the calibrated parameters may provide an outer rather than an inner solution. However, these settings are sufficient to draw the conclusion of interest in this study: XGBoost has a high performance.

e) DNN

Several hyperparameters must be determined for DNNs. This study adopted an efficient optimization method called the tree-structured Parzen estimator (TPE)⁶⁰⁾, which can appropriately handle the parameter space of the DNN tree structure, which has been extensively adopted, with its performance being proven to a certain extent^{60,61}. The traditional sigmoid, hyperbolic tangent, softmax, and recently popularized rectified linear unit (ReLU) are a few of the typical activation functions. ReLU offers a computational advantage compared to the others as it induces sparsity in the hidden units⁶²; furthermore, it accelerates convergence owing to the non-saturation of its gradient⁶³⁾. Thus, this study adopted ReLU. For the optimizer of the DNN, the results obtained using the typical algorithms, RMSprop⁶⁴⁾, and adaptive moment estimation (Adam)⁶⁵⁾ have been presented. However, techniques designed to prevent overtraining, such as the introduction of regularized terms and dropouts, have not been employed in this study.

Based on the method described by Seya and Shiroi $(2021)^{1}$, the learning procedures were performed as follows. First, considering the *t*th hyperparameter candidate vectors δ_t coupled with the results of applying five-fold CV with training data for each δ_t (MSE, Eq. 8), a 50-fold search was performed using the TPE. Second, using the optimal hyperparameter vector and all the training data to assess the predictive accuracy of the testing data a model was created once again. Moreover, the explanatory variables were standardized in advance. We employed Keras for the development of a DNN, while Optuna was used for the implementation of TPE using Python.

(4) Prediction results and discussion a) Regression-based versus ML-based models

The predictive accuracy based on the sample size for each model is illustrated in Figs. 1 (MAE, RMSE, and MAPE for log-scale) and 2 (MAPE for realscale). For RF, the results for RF_non_spatial and RF_coordinates are shown here. No evident differences were observed in the predictive accuracy of OLD, even if the sample size was increased because OLS does not use local spatial information and thus has a simple model structure such that $n = 10^4$ was sufficiently large to determine the parameters. Moreover, in the case of the real scale, increasing the sample size resulted in the predictive accuracy reducing further. This is because of the increase in the number of high-priced properties in the test data. Seya and Shiroi $(2021)^{11}$, who focused on OLS, DNN, and NNGP, showed that NNGP performed the best when considering the three models for all sample sizes and error measures. They concluded that, for rent price prediction models using standard explanatory variables, kriging (NNGP) is useful, provided the sample size is moderate ($n = 10^4$, 10^5), whereas DNN may be

However, Figs. 1 and 2 show another story. ML models—RF_coordinates and XGBoost—performed considerably better than NNGP, particularly when the sample size was large ($n = 10^6$). In fact, XGBoost achieved the highest prediction accuracy for all sample sizes and error measures for both logarithmic and real scales and for all price bands (when $n = 10^5$, 10^6) (Fig. 3). According to Fig. 1, the MAE of XGBoost is less than half that of NNGP when $n = 10^6$. These results show that although regression-based approaches have merit in terms of *attribution*, for pure prediction purposes, ML approaches, specifically XGBoost, have an advantage.

promising if a sufficient sample size is secured (10^6) .

b) Differences by method for considering the spatial dependence

The predictive accuracy by sample size for each method considering spatial dependence is shown in Fig. 4 (log-scale). It is evident that RF_coordinates performed the best (or at least the second-best) for all sample sizes and error measures. For all methods, except RF_esf_app, considering spatial dependence improved the predictive accuracy. RF_si outperformed the RF sar. This implies that the weighted average need not be considered when introducing observations at neighboring sites. In fact, the introduction of Wy worsens the predictive accuracy for higher- and lower-end markets (Fig. 5). RF_esf_app performed poorly, although RF_esf performed better than RF coordinates for $n = 10^4$. These results may have shown that simply adding spatial coordinates to explanatory variables would be a plausible option to consider spatial dependence in RF.



Fig.1 Prediction results by sample size for each model (log-scale): (a) MAE, (b) RMSE, and (c) MAPE



Fig.2 Prediction results by sample size for each model (real-scale)



Fig.3 MAPE per log rent range (in the case of $n = 10^6$)



Fig.4 Prediction results by sample size for each method of considering spatial dependence (log-scale): (a) MAE, (b) RMSE, and (c) MAPE



Fig.5 MAPE per log rent range for each method of considering spatial dependence (in the case of $n = 10^6$)

5. CONCLUSION

The limitations of existing studies, which have the predictive accuracy of real estate sales and rent prices between regression- and ML-based approaches, are the use of small sample sizes and the disregard for spatial dependence, which is an essential characteristic of real estate properties. This study compared and discussed the rent price prediction accuracy of regression- and ML-based approaches by extending the work of Seya and Shiroi (2021)¹⁾ and employing varying sample sizes in a varying order ($n = 10^4$, 10^5 , and 10^6).

For the regression-based approach, the NNGP model, which enables the application of kriging to large data, was employed. Meanwhile, for the ML-based approaches, certain representative models, namely, XGBoost, RF, and DNN, were employed. To achieve empirical validation, the LIFULL HOME dataset for apartment rent prices in Japan was used in this study. The dataset includes the following variables: rent, lot size, location (municipality, zip code, nearest station, and walk time to the nearest station), year built, room layout, and building structure. Further, the out-of-sample predictive accuracies of the models were compared.

Although Seya and Shiroi (2021)¹⁾ found that NNGP outperformed DNN, particularly when rent prices were around the median, our comparison revealed another story. Our analysis results showed that, with an increase in sample size, the out-of-sample predictive accuracy of XGBoost and RF was higher than that of NNGP. In fact, the performance of XGBoost was the best for all sample sizes. Thus, the results suggest that, although regression-based approaches have merit in terms of *attribution*, ML approaches, specifically XGBoost, have an advantage for pure prediction purposes. We also compared several methods to consider the spatial dependence with RF and found that simply adding spatial coordinates to explanatory variables can be a plausible option.

In future work, it may be important to establish an effective means to set NNGP hyperparameters. Further, it may also be interesting to use other neural network models, including graph convolutional networks. In addition, it is important to conduct experiments with several explanatory variables by using, for instance, pictures that show the floor plans and interior details of each property. Finally, it may be useful to consider spatial dependence when conducting validation⁶⁶.

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