Effects of Spatial Distribution of Amenities on Residential Land Use Patterns and Utilities with a Three-generation Model

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Spatial distribution of urban amenities brings spatially different benefits to young, middle-aged, and elderly people because people have different preferences for local amenities among generations. Therefore, the spatial distribution of urban amenities affects residential location patterns. Using an overlapping generations model in a closed city with two zones with different amenity levels, this paper demonstrates how young, middle-aged, and elderly generations reside in the two zones and analyzes the residents' welfare levels. As a result, we show that residential spatial patterns depend on the amenity levels and transportation costs, and clarify the condition in which multiple patterns can occur with the same distribution of amenities. Next, exploring the utilities in the multiple pattern cases, we show that there is a case in which the difference in the lifetime utility is large, and that there is a trade-off relationship among the utilities of generations in most cases.

Key Words : *urban land use, overlapping generations model, urban amenities*

1. INTRODUCTION

Recently, the tendency of young and elderly people to reside downtown has been found in many Japanese cities.[1)] In the USA also, the difference in residing patterns among generations has been reported. Such recent residential patterns have been caused by the spatial distribution of amenities, differences in preference for amenities among generations, and some dynamic change in the preference for amenities.[2,3,4)]

In such a recent trend, there is a circular causality between the 'amenity changes' and 'change in spatial residential pattern'. That is, if amenities preferred by a certain generation is provided in the center, the generation is attracted to the center. That might change the spatial distribution among generations. When some specific generation agglomerates at some locations, providing amenities for the generation there is efficient. In this causality, there are heterogeneity in preference for amenities among generations. Likewise, the spatial distribution of urban amenities is a key factor to determine residential land use with multiple generations.

The allocation of residents in an urban area is an important topic in urban economics for a long time. So, there is vast relevant literature. Among them, a recent study by Tabuchi (2019) shows a novel mechanism in which different income households are ranked according to the distance from the CBD. In addition, he empirically shows that such a collocation is really observed in the Tokyo metropolitan area.[5)] However, in the literature including Tabuchi (2019), there are no papers focusing on the spatial sorting among generations. Shimizu et al. (2014) classifies the amenities into twenty-four categories, and examines how a concentration of these amenities affect housing rents and population concentration in Tokyo by empirical analysis.[6)] But, Shimizu et al. (2014) has not shown the mechanism in the background.

The adoption of an overlapping generations (OLG) model is the most natural way to express the dynamics of young, middle-aged, and elderly generations. However, the combination of an OLG model and a multiple zonal model would be complex. As models to explore urban land use, several papers, construct OLG models.[7)] Duranton (2000) constructs an OLG model with two zone types (urban and suburban zones).[8)] However, in Duranton (2000) model, young people live in urban zones and elderly people live in the suburbs. This perfect segregation is derived from the assumption that the workplaces exist only in urban areas and that no amenities exist other than the workplaces.

An OLG model with two zones having different amenities types is constructed by Kono et al. (2012) to explore how young and elderly generations reside in the zones and to demonstrate the equilibrium utility paths of young and elderly people.[9)] Kono et al. (2012) consider two zones and durable housing stock. However, people do not make trips to the CBD in the model. Furthermore, there are only two generations.

Using an overlapping generations model in a closed city with two zones, this paper demonstrates how young, middle-aged, and elderly generations reside in the two zones and classifies the residential patterns. Results of analyses underscore how residential patterns arise according to the distribution of zone amenities. Furthermore, by exploring the difference in equilibrium utilities among the residential patterns, we show that there is a desirable residential patterns.

2. THE MODEL

The model is an OLG model in a closed city composed of the CBD (zone 0) and two residential zones labelled 1 and 2. The two zones are characterized by their different amenities. The CBD is located on the left and the two zones line up in the numerical order. Each zone is connected to the next one by a bridge. The travel cost from zone 1 to zone 0 is assumed to be zero. Fig. 1 shows the shape of the city. Residents are assumed to migrate freely between the two zones. The cost of migration can be negligible if the cost is spread over a long time. Agents of the model are residents, developers, and absentee landowners. All agents behave with perfect foresight.

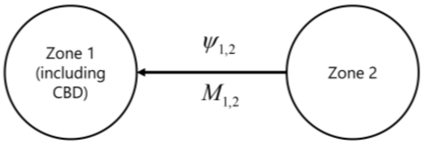

Fig. 1 The city

Residents live for three periods. During the first period, they are young and they work; in the second, they are middle-aged and they work; in the third, they are elderly and retired. The number of people who are young in period t is expressed as $\bar{N}^{yt}$. Everyone lives in three periods. The population is given exogenously to focus on a constant population.

Every resident in zone $i \in \{1,2\}$ consumes some quantity of a composite good c_i, leisure time l_i, and floor area h_i during lifetime. The utility of a person during young in period t is

$$\max_{i,c,l,h} u^{y,t} = V^y\left(c_i^{y,t}, l_i^{y,t}, h_i^{y,t}, e_i^{y}\right) \tag{1}$$

The utility of a person from the middle to the elderly, who is middle in period *t*, is

$$\max_{i,j,c,l,h} u^{mo,t} = V^m\left(c_i^{m,t}, l_i^{m,t}, h_i^{m,t}, e_i^{m}\right) + \frac{1}{1+\rho} V^o\left(c_j^{o,t+1}, l_j^{o,t+1}, h_j^{o,t+1}, e_j^{o}\right) \tag{2}$$

where ρ is the exogenously given discount rate, e_i^y, e_i^m, and e_i^o respectively represent the amount or level of amenities for young, middle-aged and elderly person.

The budget constraint of each generation at period *t* are expressed as

$$w^{y,t} L_i^{y,t} = c_i^{y,t} + r_i^{y,t} h_i^{y,t} + \sum_{k=1}^{i} \Psi_{k-1,k} \ , \tag{3}$$

$$w^{m,t} L_i^{m,t} = c_i^{m,t} + r_i^{m,t} h_i^{m,t} + s^{m,t} + \sum_{k=1}^{i} \Psi_{k-1,k} \ , \tag{4}$$

$$\left(1+\pi\right) s^{m,t} = c_i^{o,t+1} + r_i^{o,t+1} h_i^{o,t+1} + \theta \sum_{k=1}^{i} \Psi_{k-1,k} \ . \tag{5}$$

where w is the exogenous labor income, L_i is working time, r_i is residential floor rent per a unit area and $\sum_{k=1}^{i} \Psi_{k-1,k}$ is the total travel expenses from zone *i* to the CBD. The composite good price is assumed to be constant at 1 as numeraire.

The middle-aged person of period *t* becomes an elderly person of period *t*+1. The income of elderly person is equal to the savings accumulated during the middle period, i.e. $\left(1+\pi\right)s^{m,t}$, where π is the exogenously given interest rate.

We assume about the budget constraint. Young people spend all of their income for consuming goods during the young age. In other words, young people have no savings. Indeed, the median of the financial asset held by the twenties in Japan is only 50,000 yen, and 45.4% of the twenties have no financial assets (Survey of household finances (2018))[1]. So, in young periods, there are no variables connected to middle-aged or elderly. In contrast, the

[1] We can take account of youngers' savings by combining youngers' utility maximization behaviors with the middle-aged and elderly utility maximization behaviors, although it makes the solution complex.

middle-aged savings is connected to the elderly.

Each resident's total time $\bar{T}$ is fixed and assigned among working time L_i^t, leisure time l_i^t, and commuting time $\sum_{k=1}^{i} M_{k-1,k}$. Noting that a elderly person's working time is zero because of retired. The time constraint of young, middle-aged, and elderly people at period t are expressed as

$$\bar{T} = l_i^{y,t} + L_i^{y,t} + \sum_{k=1}^{i} M_{k-1,k} ,$$

$$\bar{T} = l_i^{m,t} + L_i^{m,t} + \sum_{k=1}^{i} M_{k-1,k} ,$$

$$\bar{T} = l_i^{o,t+1} + \theta \sum_{k=1}^{i} M_{k-1,k} . \tag{6}$$

Developers rent land from absentee landowners and construct housing, which might be multi-story buildings in which many residents dwell.
Residential buildings are durable, but not permanent. We assume that the period duration of use is two periods, and the future rent affects the present supply. The revenue from the building constructed at period t is the built floor space Q_i^t at period t times the present value of the rents over the two periods t and t+1. The building cost comprises those for the building materials y_i^t assumed at 1 and land H_i^t. Consequently, the profit from a building constructed at period t is expressed as

$$max\Pi^t = \sum_i^2 \left[\left(r_i^t + \frac{r_i^{t+1}}{1+\pi} \right) Q_i^t - y_i^t - R_i^t H_i^t \right], \tag{7}$$

where the land rent is R_i^t. The floor production function in zone i is represented as

$$Q_i^t = \left(y_i^t\right)^{\eta} \left(H_i^t\right)^{1-\eta}, \eta \in (0,1) \cdot \tag{8}$$

For simplicity, the total available area of each zone is assumed to be 2H. The building is built every period in half the zone area. We set $\eta = 1/2$ and define $\bar{H} \equiv H^{1/2}$.

To concretely solve the equilibria, we specify the utility function as the Cobb-Douglas utility function.

$$V^{y,t} = \delta_1 \left(\alpha^y \ln c_1^{y,t} + \beta^y \ln l_1^{y,t} + \gamma^y \ln h_1^{y,t} + \ln e_1^{\ y} \right) + \delta_2 \left(\alpha^y \ln c_2^{y,t} + \beta^y \ln l_2^{y,t} + \gamma^y \ln h_2^{y,t} + \ln e_2^{\ y} \right),$$

$$V^{m,t} = \delta_1 \left(\alpha^m \ln c_1^{m,t} + \beta^m \ln l_1^{m,t} + \gamma^m \ln h_1^{m,t} + \ln e_1^{\ m} \right) + \delta_2 \left(\alpha^m \ln c_2^{m,t} + \beta^y \ln l_2^{m,t} + \gamma^m \ln h_2^{m,t} + \ln e_2^{\ m} \right),$$

$$V^{o,t+1} = \delta_1 \left(\alpha^o \ln c_1^{o,t+1} + \beta^o \ln l_1^{o,t+1} + \gamma^o \ln h_1^{o,t+1} + \ln e_1^{\ o} \right) + \delta_2 \left(\alpha^o \ln c_2^{o,t+1} + \beta^o \ln l_2^{o,t+1} + \gamma^o \ln h_2^{o,t+1} + \ln e_2^{\ o} \right),$$

$$\alpha^y + \beta^y + \gamma^y = 1, (\alpha^o + \gamma^o)/(1+\rho) + \alpha^m + \beta^m + \gamma^m = 1,$$

$$\delta_1 + \delta_2 = 1 \text{ and } \delta_1 \delta_2 = 0 , \tag{9}$$

where δ_i indicates that the person lives in zone i.

3. RESIDENTIAL PATTERNS

In this section, we explore the residential patterns by theoretical analysis. There are two population patterns: dynamic, and static. Many cities in advanced countries have a nearly static population. Thus, We focus on the static state.

We define the notation of residential patterns as follows: "Y","M", and "O" mean young, middle-aged, and elderly: "/" is the border of the two zones; "Y/" implies that young people live in zone1, and "/O" implies that elderly people live in zone 2. Using the notation, we enumerate and classify residential patterns into three groups in Table 1.

Residential patterns in group I have no generation residing in both zones. As we see residential patterns in group II (Y/Y), young people reside in both zones. Residential patterns in group III (O/O) have elderly generations residing in both zones.

Table 1 Residential patterns which can emerge

Group I	Group II (Y/Y)	Group III (O/O)
MO/Y, Y/MO, YO/M,	YMO/Y, YM/YO,	YMO/O, YO/MO,
M/YO, YM/O, O/YM	YO/YM, Y/YMO	MO/YO, O/YMO

We obtain Proposition 1.

Proposition 1 . *Multiple residential patterns in group I cannot emerge at any distribution of amenities if*

$$\frac{B_1\gamma^y}{D_{2,2}(\gamma^m+\gamma^o)} < \frac{B_1\gamma^y + D_{2,1}\gamma^o}{D_{2,1}\gamma^m} \text{ and }$$

$$\frac{B_1\gamma^y}{D_{2,2}(\gamma^m+\gamma^o)} < \frac{B_1\gamma^y + D_{1,2}\gamma^m}{D_{1,2}\gamma^o} . \tag{10}$$

4. NUMERICAL SIMULATIONS

To compare the equilibrium utility among residential patterns, we calibrate the parameters. We simulate the model by changing the distribution of amenities for each generation. As a result, we obtain Proposition 2.

Proposition 2.

(1) Necessary conditions of each patterns emergence are composed by the ratios of rents.

(2) Residential patterns emergence depends on the distribution of amenities, and the patterns are clarified at certain sets of parameters when the pattern are depicted in Fig. 2.

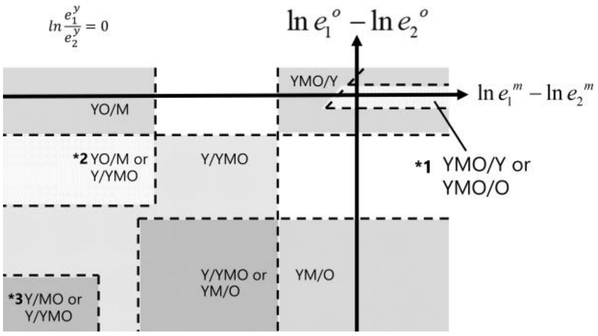

Fig. 2. Residential patterns depending on amenities (for the case of $\ln(e_1^y / e_2^y) = 0$)

Fig. 2 graphically depict the conditions of residential patterns. The horizontal axis in Fig. 2 shows the difference of the zone amenity for middle-aged people between zones and several terms which is composed by the travel cost, time, and so on. The vertical axis portrays the difference of zone amenities for elderly people between zones and several terms. In addition, it is necessary to set the difference of zone amenities for young people in z axis. However, it is hard to see a 3D figure has; thus, we set it as as $\ln(e_1^y / e_2^y) = 0, -0.1, -0.2$ to depict a 2D figure. We show the one of the three figure as Fig. 2.

Table 2 shows the utilities in the residential patterns which emerge at a certain distribution of amenities (i.e., *2 in Fig. 2). It shows that lifetime utility in "Y/YMO" is larger than one in "YO/M".

Table 2 Utilities in "YO/M" and "Y/YMO" (*2: $\ln(e_1^y / e_2^y) = 0$, $\ln(e_1^m / e_2^m) = -10.697$, $\ln(e_1^o / e_2^o) = -3.373$)

	YO/M	Y/YMO
V^y	10.908	**11.008**
V^m	**7.926**	7.889
V^o	4.490	**7.837**
$V^m + V^o / (1+\pi)$	12.403	**15.702**
$V^y + V^m / (1+\pi) + V^o / (1+\pi)^2$	23.310	**26.661**

5. CONCLUSION

This paper clarifies, using an OLG model in a closed city with two zones, how the distribution of amenities for young, middle-aged, and elderly generations affects the residential location patterns and their utilities.

Our model has many residential patterns reflecting how young, middle-aged, and elderly people reside in two zones. Results show that which residential pattern emerges depends on the distribution of zone amenities. For residential patterns in group I, this study shows that multiple residential patterns in group I cannot emerge at any distribution of amenities. In addition, for all residential patterns, this paper checks the difference of utility among the residential patterns which can occur at a certain distribution of amenities. Results show that there is the equilibrium producing the higher level of utility than the others. That implies that the residential pattern is important from perspectives of fairness and other policy objectives.

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