# How Should River Embankments Be Spatially Developed?: From the Upstream Section or the Downstream Section

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This paper explores the efficient heights of embankments from two viewpoints: 1) which section of a river should be constructed first, the upstream or downstream section, and 2) how tall the embankments should be constructed depending on the section. The results of numerical simulations reveal that the construction from the upstream section can be efficient if the upstream section has a large population size compared with the downstream section, and show that the constructing higher embankments in the upstream section give negative externalities in the downstream section because it increases the possibility of flood in the downstream section. In addition, this paper shows that even if we construct efficient heights of embankments in both sections, residents in the downstream section is sufficiently large compared to that into the upstream section, embankments with efficient heights bring about Pareto improvements, giving benefits to residents in the downstream section as well as the upstream section. Moreover, we show that the efficient heights of embankments depend on which section the construction starts from.

Key Words : Flood control planning, Cost-benefit analysis Kinematic wave method

## **1. INTRODUCTION**

There have been many torrential rains that we have never experienced before. In order to protect cities from such flood disasters, construction of river embankments are urgently necessary.

It is important for planning the construction of embankments to consider two viewpoints: 1) which section of a river should be constructed first, and 2) how tall the embankments should be depending on the section. For instance, if the population size is large in the upstream section, the construction of embankments from the upstream section is efficient. In addition, the efficient height of embankments depends on river conditions and population distribution conditions.

In this study, we formulate inundation damage using a runoff model and explore the efficient height of embankments and the construction sequence using cost-benefit analysis. Then, we discuss the fairness depending on the conditions with implementing the efficient construction.

## **2. THE MODEL**

In this model, the river basin is divided into two geographical sections: the upstream section (x=1) and the downstream section (x=2). In each section, several tributaries flow into the main river. Simultaneous construction of embankments cannot be done at once in the upstream section and the downstream section. For simplicity, we do not consider embankment collapses.

The flow of the main river is expressed using the kinematic wave method. Using the equation of continuity, the depth of the main river at each time can be solved successively. This can be expressed as,

$$h_x^t = h_x^{t-\Delta t} + r_x^t \Delta t + q_{in_x}^t \frac{\Delta t}{\Delta x} - q_{out_x}^t \frac{\Delta t}{\Delta x}, \qquad (1)$$

where  $h_x^t$  is the depth of the main river in section x,  $r_x^t$  is the amount of rainfall per second (m/s),  $q_{in_x}^t$ 

is the unit-width flow rate into section x,  $q_{out}_x^t$  is the flow rate at section x. Here,  $q_{out}_x^t$ , is derived by using the Manning formula.

Flooding occurs when water depth  $h_x^t$  exceeds the height of the embankment. At each time, the volume that exceeds the embankments height floods the outside of the main river. The total flood water from the main river,  $M_{total_x}$ , can be obtained by adding these volumes during the time when flooding occurs. This volume can be expressed as

$$M_{total_x} = \sum_{t_s}^{t_l} B_x (h_x^t - b_x) \times \Delta x, \qquad (2)$$

where  $M_{total_x}$  is the total flood water in each section,  $t_s$  is the time when flooding starts,  $t_l$  is the time when flooding finishes,  $B_x$  is the width of the main river,  $b_x$  is the height of the embankment, and  $\Delta x$  is the length of the main river.

The flood water from the main river  $M_{total_x}$  and rainfall  $\sum_{t=1}^{\tau} r_x^t \Delta t$  are pooled on the possible flooded area  $A_x$ . (the volume that the sewerage system can drain is subtracted.)

According to this pooled volume, inundation depth  $H_x$  is determined endogenously.

$$H_x = \frac{M_{total_x}}{A_x} + \sum_{t=1}^{\tau} r_x^t \Delta t - \frac{V_{drainage}}{A_x}, \qquad (3)$$

where  $\tau$  is the duration time of rainfall,  $V_{drainage}$  is possible drainage volume per day.

Using Eq. (3), the damage cost by flood from the main river and rainfall on the possible flooded area can be defined. This damage cost can be expressed as

 $D_x(b_x, r_S) = A_{r_x} \times N \times d_x \times p(H_x), \quad (4)$ 

where  $D_x$  is the damage cost, subscript *s* is recurrence interval,  $r_s$  is rainfall of each recurrence interval,  $A_{r_x}$  is building area per household, *N* is the total number of households,  $d_x$  is the ratio of household in each section to all households living along the main river,  $p(H_x)$  is the damage cost per square meter according to the inundation depth.

By calculating the expected value for the damage costs of each recurrence interval, the expected damage cost can be expressed as,

$$E_x = \sum_{s \in S} \frac{1}{s} \times D_{(x,s)}.$$
 (5)

The most efficient construction of embankments is to minimize the total value of the expected damage cost in the whole basin and the construction cost of embankments. Here, we define this total value as the total expected social  $\cos t \Theta$ . Minimizing  $\Theta$  determines the efficient embankment heights and from which section the construction starts. The total expected social  $\cos t \Theta$ . consists of three terms: 1) the expected damage cost when construction in only either one section is completed, 2) the expected damage cost when construction in both sections are completed and 3) the construction cost of embankments according to the heights of embankments.  $\Theta$  is expressed as

$$\min_{b_{x},\delta} \quad \theta = \sum_{T=1}^{T_{c}} (\frac{1}{1+i})^{T} \left\{ \delta \left( \dot{E}_{1} + E_{2(before)} \right) + (1-\delta) \left( E_{1(before)} + \dot{E}_{2} \right) \right\} \\
+ \sum_{T=T_{c}}^{T_{B}} (\frac{1}{1+i})^{T} (E_{1} + E_{2}) \\
+ \delta C(b_{1}) + (1-\delta) C(b_{2}) + \left( \frac{1}{1-i} \right)^{T_{c}} \left\{ (1-\delta) C(b_{1}) + \delta C(b_{2}) \right\}, \quad (6)$$

where  $\delta$  is dummy variable about from which section construction starts (if the construction starts from the upstream section,  $\delta$  is 1. If the construction starts from the downstream section,  $\delta$  is 0),  $\vec{E}_x$  is the expected damage cost when construction at only its own section is completed,  $E_x$  (*before*) is the expected damage cost when construction at only the other section is completed,  $E_x$  is the expected damage cost when construction at both sections are completed,  $C(b_x)$  is the construction cost of embankment according to the height of embankments, *i* is discount rate, *T* expresses years,  $T_c$  is the construction duration per one section, and  $T_B$  is the years considering the benefit by construction of embankments.

#### **3. SIMULATIONS AND RESULTS**

First, we set the parameters that match the Kitakami River to some extent and explore the efficient construction sequence depending on the population distribution. As a result, we obtain Main Finding 1.

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#### Main Finding 1

Construction from the upstream section to the downstream section can be efficient if the ratio of households in the upstream section to the households in the downstream section is larger than a certain value.



Fig. 1. Total expected social cost  $\Theta$  according to which section construction starts

Next, we obtain proposition 1 (Optimal conditions for determining  $b_1$  and  $b_2$ ).

## **Proposition 1**

- (1) Height  $b_1$  must be decided based on  $E_1$  and  $E_2$ , while  $b_2$  is decided based on only  $E_2$ .
- (2) Heights b<sub>1</sub> and b<sub>2</sub> should be determined in consideration of their mutual interdependence because E<sub>2</sub> is functions of both b<sub>1</sub> and b<sub>2</sub>.

Next, we explore the amount of benefits when constructing the efficient heights of embankments with the efficient construction sequence, depending on how many tributaries each section has. As a result, we obtain Main finding 2.

#### **Main Finding 2**

Depending on how many tributaries each section has, the construction of embankments with the efficient heights and the efficient construction sequence result in Pareto improvement in some cases. Whereas in the other cases, residents in the downstream section suffer negative benefits due to the negative externalities caused by the construction in the upstream section. The upstream section always obtains benefits from the construction.

In addition, we explore the relationship between the construction sequence and the height of embankments. The result can be summarized as Main Finding 3.



Fig. 2. Efficient heights of embankments

#### Main Finding 3

The efficient height of embankment can depend on

#### which section the construction starts from.

In particular, when the ratio of the households in the upstream section is large (except 1.0), if the construction starts from the upstream section, higher embankments in the downstream section are efficient. This is because constructed embankments in the upstream section increases the expected cost in the downstream section, and accordingly the downstream section requires higher embankments.

## **4. CONCLUSION**

This paper clarifies, using a Kinematic wave method and cost-benefit analysis, that if the population size is large in the upstream section, the construction of embankments from the upstream section is efficient. In addition, we show how fairness of the improvement depends on the river conditions.

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