Dynamic User Equilibrium in Many-to-One Corridor Network with Irregular Ordered Capacity Patterns

Yang LIU[1], Takashi NAGAE[2] and Koki SATSUKAWA[3]

[1]Master Student, Graduate School of Eng., Tohoku University
(6-6-11-816, Aoba, Aramaki, Aoba, Sendai 980-8579, Japan)
E-mail: liu.yang.q5@dc.tohoku.ac.jp
[2]Member of JSCE, Associate Professor, Graduate School of Eng., Tohoku University
(6-6-11-814, Aoba, Aramaki, Aoba, Sendai 980-8579, Japan)
E-mail: nagae@tohoku.ac.jp
[3]Member of JSCE, Assistant Professor, New Industry Creation Hatchery Center, Tohoku University
(Aoba, Aramaki, Aoba, Sendai 980-8579, Japan)
E-mail: koki.satsukawa.e1@tohoku.ac.jp

This article reveals some unstudied solutions patterns in dynamic user equilibrium with route and departure-time choice (DUE-RDC) traffic assignment in many-to-one corridor networks with irregular ordered bottleneck capacities. We develop a critical network obtained from the original network and give some numerical examples and a case-specific theoretical proof of the resemblance between the critical and original network. These enable us to expand the understandings of the DUE-RDC model in many-to-one corridor networks, which could be a building block for analyzing general dynamic assignment problems.

Key Words : *dynamic user equilibrium with route and departure-time choice, many-to-one corridor network, critical network, irregular ordered capacity patterns*

1. INTRODUCTION

Dynamic User Equilibrium with Route and Departure-Time Choice (DUE-RDC) assignments for a large-scale network is difficult to obtain for its nonlinear characteristic, and thus a great deal of research has been conducted to develop an efficient and general solution method for this problem. In the present article, we aim to show different assignment patterns in a corridor network to help better understand the essential aspect of DUE-RDC and find a possible direction for some efficient methods in the future research.

Some of related literatures are in order. Akamatsu et al. (2015) analyzed corridor network with multiple bottlenecks under the Lagrangian-like coordinate formulation and provided rigorous results on the existence and uniqueness of equilibria. Fu et al. (2021) proved DSO with a certain route pass permit price is identical with DUE in corridor networks under certain circumstances regarding to the schedule cost function. Nagae et al. (2020) proposed *generalized linear complementarity problem* (GLCP) and solved the DUE problem with route and departure time choice in Many-to-One (M2O) and One-to-Many (O2M) traffic networks.

The present article reveals that some unresearched DUE-RDC solution patterns could exist for many-to-one corridor networks with irregular order of bottleneck capacities. Our analyses show that some "exclusive" departure time choices could happen and there is connection between original network and reduced critical network when network has irregular capacity patterns. This study could expand the understanding of the DUE-RDC assignment in simple networks for future method development.

The remaining of the present article is organized as follows. In section 2, we describe the network model that is studied in this article and introduce the related notations. Section 3 then illustrates the solutions patterns that are studied in former research and that are found in our research. In section 4, we introduce the critical network to analyze the irregular solution patterns and show some numerical examples. Section 5 concludes.

2. MODEL

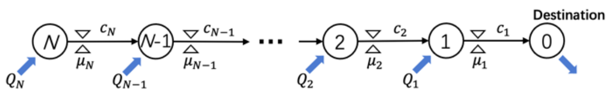

Fig.1 Corridor network with N origins and 1 destination.

Suppose a many-to-one (M2O) corridor network illustrated as Fig.1 with N origins, N links and one destination. Let $i=0$ be the (single) destination and let $\mathcal{N}$ and $\mathcal{L}$ denote the set of origins and links. Q_i $(i=1,2,\ldots,N)$ is the total demand from origin i to the destination in the planning horizon $\mathcal{T}=[T_1,T_2]$. c_i and μ_i $(i=1,2,\ldots,N)$ denote the free flow travel time and capacity of link i $(i\in\mathcal{L})$. $\hat{\rho}_i$ denotes the equilibrium travel disutility of origin node i in DUE-RDC assignment. $\psi(t)$ denotes the schedule cost function that describes the cost of users arrive at the destination at time t, which might be either earlier or later than the desired arrival time t_D.

In a Lagrangian-like coordinate system, this article uses the notations of link flow $y_i(t)$ on link i, departure flow $q_i(t)$ from node i, shortest travel time $\hat{\pi}_i(t)$ from node i in DUE assignment, bottleneck departure time $\hat{\sigma}_i(t)$ from μ_i and queue waiting time $w_i(t)$ at bottleneck μ_i; time t in these notations are denoted as the destination arrival time instead of clock time. As free flow travel time being constant, this article also uses $\pi_i(t)=\hat{\pi}_i(t)-\hat{c}_i(t)$,$\sigma_i(t)=\hat{\sigma}_i(t)-\hat{c}_i(t)$ and $\rho_i(t)=\hat{\rho}_i(t)-\hat{c}_i(t)$ in which

$$\hat{c}_i(t)=\sum\nolimits_{j\le i}c_j(t)$$

to denote the variable factors to exclude the constant values. The relationship between travel time and destination arrival time is illustrated in Fig. 2.

Under this network definition, we have:

$$\hat{\sigma}_i(t)=t-\hat{\pi}_i(t)+w_i(t)=t-\hat{\pi}_{i-1}(t)-c_i \quad (2a)$$

And also, the differential of equation (2a):

$$\begin{aligned}\Delta\hat{\sigma}_i(t)=\Delta\sigma_i(t)&=1-\Delta\pi_i(t)+\Delta w_i(t)\\&=1-\Delta\pi_{i-1}(t)\end{aligned} \quad (2b)$$

In the above framework, the DUE-RDC assignment model is formulated as a *generalized linear complementarity problem* (GLCP) from Eq. (2c) to Eq. (2g). The DUE-RDC conditions consist of the conditions for the optimum departure time choice (2c), the optimum route choice (2d), the queue waiting time dynamics (2e), the link flow reservation (2f), the total flow reservation (2g) and the fist-in-first-out constraint (2h).

$$0\le q_i(t)\perp\{\pi_i(t)+\psi(t)-\rho_i\}\ge 0 \quad (2c)$$

$$0\le y_i(t)\perp\{-\pi_i(t)+\pi_{i-1}(t)+w_i(t)\}\ge 0 \quad (2d)$$

$$0\le w_i(t)\perp\{\mu_i(1-\Delta\pi_{i-1}(t))-y_i(t)\}\ge 0 \quad (2e)$$

$$0\le\pi_i(t)\perp\{y_{i+1}(t)+q_i(t)-y_i(t)\}\ge 0 \quad (2f)$$

$$0\le\rho_i\perp\{\int_{T_1}^{T_2}q_i(t)dt-Q_i\}\ge 0 \quad (2g)$$

$$\Delta\pi_i(t)\ge -1 \quad (2h)$$

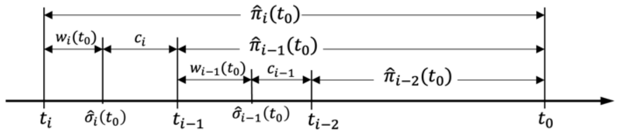

Fig.2 $\hat{\pi}_i(t)$ with respect to destination arrival time.

3. SOLUTION PATTERNS

In this section, we show several relevant solution patterns of the DUE-RDC in a three-link corridor network. Several numerical analyses by using Nagae et al. (2020) revealed that the DUE-RDC solution pattern could be categorized into (i) regular-ordered-DSO-equivalent solution pattern; (ii) regular-ordered-DSO-inequivalent solution pattern; and (iii) irregular-ordered solution patterns.

Fig.3 shows (i) regular-ordered-DSO-equivalent solution pattern, while Fig.4-6 shows the (ii) regular-ordered-DSO-inequivalent solution patterns. The situation in Fig.3 is analyzed by Fu et al. (2021) which is identical to the solution of DSO, when there is no false bottleneck and $\Delta\psi(t)\le\mu_i/\mu_{i+1}-1$ $(\forall i\in\mathcal{L}\backslash\{N\}, t\in\mathcal{T}_i\backslash\mathcal{T}_{i-1})$. $\mathcal{T}_i=[t_i^-,t_i^+]$ is the time period of users from node i arriving at the destination and $\boldsymbol{\tau}_i=(\tau_i^-,\tau_i^+)$ is defined as the time period when $\pi_i(t)>0$.

Situation in Fig.4-6 shows the DSO-inequivalent DUE-RDC solutions. In Fig.4, $t_1^+=t_2^+$, that is, the latest arrival time from node 1 and 2 are identical. This case occurs when

$$\Delta\psi(t)\le\frac{\mu_1}{\mu_2}-1,\qquad t\in[t_D,t_1^+]$$

$$\Delta\psi(t)>\frac{\mu_2}{\mu_3}-1,\qquad t\in[t_1^+,t_2^+].$$

In this case, the second condition makes the decreasing rate of the queue waiting time on link 2 smaller than the increasing rate of the schedule cost, and thus no users from node 2 arrives at the destination after t_1^+.

In Fig.5, $t_1^+=t_D$ and $t_2^+=\tau_1^+$, that is, no user from node 1 arrives after the desired arrival time and the users from node 2 arrive before the queue on link 1 is vanished. This case occurs when

$$\Delta\psi(t)>\frac{\mu_1}{\mu_2}-1,\qquad t\in[t_D,\tau_1^+]$$

$$\Delta\psi(t)\le\frac{\mu_1}{\mu_3}-1,\qquad t\in[t_D,\tau_1^+]$$

$$\Delta\psi(t)>\frac{\mu_2}{\mu_3}-1,\qquad t\in[\tau_1^+,t_3^+].$$

In this case, the first and the third condition make the decreasing rate of the queue waiting time on link 1 and link 2 smaller than the increasing rate of the schedule cost after t_D and τ_1^+, respectively. Thus, no users from node 1and 2 arrives at the destination after t_D and τ_1^+.

In Fig.6, $t_1^+ = t_2^+ = t_D$, that is, whole of the users from node 1 nor node 2 arrive the destination before the desired arrival time. This case occurs when

$$\Delta\psi(t) > \frac{\mu_1}{\mu_2} - 1, \qquad t \in [t_D, \tau_1^+]$$

$$\Delta\psi(t) > \frac{\mu_1}{\mu_3} - 1, \qquad t \in [t_D, \tau_2^+].$$

In this case, both two conditions make the decreasing rate of the queue waiting time on link 1 and link 2 smaller than the increasing rate of the schedule cost after t_D, so no users from node 1 and node 2 arrives at the destination after t_D.

We refer the above cases as the *regular-ordered* (RO) capacity patterns, in which $\mu_1 > \mu_2 > \mu_3$. In this situation, we always have $\mathcal{T}_1 \subset \mathcal{T}_2 \subset \mathcal{T}_3$ and also $\boldsymbol{\tau_1} \subset \boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$. And $\mathcal{T}_i = \mathcal{T}_{i+1}$ $(i, i+1 \in N)$ happens when bottleneck $i+1$ is vanished, which is not discussed in this article.

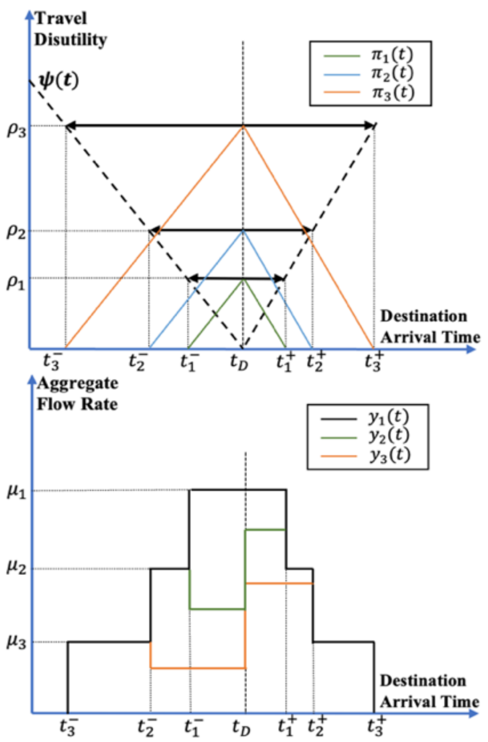

Fig.3 Regular-ordered-DSO-equivalent pattern.

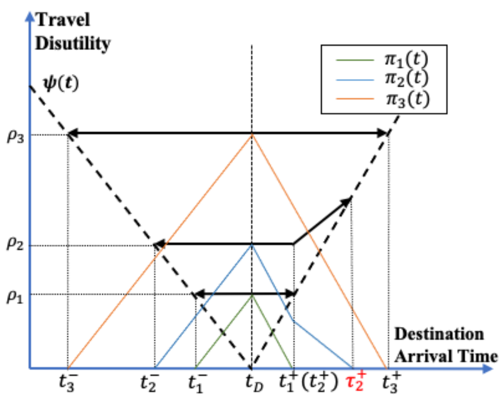

Fig.4 Regular-ordered-DSO-inequivalent patterns case 1.

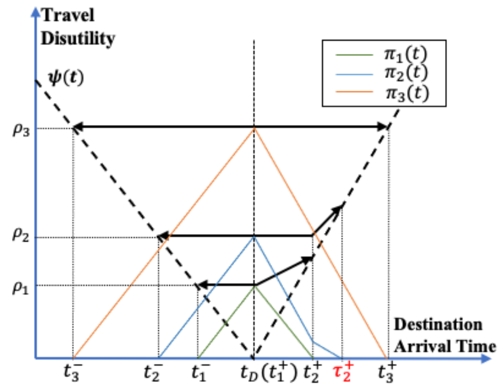

Fig.5 Regular-ordered-DSO-inequivalent patterns case 2.

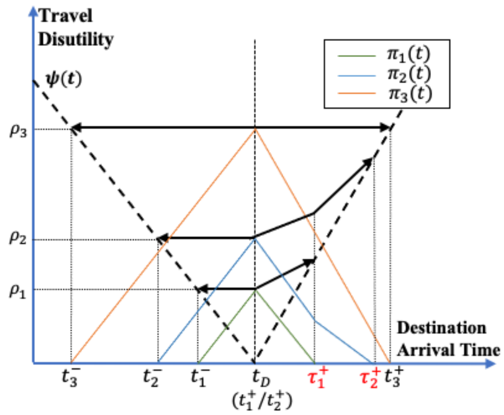

Fig.6 Regular-ordered-DSO-inequivalent patterns case 3.

In the general understanding, the corridor network should be a RO network, which means the traffic link that close to the destination must have larger traffic capacity because of the flow accumulation from upstream, else it would be a false bottleneck with no queue happening at any time point. But with a certain demand pattern, some non-trivial DUE-RDC solution exist in the *irregular ordered* (IrO) capacity patterns where no bottleneck is false.

Fig.7 shows an illustrative example in the IrO capacity patterns, like $\mu_1 < \mu_2 < \mu_3$, in which the arrival time window of node 1 and 2 are "exclusive" each other, i.e., $\mathcal{T}_1 \cap \mathcal{T}_2 = \emptyset$. And also, we can have $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} = \boldsymbol{\tau_3}$ (proof see Appendix A).

Fig.8 shows another IrO case with $\mu_2 > \mu_1 > \mu_3$, where $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$ (proof see Appendix B), and the arrival time windows are not exclusive but the fastest arrival time does not fit to the schedule cost function.

In the flow pattern of Fig.7 and Fig.8, the waiting time on some downstream links grows before there is departure flow on the origin of this link, and it is because the upstream link has greater link capacity and the outflow of the fully occupied $(w_i(t) > 0)$ upstream link exceeds the downstream link capacity.

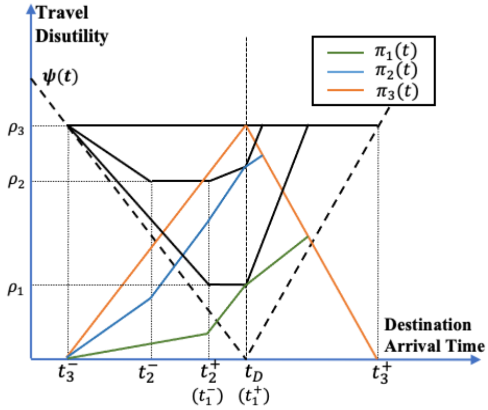

Fig.7 IrO capacity pattern $\mu_1 < \mu_2 < \mu_3$.

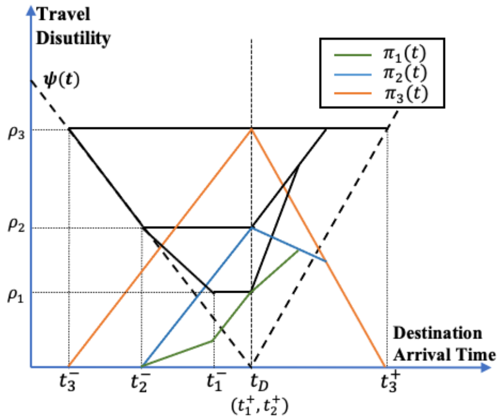

Fig.8 IrO capacity pattern $\mu_2 > \mu_1 > \mu_3$.

4. CRITICAL NETWORK ANALYSES FOR THE CASE WITH IRREGULAR ORDRED CAPACITIES

We also found that the above irregular solution patterns correspond to those of "***critical network,***" which can be regarded as a reduced form of the origin corridor network and consists of only critical bottlenecks in the original network (every bottleneck in the original network can be either critical bottleneck or non-critical bottleneck).

The critical network can be constructed by the following procedure. We first pick up the critical bottlenecks from the original network. A bottleneck i is defined as "critical" if its capacity is smaller than those of the downstream nodes, i.e.:

$$\mu_i = min\{\mu_j \mid j \in \mathcal{L}, j \leq i\}$$

Let $\boldsymbol{r} = (r_1, r_2, \dots, r_k)$ be the list of the critical bottlenecks, and it is obvious that $k \leq N$. We then aggregate the demand from the non-critical node to the critical node via the following rule:

$$\begin{cases} \bar{Q}_{r_i} = \sum_{r_i \leq j < r_{i+1}} Q_j, \forall i < k \ and\ j \in \mathcal{N} \\ \bar{Q}_{r_k} = \sum_{r_k \leq j \leq N} Q_j \qquad , j \in \mathcal{N} \end{cases}$$

It is obvious that (i) the bottleneck capacity is always in regular order in the critical network; and (ii) the critical network of a RO network is identical to the original network.

By using the critical network with k links and k origins with link capacity and traffic demand of μ_{r_i} and $\bar{Q}_{r_i}$ $(i = 1, \dots, k)$, we can "outline" the DUE-RDC solution of the original network.

Fig.9 is an example of critical network obtained from a 5-origin corridor network using the method discussed above.

From the GLCP method of the original network and the critical network, we give a theoretically prove of the relationship between original and critical network with the original capacity pattern of $\mu_1 < \mu_2 < \mu_3$ (see Appendix D).

This article gives some numerical examples to show the relationship mentioned above. Consider a three-link M2O corridor network with bottleneck capacity $(\mu_1, \mu_2, \mu_3) = (60, 75, 90)$, demand $(Q_1, Q_2, Q_3) = (3, 5, 60)$, and early arrival cost (*Ec*) and late arrival cost (*Lc*) being (*Ec, Lc*) = (0.5, 1.2) (Ec and Lc are the coefficients of schedule cost function in a linear situation with $\Delta\psi(t) = -Ec, t < t_D$ and $\Delta\psi(t) = Lc, t > t_D$). We can easily obtain the critical network of this three-link network, which is a one-link corridor network with $\mu_1 = 60$ and $\bar{Q}_1 = 68$. Traffic assignment of original and critical network is illustrated in Fig.10 and Fig.11. From the output we can have:

$$\rho_3 = \bar{\rho}_1 = 24$$
$$y_1(t) = \bar{y}_1(t), \qquad for\ t \in \mathcal{T}$$

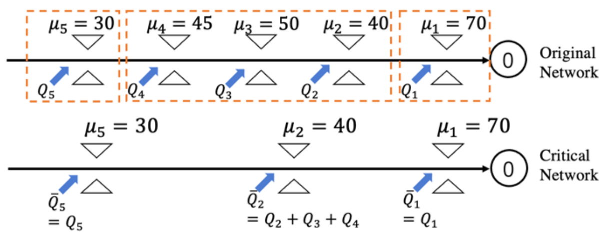

Fig.9 An example of critical network.

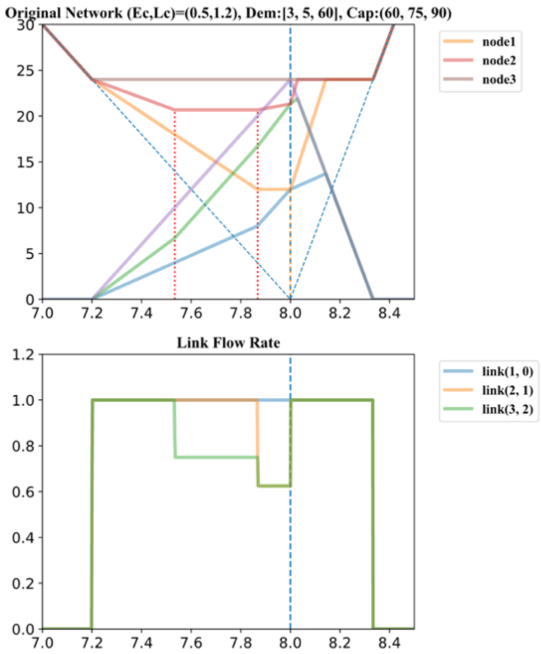

Fig.10 Origin network assignment with $\mu_1 < \mu_2 < \mu_3$

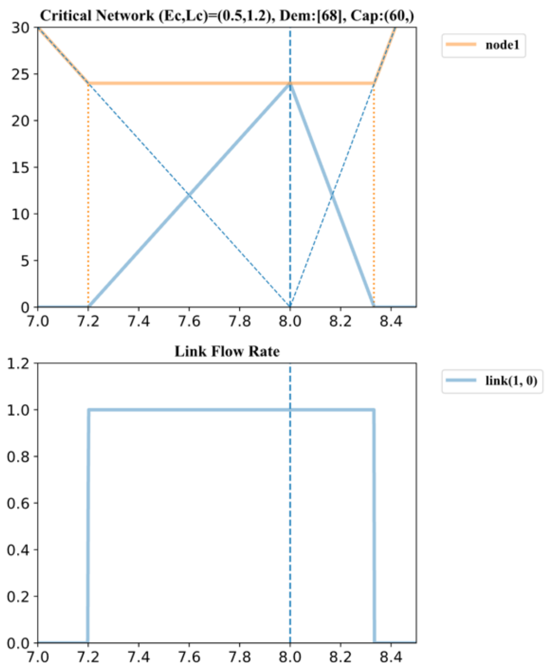

Fig.11 Critical network assignment with μ_1

In the case of a network with link capacity $(\mu_1, \mu_2, \mu_3) = (60, 75, 50)$ which is the case of $\mu_2 > \mu_1 > \mu_3$, and $(Q_1, Q_2, Q_3) = (5, 10, 60)$, the critical network is demonstrated as $(\mu_1, \mu_3) = (60, 50)$ and $(\bar{Q}_1, \bar{Q}_3) = (15, 60)$. The assignment of original network and critical network is illustrated in Fig.12 and Fig.13. And also, the relationship between these two networks is denoted as follow:

$$y_1(t) = \bar{y}_1(t), \quad for\ t \in \mathcal{T}$$

$$y_3(t) = \bar{y}_3(t), \quad for\ t \in \mathcal{T}$$

$$\rho_2 = \bar{\rho}_1 = 12.84$$

$$\rho_3 = \bar{\rho}_3 = 28.26$$

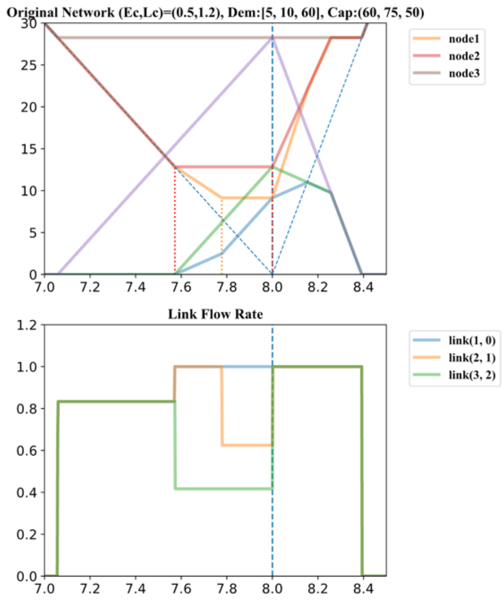

Fig.12 Original network assignment with $\mu_2 > \mu_1 > \mu_3$

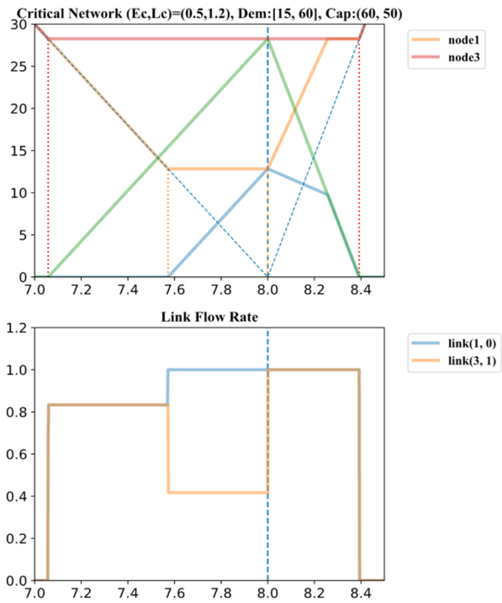

Fig.13 Critical network assignment with $\mu_1 > \mu_3$

The relationship between the origin network and the critical network can be described as the critical network is the main structure of origin network. Every bottleneck in the critical network is the most downstream bottleneck and also has the minimum capacity on the critical link, so that it decides the flow departure rate and the departure time window on this link. Other links and nodes that are reduced in obtaining the critical network can be regarded as sharing the bottleneck capacity in a certain time period. Solving an IrO capacity pattern corridor network can be reduced to solve its critical network instead.

5. CONCLUSION

In this article, we showed several DUE-RDC solution patterns that cannot be explained in analogy of the DSO in the many-to-one corridor network. Especially, we showed that the DUE-RDC solution could have "exclusive" solution pattern in the network with irregular-ordered bottleneck capacities. We also proposed a critical network analyses by reducing the original network. Numerical results shows that the critical network could outline the DUE-RDC solutions of the original network. The detailed analyses in the traffic assignment on the links with non-critical bottlenecks is surely important and necessary though, it is beyond of the scope of the present article.

With the study of the irregular-ordered capacity pattern, we can extend our understanding to the DUE assignment. Future study in generalized networks should pay attention to the irregular-ordered capacity patterns. And also, by applying the critical network, we may be able to simplify our work in finding the efficient DUE solution.

APPENDIX A

Proof of $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} = \boldsymbol{\tau_3}$ when $\mu_1 < \mu_2 < \mu_3$.

In a corridor network defined in section 2, we always have $\pi_i(t) \geq \pi_j(t)$ if $i > j$. So, in the above mentioned three-link network, $\pi_3(t) \geq \pi_2(t) \geq \pi_1(t)$, and $\boldsymbol{\tau_1} \subseteq \boldsymbol{\tau_2} \subseteq \boldsymbol{\tau_3}$ is always true.

Using proof by contradiction, assume $\boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$, so we can find a time point t_α that:

$$\pi_3(t_\alpha) > \pi_2(t_\alpha) = \pi_1(t_\alpha) = 0.$$

Since $w_3(t_\alpha) = \pi_3(t_\alpha) - \pi_2(t_\alpha) > 0$, according to equation (2e):

$$y_3(t_\alpha) = \mu_3\big(1 - \Delta\pi_2(t)\big) = \mu_3.$$

Also, $y_2(t_\alpha) = y_3(t_\alpha) + q_2(t)$ derives $y_2(t_\alpha) \geq y_3(t_\alpha) = \mu_3$.

And since $w_2(t_\alpha) = \pi_2(t_\alpha) - \pi_1(t_\alpha) = 0$, flow on link 2:

$$y_2(t_\alpha) < \mu_2\big(1 - \Delta\pi_1(t)\big) = \mu_2.$$

And because of $\mu_3 > \mu_2$, there is contradiction, so that $\boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$ is false. So, $\boldsymbol{\tau_2} = \boldsymbol{\tau_3}$. In the same method, we can obtain $\boldsymbol{\tau_1} = \boldsymbol{\tau_2}$.

APPENDIX B

Proof of $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$ when $\mu_3 < \mu_1 < \mu_2$.

The proof of $\boldsymbol{\tau_1} = \boldsymbol{\tau_2}$ is similar to Appendix A. Here we use proof by contradiction to prove $\boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$.

Assume $\boldsymbol{\tau_2} = \boldsymbol{\tau_3}$, because of $\boldsymbol{\tau_2} = \boldsymbol{\tau_1}$, we have $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} = \boldsymbol{\tau_3}$. Under this assumption, there must be $\pi_3(t) \geq \pi_2(t) \geq \pi_1(t) > 0$ for any $t \in \boldsymbol{\tau_3}$. For there is no bottleneck vanishment in this network, then $\rho_3 > \rho_2 > \rho_1$. So, there must exist a time point $t_\omega \in \boldsymbol{\tau_3}$ that $q_3(t_\omega) > 0$ and $q_2(t_\omega) = q_1(t_\omega) = 0$. And in time t_β, link flow:

$$y_1(t_\omega) = y_2(t_\omega) = y_3(t_\omega) = q_3(t_\omega) \leq \mu_3.$$

And because $\pi_1(t_\omega) = w_1(t_\beta) > 0$, according to equation (2e):

$$y_1(t_\omega) = \mu_1 > \mu_3.$$

There is contradiction, so the assumption is false, $\boldsymbol{\tau_2} \subset \boldsymbol{\tau_3}$.

APPENDIX C

Proof of $q_3(t) > 0$ $(\forall t \in \boldsymbol{\tau_3})$ in a three-link corridor network.

Using proof by contradiction, assume there is time point t_ε that $t_\varepsilon \in \boldsymbol{\tau_3}$ and $q_3(t_\varepsilon) = 0$. Because link 3 has no upstream links, $y_3(t_\varepsilon) = q_3(t_\varepsilon) = 0$, which derives $w_3(t_\varepsilon) = 0$. According to equation (2d):

$$\pi_3(t_\varepsilon) < \pi_2(t_\varepsilon).$$

There is contradiction, so the assumption is false.

APPENDIX D

For a three-link corridor network with $\mu_1 < \mu_2 < \mu_3$ and (Q_1, Q_2, Q_3), its critical network is a one-link corridor with link capacity μ_1 and demand $\bar{Q}_1 = Q_1 + Q_2 + Q_3$. As we obtained in section 3, $\boldsymbol{\tau_1} = \boldsymbol{\tau_2} = \boldsymbol{\tau_3}$. Link flow $y_1(t)$ on original network, according to equation (2e):

$$y_1(t) = \begin{cases} \mu_1, & for\ t \in \boldsymbol{\tau_1} \\ 0, & for\ t \in \mathcal{T} \backslash \boldsymbol{\tau_1} \end{cases}.$$

And also,

$$y_1(t) = q_1(t) + q_2(t) + q_3(t).$$

$\bar{\rho}_1$, $\bar{y}_1(t)$ and $\bar{q}_1(t)$ denote the minimum travel cost, link flow and departure flow in the single link of critical network with respect to the destination arrival time, $\bar{\boldsymbol{\tau}}_1$ denotes the time period minimum travel cost of node 1 is positive in critical network. We can easily have:

$$\bar{y}_1(t) = \begin{cases} \mu_1, & for\ t \in \bar{\boldsymbol{\tau}}_1 \\ 0, & for\ t \in \mathcal{T} \backslash \bar{\boldsymbol{\tau}}_1 \end{cases}.$$

So that $y_1(t) = \bar{y}_1(t)$ $(t \in \mathcal{T})$. And $\bar{y}_1(t) = \bar{q}_1(t)$.With the total flow reservation rule (2.7):

$$\int_{t \in \boldsymbol{\tau_1}} \{q_1(t) + q_2(t) + q_3(t)\}\, dt = Q_1 + Q_2 + Q_3$$

and also,

$$\int_{t \in \bar{\boldsymbol{\tau}}_1} \bar{q}_1(t)\, dt = Q_1 + Q_2 + Q_3.$$

So that we have $\boldsymbol{\tau_1} = \bar{\boldsymbol{\tau}}_\mathbf{1}$. In the last bottleneck of the original network, $q_3(t) > 0$ $(\forall t \in \boldsymbol{\tau_3})$ (proof see Appendix C). Also, $\bar{q}_1(t) > 0$ $(\forall t \in \bar{\boldsymbol{\tau}}_\mathbf{1})$. According to (2.3) we can have $\rho_3 = \bar{\rho}_1$.

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