NON-SORTING SOLUTIONS OF DYNAMIC USER EQUILIBRIUM WITH ROUTE AND DEPARTURE-TIME CHOICE IN ONE-TO-MANY CORRIDOR NETWORK

Joshua Teguh SANTOSO[1] ・ Takeshi NAGAE[2]

[1]Non-member of JSCE, Student Researcher, Dept. of Mechanical and Aerospace Engineering, Tohoku University
(6-6-11-816, Aramaki-Aza Aoba, Aoba-ku, Sendai 980-8579, Japan)
E-mail: joshua.teguh.santoso.r5@dc.tohoku.ac.jp
[2]Member of JSCE, Associate Professor, Dept. of Management Science and Technology, Tohoku University
(6-6-11-814, Aramaki-Aza Aoba, Aoba-ku, Sendai 980-8579, Japan)
E-mail: nagae@tohoku.ac.jp

This article demonstrates that non-sorting dynamic user equilibrium assignment with route and departure-time choice (DUE-RDC) solutions can be generated in one-to-many corridor networks. Such non-sorting solutions can exist regardless of the order of the bottleneck (BN) capacities. The non-sorting solution patterns consist of two types: shifting and separated. Based on our sensitivity analysis, congestion effect is one of the reasons that these solution patterns are generated. It is hoped that from the discovery of non-sorting solution patterns, it could expand our understanding in the DUE-RDC model in one-to-many corridor networks, which is expected to become a building block for general analysis and eventually for the development of a more accurate, robust and efficient solution methods in the future.

Key Words: *dynamic user equilibrium with route and departure-time choice, corridor networks, evening commute, non-sorting solutions*

1. Introduction

Investigation of user equilibrium solution patterns of dynamic traffic assignment is important in order to gain deeper understanding the nature of user-choice can affect the traffic state in a network. A few studies about the solution pattern have been conducted thoroughly, for instance, Nagae et al.[1)] have found that solution patterns could be considerably different in one-to-many (O2M) cases and many-to-one (M2O) cases. Osawa et al.[2)] have also found that the equilibrium solution pattern has sorting regularities which means that, for instance, in O2M case, users who have a farther destination have longer departure time window. However, this present study demonstrates that *dynamic user equilibrium with route and departure-time choice* (DUE-RDC) in O2M corridor network could have some ”non-sorting” solutions.

The DUE-RDC model in corridor networks have been analysed extensively in the previous studies (Akamatsu et al.[3)] and Fu et al.[4)] to name a few) but there were no non-sorting solution patterns discovered. The discovery of such non-sorting solution patterns could expand our understanding in the DUE-RDC model in one-to-many corridor networks, which is expected to become a building block for general analysis and eventually for the development of a more accurate, robust and efficient solution methods in the future.

The purpose of this study is to exhibit DUE-RDC could have 2 types of non-sorting solution patterns: ”shifting” and ”separated”. Usually, in the sorting solution pattern, the departure time window for users with destination to node i (T_i) is fully included in the time window of a destination with farther distance i.e. $T_i \subset T_{i+1}$. In shifting solution pattern, the departure time window shifts so that the time window for each destination is not fully overlapping i.e. $T_i \cap T_{i+1} \neq \emptyset$, whereas in separated solution pattern, the departure time window for each destination is completely separated with one another i.e. $T_i \cup T_{i+1} = \emptyset$.

The present article is organised as follows. Section 2 reviews the formulation of the DUE-RDC model in O2M corridor network, that is originally introduced by Akamatsu et al.[3)]. In Section 3, the two non-sorting solution patterns found in O2M corridor network with regular order (RO) bottleneck (BN) capacities are analysed. In order to understand as to why these solution patterns are generated, a sensitivity analysis of the solution patterns with respect to the total demand is also presented. Subsequently, the solution patterns found in O2M corridor network with non-regular order (NRO) bottleneck (BN) capacities are also analysed in Section 4. Finally, Section 5 will present conclusion of this discovery.

2. Model

In this section, we review the formulation of the DUE-RDC in O2M corridor network. The readers are referred to Akamatsu et al.[3], Nagae et al.[1] and Fu et al.[4] for more details.

(1) Framework

Consider an O2M or evening corridor network with N nodes as shown by **Fig. 1**. We denote a set of nodes as $\mathcal{N}$ with node $i = 0$ as the origin and the rest (i.e. $i = 1, 2, \ldots, N$) as the destination. Each destination i has a total demand denoted by Q_i. Let link ($i-1$,i) to be denoted as link i where each link i has a BN capacity of μ_i and free flow travel time given by c_i. The queueing dynamics at each BN can be described by the standard point queue model along with the First-In-First-Out principle (FIFO), which means that a queue is accumulated vertically when the link arrival flow exceeds the BN capacity.

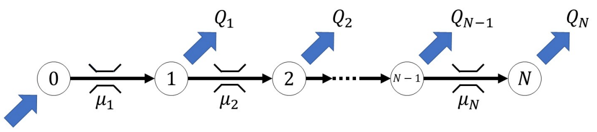


Fig.1: One-to-many(O2M)/Evening commute in a corridor network.

The origin departure time space can be expressed as $\mathcal{T} = [t^-, t^+]$ where t^- is the time space before the desired departure time t_D while t^+ is the time space after t_D. All users in the network have schedule cost $\psi(t)$ with respect to the origin departure time t that can be expressed by the piecewise linear equation:

$$\psi(t) = \begin{cases} \Delta\psi^-(t_D - t), & \forall t \in t^- \quad \text{(1a)} \\ \Delta\psi^+(t - t_D), & \forall t \in t^+ \quad \text{(1b)} \end{cases}$$

where $\Delta\psi^-$ is called as early cost and $\Delta\psi^+$ as late cost.

From the study conducted by Akamatsu et al.[3] and **Fig. 2**, the equilibrium arrival time $\sigma_i(t)$ at destination i which is also the departure time from BN i for users departing from origin at time t can be defined as,

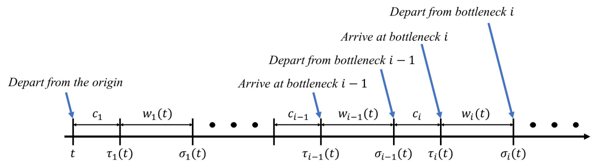


Fig.2: Arrival/departure time at bottlenecks for a user departing at time t from the origin.

$$\sigma_i(t) = t + \sum_{j=1}^{i}(w_j(t) + c_j) = t + \pi_i(t), \qquad \forall\, t \in \mathcal{T},\ \forall\, i \in \mathcal{N} \quad (2)$$

where w_i is the queueing delay at bottleneck i. While the arrival time $\tau_i(t)$ at BN i for users departing from origin at time t satisfies:

$$\tau_i(t) = \sigma_i(t) - w_i(t) = \sigma_{i-1}(t) + c_i \quad (3)$$

The derivative of Equation (2) with respect to the origin departure time t can be obtained as,

$$\Delta\sigma_i(t) = 1 + \Delta\pi_i(t), \quad \forall\, t \in \mathcal{T},\ \forall\, i \in \mathcal{N} \quad (4)$$

Here, $\Delta = \frac{d}{dt}$ denotes the derivative operation with respect to the origin departure time.

We define $y_i(t)$ as the link flow at BN i for users departing the origin at time t which can be expressed as:

$$y_i \leq \mu_i \Delta\sigma_i(t) = \mu_i(1 + \Delta\pi_i(t)) \quad (5)$$

Also, q_i can be defined as the departure flow from the origin at time t with destination to node i.

(2) Equilibrium Conditions

In our DUE-RDC model, equilibrium solution can be obtained when it satisfies the following conditions:

1. Optimum departure time equilibrium condition: This condition assumes that each user with destination node $i \in \mathcal{N}$ choose the departure time from the origin $t \in \mathcal{T}$ that minimises his/her travel disutility which is the sum of the shortest travel time to the destination and the given schedule cost.

$$0 \leq q_i(t) \perp \{\pi_i(t) + \psi(t) - \rho_i\} \geq 0 \quad \text{(6a)}$$

2. Optimum route choice equilibrium condition: This condition assumes that each user departing the origin at time $t \in \mathcal{T}$ to the destination $i \in \mathcal{N}$ chooses the shortest path where the total link travel time for the user is the sum of the given free flow travel time and the queueing delay.

$$0 \leq y_i(t) \perp \{\pi_{i-1}(t) + w_i(t) - \pi_i(t)\} \geq 0 \quad \text{(6b)}$$

3. Link waiting time equilibrium condition:
   This condition represented the dynamics of queueing delay based on FIFO condition which is represented by the link flow and the shortest travel time to the downstream node.

$$0 \le w_i(t) \perp \{\mu_i(1 + \Delta\pi_i(t)) - y_i(t)\} \ge 0 \quad (6c)$$

4. Departure flow conservation law:

$$\int_{t \in \mathcal{T}} q_i(t)dt = Q_i \quad (6d)$$

5. Link flow conservation law:

$$y_i(t) = \begin{cases} y_{i+1}(t) + q_i(t), & i \neq N \\ q_i(t), & i = N \end{cases} \quad (6e)$$

6. FIFO condition:
   This condition means that users that depart the origin at time t can not reach to destination node i before users that depart at time $t' < t$[5]

$$t > t' \quad \rightarrow \quad [t + \pi_i(t)] \ge [t' + \pi_i(t')], \quad \forall(t, t') \in \mathcal{T}$$

   which can be equivalently written as,

$$\Delta\pi_i(t) \ge -1 \quad (6f)$$

3. Solution Patterns

Solving the DUE-RDC with various parameters by using the latest solution method[1], we found that based on our model, DUE-RDC problem could have non-sorting solution in a O2M corridor networks. The non-sorting solutions includes shifting and separated pattern in which it has interesting impact towards the departure flow to the respective destination nodes.

From our observation, it also can be found that the gradient for the shortest travel time to destination node $i \neq N$ can be expressed as:

$$\Delta\pi_i(t) = \frac{\mu_{i+1}}{\mu_i}(1 + \Delta\pi_{i+1}(t)) - 1$$
$$\text{if } \Delta\pi_{i+1}(t) > \frac{\mu_i}{\mu_{i+1}} - 1, \quad (7a)$$

or

$$\Delta\pi_i(t) = -\Delta\psi(t)$$
$$\text{if } \Delta\pi_{i+1}(t) \le \frac{\mu_i}{\mu_{i+1}} - 1, \quad (7b)$$

As for destination node $i = N$ which is the farthest destination node, the gradient for the shortest travel time will always be equal to $-\Delta\psi(t)$.

(1) Sorting

Sorting solution pattern is the DUE-RDC solution pattern that is assumed to be the only solution pattern for a DUE-RDC solution that has achieved equilibrium[2]. In the sorting solution pattern, the departure time window for users whose destination is to node i is always smaller than for the users whose destination is farther than node i i.e. $i+1$ or $i+2$. Also, the departure time window for destination node i is included in the departure time window for destination node $i + 1$.

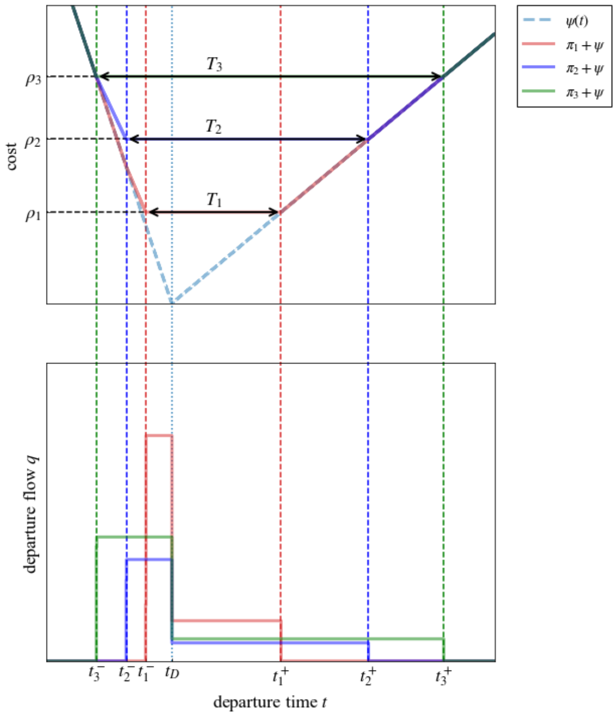


Fig.3: Sorting solution pattern.

Fig. 3 shows the sorting solution pattern in a 3-link O2M corridor network. The top figure depicts the users' cost, $\pi_i(t) + \psi(t)$, with respect to the origin departure time t. The red, blue and green lines describe the $\pi_i(t) + \psi(t)$ for $i = 1, 2$ and 3, respectively, while the light blue dashed line indicates the schedule cost $\psi(t)$. ρ_1, ρ_2 and ρ_3 are the equilibrium cost of users for the corresponding destination. The bottom figure of **Fig. 3** shows the corresponding departure flow, where the red, blue and green solid lines indicate $q_i(t)$ for $i = 1, 2$ and 3, respectively. The departure time window for destination i, i.e. $T_i = t : q_i(t) > 0$ is described as the red, blue and green dashed vertical lines while the light blue dotted vertical line describe the desired departure time t_D. It can be observed that the users' cost $\pi_i(t) + \psi(t)$ is "flat" on the corresponding departure time window, T_i. In the sorting solution pattern, $T_i \subset T_{i+1}$.

(2) Shifting

Shifting solution pattern is one of the newly discovered non-sorting DUE-RDC solution pattern. As the name implies, the departure time window for a particular node except for node $i = N$ shifts whether to early departure time window or to late departure time window. The departure time window for destination node i overlaps with the departure time window for node $i+1$ but it is not fully included as shown in sorting solution pattern.

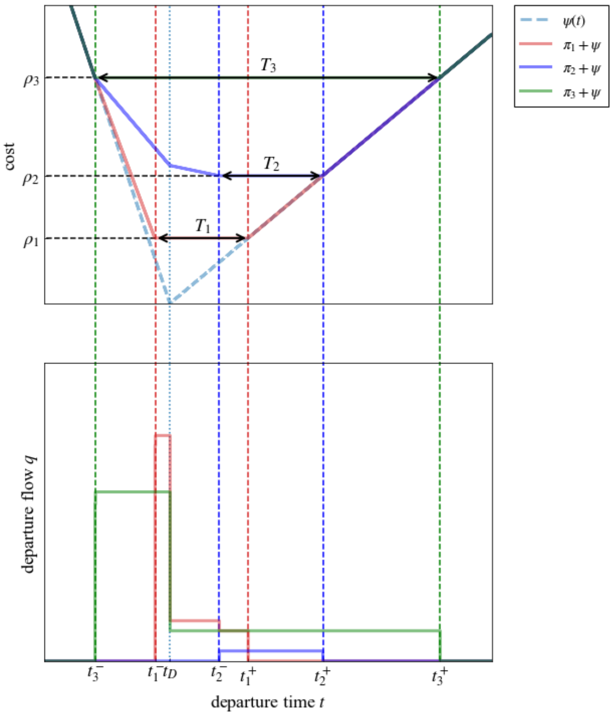


Fig.4: Shifting solution pattern.

Fig. 4 shows the shifting solution pattern in a 3-link O2M corridor network, where the T_1 is not included in T_2 and vice versa, while there is still intersection between T_1 and T_2, i.e. $T_1 \cap T_2 \neq \emptyset$. Similar to **Fig. 3**, the top figure of **Fig. 4** depicts the users' cost, $\pi_i(t) + \psi(t)$, with respect to the origin departure time t whereas the bottom figure of **Fig. 4** shows the corresponding departure flow, where the red, blue and green lines indicate $q_i(t)$ for $i = 1, 2$ and 3, respectively.

(3) Separated

Separated solution pattern is another newly discovered non-sorting DUE-RDC solution pattern. Similar to the shifting solution pattern, as the name implies, the departure time window for a particular node except for node $i = N$ is completely separated with the departure time window of another destination node whether it is the previous or the following node.

Fig. 5 shows the separated solution pattern where the departure time durations T_1 and T_2 are completely separated i.e. $T_1 \cup T_2 = \emptyset$. Similar to **Figs.3** and **4**, the top figure of **Fig. 5** depicts the users' cost, $\pi_i(t) + \psi(t)$, with respect to the origin departure time t whereas the bottom figure of **Fig. 5** shows the corresponding departure flow, where the red, blue and green solid lines indicate $q_i(t)$ for $i = 1, 2$ and 3, respectively. It can

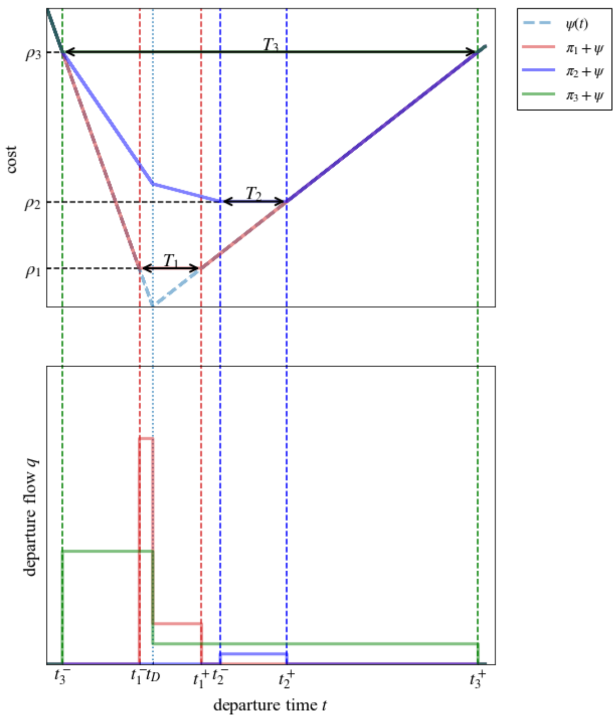


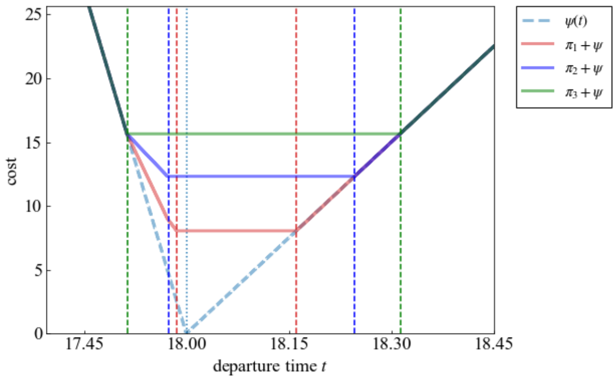
Fig.5: Separated solution pattern.

be seen from the bottom figure of **Fig. 5**, the impact of the this separation is that there is a time window where there is no departure flow for all destination nodes except $i = N$.

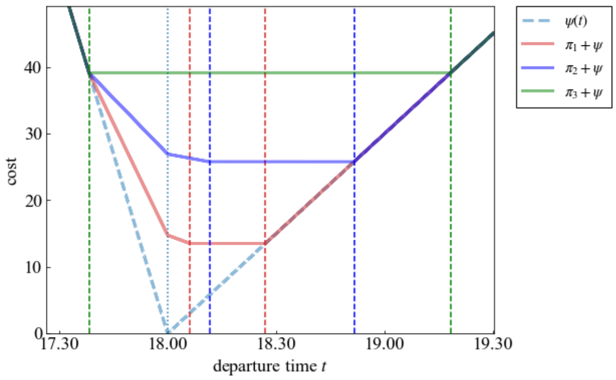
(4) Sensitivity analysis of the solution patterns with respect to total demand

From the discovery of the non-sorting solution patterns, the next step is to find the conditions and to understand how these solution patterns are generated. Based on our sensitivity analysis of the solution patterns with respect to the total demand, we found that congestion effect due to increase in demand can be one of the reasons that these solution patterns are generated. Numerical examples in a corridor network with capacities of $\mu_i = (100, 75, 60)$ and schedule cost of $(\Delta\psi^-, \Delta\psi^+) = (1.8, 0.5)$ with different Q_3 could demonstrate that the solution patterns is changed due to increasing demand.

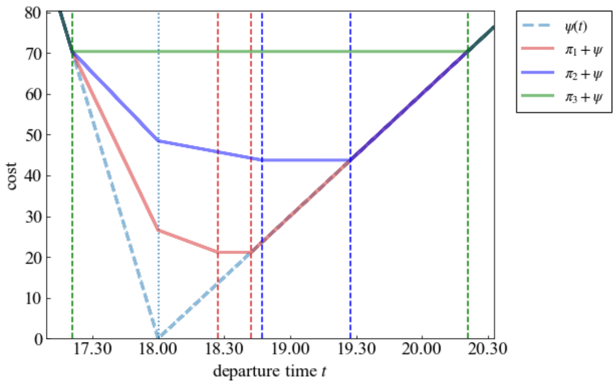
Fig. 6 shows the numerical examples of the sensitivity analysis of the solution patterns with respect to total demand in a 3-link O2M corridor network. **Figs.6a, 6b** and **6c** depict the users' cost, $\pi_i(t) + \psi(t)$, with respect to the origin departure time t for node $i = 1, 2, 3$ with $Q_3 = 40, 100$ and 180, respectively. It can be seen that the solution pattern changed from sorting pattern to shifting pattern and finally to separated pattern as the demand increases. We know that as the demand increases with a fixed BN capacities, it will increases the queue in each bottleneck which



(a) $Q = (5, 5, 40)$



(b) $Q = (5, 5, 100)$



(c) $Q = (5, 5, 180)$

Fig.6: Sensitivity analysis of the solution patterns with respect to total demand. Different Q_3 generate different solution pattern: (a) the sorting ($Q_3 = 40$); (b) the shifting ($Q_3 = 100$), and (c) the separated ($Q_3 = 180$).

increases the travel time for the users. Hence, we can observed vertical shift in the equilibrium cost for each destination nodes. However, as the demand increases, there is also horizontal shifts of the departure time window for every destination node i except $i = N$. In this case, the departure time window shifts to late-departure time space (t^+) until there is no-early departure and become shifting pattern as shown by **Fig. 6b**. As the demand increases more, the departure time shifts further until it becomes separated pattern as shown by **Fig. 6c**. Therefore, it can be said that congestion effect is one of the reasons these solution patterns are generated.

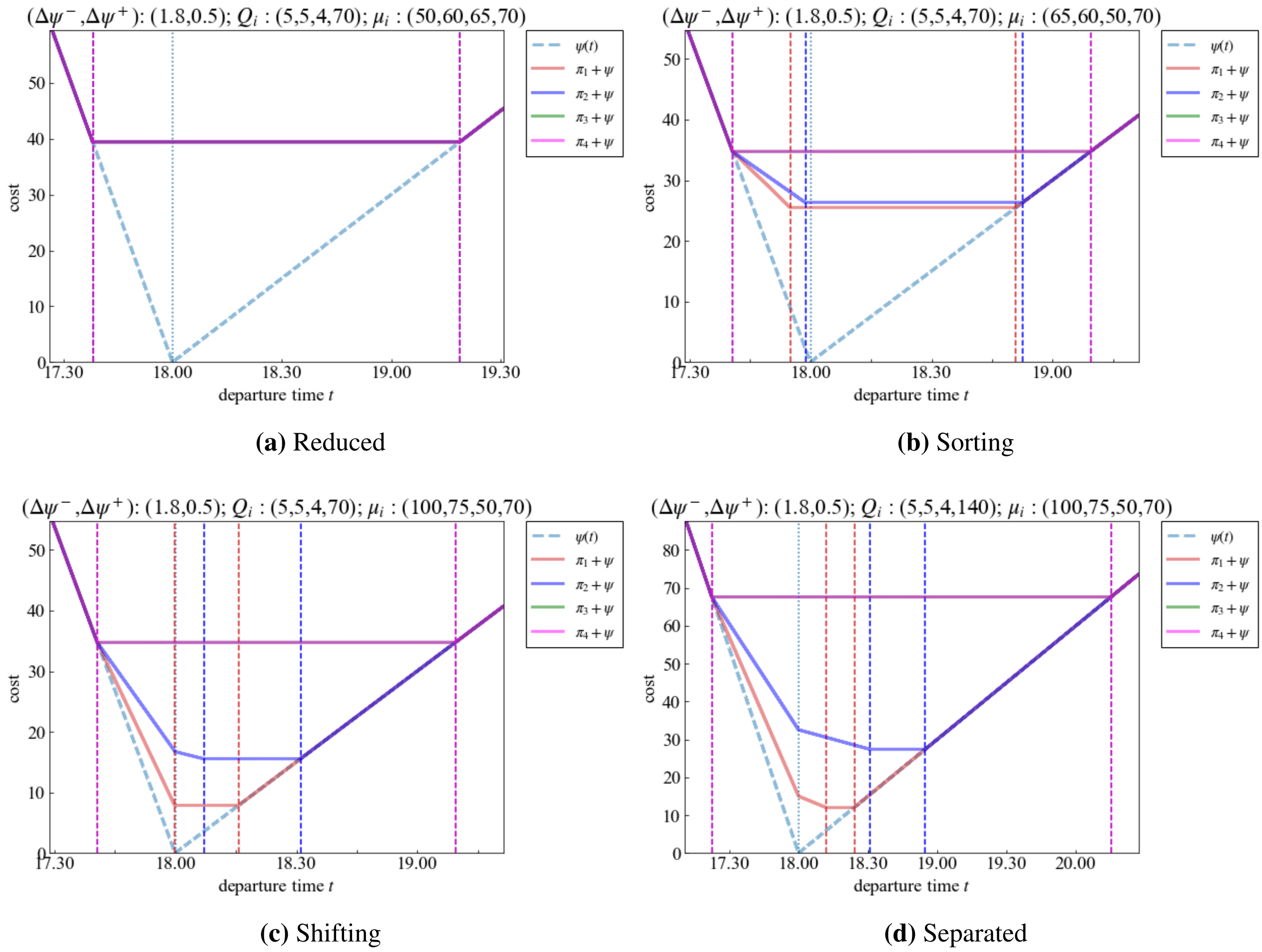
4. Cases with Non-regular Ordered BN Capacity

As for the case in a corridor network with non-regular ordered (NRO) BN capacity which can be expressed as:

$$\mu_i < \mu_{i+1} < \mu_{i+2} < \cdots < \mu_N, \tag{8}$$

both sorting and non-sorting solution pattern can be observed as well. NRO itself can be categorised into two types: fully NRO and partial NRO. Fully NRO means that the order of BN capacity strictly follows the definition shown by Equation (8). On the other hand, partial NRO does not necessarily follows the definition in Equation (8). This means that there exists a RO BN capacity pattern in the midst of NRO BN capacity pattern corridor network or vice versa e.g. $\mu_i - 1 < \mu_i > \mu_{i+1} < \mu_{i+2}$. Numerical examples of DUE-RDC solution patterns in a 4-link O2M corridor network with both fully and partial NRO BN capacity could demonstrate the patterns of the solution that are being generated.

Fig. 7 shows different solution patterns that are generated with various numerical examples of NRO capacity and demand pattern in 4-link corridor network. It depicts the users' cost, $\pi_i(t) + \psi(t)$, with respect to the origin departure time t. In addition of another link, magenta solid line as well as magenta dashed line describe $\pi_i(t) + \psi(t)$ and the departure time window for node $i = 4$, respectively. It can be seen that in addition with all solution patterns i.e. sorting, shifting and separated pattern, "reduced" solution pattern can be generated as well where $\rho_i = \rho_{i+1} = \cdots = \rho_N$. From **Fig. 7a**, in a fully NRO BN capacity, the solution pattern will always be equal to the solution of a reduced network. This is because of since smaller BN capacity located in the upstream node while larger BN capacity is located in the downstream node, this create a "false bottleneck"[4)] as the outflow from the smaller BN capacity will not create a queue in the downstream node with larger BN capacity. Similarly, in partial NRO BN capacity, consecutive bottlenecks that have non-regular order in the network will be reduced. For instance, in **Figs.7c** and **7d**, the original network has a capacity of $\mu_i = (100, 75, 50, 70)$ will become a reduced network with BN capacity of $\mu_i = (100, 75, 50)$.



(a) Reduced (b) Sorting (c) Shifting (d) Separated

Fig.7: Solution patterns in a 4-link O2M corridor network with NRO BN capacities.

5. Conclusion

In the present article, we showed that DUE-RDC in O2M corridor networks could have non-sorting solution patterns, which further can be categorised into two types: shifting and separated. These solution patterns can exist regardless of the order of the BN capacities i.e. regular-order (RO) and non-regular order (NRO). Then, we present our sensitivity analysis of the solution pattern with respect to total demand. Based on the sensitivity analysis, we found that congestion effect could be one of the reasons that these solution patterns are generated. Finding concrete conditions as to why these solution patterns are generated analytically will be our main focus in our research in the future. It is hoped that from the discovery of non-sorting solution patterns, it could expand our understanding in the DUE-RDC model in one-to-many corridor networks, which is expected to become a building block for general analysis and eventually for the development of a more accurate, robust and efficient solution methods in the future.

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(Received March 5, 2021)