# Optimized Calculation of High－speed Train Braking Mode Control Curve 

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#### Abstract

Starting from the dynamic model of high－speed trains，this paper derives the calculation method of the brake control mode curve based on the force analysis under the train braking state．This method is optimized to achieve online generation of the train control curve．Finally，it is proved by the calculation that the control curve optimization with the braking distance as the shortest optimization target has certain applicability．


Key Words ：High－speed Train，Brake Control，Braking Distance，Optimization Calculation

## 0．INTRODUCTION

Increasing speed of train is the key to improve the quality of railway transportation．However，the safe operation at high speed is a top priority．Therefore， the braking technology of high－speed trains is one of the keys to ensure high－speed and safe．This paper mainly studies the basic theory of the force analysis of train during braking，the calculation method of the control mode curve，then proposes the calculation and optimization of the mode curve．At last put out an example of calculation illustrated by CRH2－300．

## 1．BASIC THEORY

The theoretical basis of the dynamics of high－ speed train operation is the force model．Considering it as a rigid system，regardless of the interaction force between couplers，the force model is shown in Figure 1．In the running high－speed trains，there are three main types of longitudinal forces that can affect the speed of train＇s operation：Train traction force F， train running resistance W （including basic running resistance $\mathrm{W}_{0}$ and additional running resistance $W_{j}$ ），train braking force $B$ ．

Therefore，the resultant force C when the train is running is：

$$
\begin{equation*}
\mathrm{C}=\mathrm{F}-\left(W_{0}+W_{j}\right)-\mathrm{B} \tag{1}
\end{equation*}
$$

Usually the traction is no longer provided when the train is braking，that is to say $\mathrm{F}=0$


Figure1．Model of train＇s force

## 1．1 Running Resistance

The running resistance W refers to an external force generated by the train during the operation due to objective conditions and opposite to the running direction of the train，which hinders the operation of the train．Experiments show that when the train speed reaches $200 \mathrm{~km} / \mathrm{h}, 70 \%$ of the total resistance is air resistance；when the speed is above $300 \mathrm{~km} / \mathrm{h}$ ， the air resistance will account for $85 \%$ ．The following table lists the basic resistance calculation for each EMU model，where $M_{x}$ represents the quality of the corresponding train and $\mathrm{V}_{\mathrm{x}}$ represents the corresponding speed ${ }^{[1]}$ ：

Table1 Basic Resistance Calculation

| Type | Basic Resistance Calculation |
| :---: | :---: |
| CRH1 | $\mathrm{W}_{1}=\left(0.53+0.0026 \mathrm{~V}_{1}+0.000069 \mathrm{~V}_{1}^{2}\right) \mathrm{M}_{1}$ |
| CRH2－300 | $\mathrm{W}_{2}=\left(0.88+0.0074 \mathrm{~V}_{2}+0.000114 \mathrm{~V}_{2}^{2}\right) \mathrm{M}_{2}$ |
| CRH3 | $\mathrm{W}_{3}=\left(0.79+0.0064 \mathrm{~V}_{3}+0.000115 \mathrm{~V}_{3}^{2}\right) \mathrm{M}_{3}$ |
| CRH5 | $\mathrm{W}_{4}=\left(0.69+0.0063 \mathrm{~V}_{4}+0.000146 \mathrm{~V}_{4}^{2}\right) \mathrm{M}_{4}$ |

The additional resistance is not closely related to the type of train and parameters，but the objective conditions of the line on which the train is running will have a decisive influence on it．The additional resistance generally refers to the additional resistance of the slope，curve，and tunnel．It is calculated according to the condition of lines and then added．The total resistance is composed of basic resistance and additional resistance，so it can be calculated according to the following formula；and then the unit resistance of the train is obtained．

$$
\begin{gather*}
\mathrm{W}=\mathrm{W}_{0}+\mathrm{W}_{\mathrm{j}}=\left(\mathrm{w}_{0}+\mathrm{i}_{\mathrm{j}}\right) \times \mathrm{M} \times \mathrm{g}  \tag{2}\\
\mathrm{w}=\frac{\mathrm{W}}{\mathrm{M} \times \mathrm{g}}=\mathrm{w}_{0}+\mathrm{i}_{\mathrm{j}} \tag{3}
\end{gather*}
$$

## 1．2 Braking Force

There are many ways to brake the train，such as friction braking，dynamic braking，and regenerative braking．According to the electric－air joint braking method commonly used in high－speed trains，this paper focuses on air brake $B_{1}$ and regenerative brake $\mathrm{B}_{0}$ ．

Formula for calculating air braking force：

$$
\begin{equation*}
B_{1}=M \times(1+\gamma) \times \beta \times 10^{3} \tag{4}
\end{equation*}
$$

Unit air braking force：

$$
\begin{equation*}
b_{1}=\frac{B_{1}}{B \times g}=(1+\gamma) \times \beta \times 10^{3} \tag{5}
\end{equation*}
$$

$\beta$－decrease speed（ $\mathrm{m} / \mathrm{s}^{2}$ ）；
$\gamma$＿－Rotary mass coefficient，CRH series high－ speed trains are all power－distributed，so $\gamma=0.1$ ．${ }^{[1]}$

The calculation of the regenerative braking force is obtained by reading and calculating the regenerative braking characteristic curve．The magnitude of the regenerative braking force is related to the train type and speed．The Lagrangian linear interpolation method is used to calculate the value of the regenerative braking force．

## 1．3 Braking Process

When the high－speed train is in braking process， the resultant force consists of braking force and running resistance．When the train is at high speed， regenerative braking and air braking work together； but when the train speed is lower than a certain value， regenerative braking is no longer effective．

The combined force of the train braking process is calculated as shown in the following formula（6）． The recursive calculation method of the running
speed and position is as shown in equation（7）：

$$
\begin{equation*}
\mathrm{C}=-\left(\mathrm{W}_{\text {res } 1}-\mathrm{W}_{\text {res } 2}-\mathrm{W}_{\text {bra }}\right) \tag{6}
\end{equation*}
$$

$\mathrm{W}_{\text {res1 }}$ ：basic resistant． $\mathrm{W}_{\text {res2 }}$ ：additional resistant
$\mathrm{W}_{\text {bra }}$ ：braking force

$$
\left\{\begin{array}{c}
S_{i+1}=S_{i}+v_{i} \times \Delta t-\frac{a_{i} \times \Delta t^{2}}{2}  \tag{7}\\
v_{i+1}=v_{i}-a_{i} \times \Delta t
\end{array}\right.
$$

## 2．BRKING MODE CURVE OPTIMIZED CALCULATION

## 2．1 Target－distance Braking

The speed－distance mode curve control is a method of controlling the train speed by continuous one－time braking．It determines and generates a mode curve according to the target distance，the target speed，the current running line condition and the train＇s own parameters．Trains continue to brake according to the curve．Figure2 is a schematic diagram of speed－distance mode curve control．


Figure2 Speed－distance Mode Curve Control

## 2．2 Brake Curve

The train is assumed to be a rigid system， regardless of the interaction force and energy loss in the system，then according to the kinetic energy theorem，the train braking process is done．The work is equal to the increase in train kinetic energy．Train braking should meet：

$$
\begin{equation*}
-(\mathrm{B}+\mathrm{W}) \cdot \mathrm{S}_{\mathrm{e}}=\frac{\mathrm{M}\left(\mathrm{v}_{\mathrm{e}}^{2}-\mathrm{v}_{0}^{2}\right)}{2} \tag{8}
\end{equation*}
$$

Considering the influence of inertia $(\gamma=0.1)$ and running resistance，then unify the unit．

$$
\begin{equation*}
\mathrm{S}_{\mathrm{e}}=\frac{1}{2} \cdot \frac{1100\left(\mathrm{v}_{\mathrm{e}}^{2}-\mathrm{v}_{0}^{2}\right)}{\left(\mathrm{b}+\mathrm{w}_{0}+\mathrm{i}+\mathrm{w}_{\mathrm{r}}+\mathrm{w}_{\mathrm{s}}\right) \times 9.8} \tag{9}
\end{equation*}
$$

B－－brake force（KN）
$\mathrm{S}_{\mathrm{e}}$－－effective brake distance（m）
M－－total weight of train（ t ）
$\mathrm{v}_{0}$－－Initial speed of braking（ $\mathrm{m} / \mathrm{s}$ ）
$\mathrm{v}_{\mathrm{e}}-$－Train braking target speed $(\mathrm{m} / \mathrm{s})$
In fact，when the train starts to brake to brake effectively（assuming that the brake system is sudden pressure to brake is completed in an instant），there is a free travel without brake force，and the process goes through the idle time $t_{k}$ The distance traveled by the train during this process is called the idle
distance $S_{k}$ ．For the sake of simplicity in calculation， it is assumed that the train speed does not change during the idle time，that is，the train speed maintains the initial speed $\mathrm{v}_{0}$ ．

Thus，the braking distance expression for the entire braking process is obtained．

$$
\begin{equation*}
\mathrm{S}=\mathrm{v}_{0} \cdot \mathrm{t}_{\mathrm{k}}+\frac{1}{2} \cdot \frac{1100\left(\mathrm{v}_{\mathrm{e}}^{2}-\mathrm{v}_{0}^{2}\right)}{\left(\mathrm{b}+\mathrm{w}_{0}+\mathrm{i}+\mathrm{w}_{\mathrm{r}}+\mathrm{w}_{\mathrm{s}}\right) \times 9.8} \tag{10}
\end{equation*}
$$

A common method is to use the inverse iteration method to inversely calculate the braking curve of the train to obtain an effective braking distance．

## 2．3 Online Optimization Calculation

In China＇s high－speed railway train control system， a number of mode curves are calculated in advance according to preset running situation．During the running process，the train selects different mode curves to control the operation according to the command information transmitted by the ground and the specific conditions．But if the train receives a temporary braking command，the environment is uncertain．It is necessary to be able to quickly obtain the information of line and quickly generate a suitable brake mode curve to ensure the safety，and meanwhile improve the operating efficiency．In the previous section，the whole train is regarded as a system，ignoring the influence of internal forces on the braking，which is actually a simplification of the calculation．The running time and distance are taken as the optimization targets to calculate the controlling curve and ensuring the train operation safety and passengers＇comfort．

$$
\begin{align*}
& \min \mathrm{T}=\sum \mathrm{T}_{\mathrm{i}} \\
& \text { s.t. }\left\{\begin{aligned}
\mathrm{T}_{\mathrm{i}} & =\mathrm{h}(\mathrm{v}) \\
\mathrm{v} & =\mathrm{g}(\mathrm{a}) \\
\mathrm{a} & =\mathrm{f}(\mathrm{c}) \\
|\mathrm{a}| & \in[0,0.8]
\end{aligned}\right. \tag{11}
\end{align*}
$$

As known the initial speed $\mathrm{v}_{1}$ and the end speed $\mathrm{v}_{2}$ in a certain interval，the breaking force is $F_{B}=c_{i}$ ． $v+d_{i}$ ，so $a=\frac{\left(c_{i}+x\right) \cdot v+\left(b_{i}+y+w_{j}\right)}{M(1+\gamma)}$ ，The speed $v=g(a)$ ， $\mathrm{v}_{2}=\mathrm{v}_{1}+\mathrm{a} \cdot \mathrm{t} . \mathrm{dt}=\frac{\mathrm{M}(1+\gamma)}{\left(\mathrm{a}_{\mathrm{i}}+\mathrm{x}\right) \cdot \mathrm{v}+\left(\mathrm{b}_{\mathrm{i}}+\mathrm{y}+\mathrm{w}_{\mathrm{j}}\right)} \mathrm{dv}$.

$$
t=\frac{M(1+\gamma)}{\left(a_{i}+x\right)} \ln \left|\left(a_{i}+x\right) v+\left(b_{i}+y+w_{j}\right)\right|+C
$$ suppose $t=0$ ，then get $C$ ：

$$
C=-\frac{M(1+\gamma)}{a_{i}+x} \ln \left|\left(b_{i}+y+w_{j}\right)\right|
$$

the running time is

$$
\begin{align*}
& \mathrm{T}=\left\{\frac{\mathrm{M}(1+\gamma)}{\left(a_{i}+x\right)} \ln \left|\left(a_{i}+x\right) v+\left(b_{i}+y+w_{j}\right)\right|-\right. \\
&\left.\frac{M(1+\gamma)}{a_{i}+x} \ln \left|\left(b_{i}+y+w_{j}\right)\right|\right\}\left|\left.\right|_{v_{1}} ^{v_{2}}\right. \tag{12}
\end{align*}
$$

The constraint $\mathrm{a} \in[-0.8,0.8]$ is the acceleration limit of the comfort table given in accordance with the ISO－2631 standard．When the time is optimized，
the passenger＇s ride comfort can be guaranteed．
According to the above model and simulation experiment，it can be known that during the running of the high－speed train，when the train fully utilizes its traction and braking force to control the train operation within the speed allowable range，the time $T$ required for the train to travel the same distance $S$ will be at the very least，the control mode curve with the shortest train running time will also be obtained．

The relationship between operation control line and speed limit line is as Figure 3.


Figure 3
In the figure，the $C D$ curve starts from $D$ and the initial speed of the train is $\mathrm{v}_{\text {min }}$ ．Inversely calculate by the braking acceleration from $v_{\text {min }}$ to $v_{\text {max }}$ ，then get the brake distance $S_{C-D}$ and running time $T_{C-D}$ ：

$$
\begin{gather*}
S_{C-D}=\left\{\frac{M(1+\gamma)}{\left(c_{i}+x\right)}\left[\begin{array}{c}
v \ln \left|\left(c_{i}+x\right) v+\left(d_{i}+y+w_{j}\right)\right|-v-\ln \left|b_{i}+y+w_{j}\right| \cdot v+ \\
\frac{\left(d_{i}+y+w_{j}\right)}{\left(c_{i}+x\right)} \ln \left|\left(c_{i}+x\right) v+\left(d_{i}+y+w_{j}\right)\right|
\end{array}\right]\right\}_{(13)}^{v_{v_{\text {min }}}} \\
T_{C-D}=\left\{\frac{M(1+\gamma)}{\left(c_{i}+x\right)} \ln \left|\left(c_{i}+x\right) v+\left(d_{i}+y+w_{j}\right)\right|\right.  \tag{13}\\
\left.\quad-\frac{M(1+\gamma)}{c_{i}+x} \ln \left|\left(d_{i}+y+w_{j}\right)\right|\right\}\left.\right|_{v_{\text {min }}} ^{v_{\text {max }}} \tag{14}
\end{gather*}
$$

## 3．CALCULATION EXAMPLE

The CRH2－300 high－speed train grouped by 8 vehicles is taken as an example for calculation．The high－speed train CRH2－300 is assumed to be braked to stop at an initial speed of $300 \mathrm{~km} / \mathrm{h}$ on a straight ramp with a slope of 0 （have no additional resistance of the tunnel and the curve）．Taken train braking time be the shortest as the optimization target（non－ emergency brake），and the calculation are as follows．

## 3．1 Braking Force

## （1）Regenerative braking

The regenerative braking force of CRH2－300 can be calculated by Lagrangian interpolation calculation according to the regenerative braking characteristic curve．Using Matlab to fit the characteristic curve of regenerative braking force， and obtains the regenerative system with certain precision．The dynamic expression is convenient to calculate here：

$$
\left\{\begin{array}{cc}
\mathrm{B}_{0}=234 & 0 \leq \mathrm{v} \leq 160  \tag{15}\\
\mathrm{~B}_{0}=\frac{37440}{\mathrm{v}} & 0 \leq \mathrm{v} \leq 300
\end{array}\right.
$$

## （2）Air braking force

The braking force of CHR2－300 is

$$
\begin{equation*}
B_{1}=M(1+\gamma) \beta=462 \beta \tag{16}
\end{equation*}
$$

$M$ is the total mass of the train，and the value is 420 t when at full load．$\beta$ s train＇s deceleration，It is commonly used that the average deceleration of $\beta$ can be taken as $0.75 \mathrm{~m} / \mathrm{s}^{2}$ ．

During the braking process，the regenerative braking is basically operated when the speed is high （ $\geq 160 \mathrm{~km} / \mathrm{h}$ ），and the air braking mode assists；when the train speed is less than $160 \mathrm{~km} / \mathrm{h}$ ，only air braking is needed to work．So the total braking force：

$$
B=B_{0}+B_{1}= \begin{cases}462 \beta & 0 \leq v \leq 160  \tag{17}\\ \frac{37440}{v}+462 \beta & 0 \leq v \leq 300\end{cases}
$$

## 3．2 Running Resistance

It can be calculated according to Table 1.
$\mathrm{w}_{0}=\left(0.88+0.0074 \mathrm{v}+0.000114 \mathrm{v}^{2}\right) \mathrm{M}$
$w_{0}=8.63+0.07295 \mathrm{v}+0.00112 \mathrm{v}^{2} \mathrm{~N} / \mathrm{t}, \quad \mathrm{M}=420 \mathrm{t}$ ．
Since the train is assumed to brake on a straight ramp，no additional resistance is considered．

## 3．3 Braking Distance

The braking force and basic resistance of the train are linearly fitted in a stepwise manner using the


Figure 4

## ACKNOWLEDGMENT

Based on the common brake mode curve，this paper optimizes the calculation method of the target－ distance mode curve by using the dynamic model and related theory of train operation control．Then taking the braking curve of CRH2－300 train as an example，the calculation is validated that it is suitable to encounter sudden situations to generate the brake control curve in real．The next step of the research could be in different environments，considering the impact of different factors on high－speed trains，and the optimization of multi－targets during braking．
least squares method．Then the relationship between the train running speed and the distance in the mode curve is：

$$
\begin{gather*}
S=\int_{v_{1}}^{v_{2}}\left\{\frac{M(1+\gamma)}{\left(a_{i}+x\right)} \ln \left|\left(c_{i}+x\right) v+\left(d_{i}+y+w_{j}\right)\right|-\right. \\
\left.\frac{M(1+\gamma)}{c_{i}+x} \ln \left|\left(d_{i}+y+w_{j}\right)\right|\right\} d v \tag{18}
\end{gather*}
$$

（1）Distance without braking force
Initial speed is $300 \mathrm{~km} / \mathrm{h}$ ，

$$
\begin{aligned}
& \mathrm{t}_{\mathrm{k}}=(4.1+0.002 \cdot \mathrm{R} \cdot \mathrm{n}) \times\left(1-0.03 \mathrm{i}_{\mathrm{j}}\right) \\
& \mathrm{i}_{\mathrm{j}}=0, \mathrm{n}=8, \quad \mathrm{R}=350 \mathrm{kPa} \\
& \quad \mathrm{t}_{\mathrm{k}}=4.1+0.002 \times 350 \times 8=9.7 \mathrm{~s} \\
& \quad \mathrm{~S}_{\mathrm{k}}=\mathrm{v} \cdot \mathrm{t}=\frac{300}{3.6} \times 9.7=808.33 \mathrm{~m}
\end{aligned}
$$

## （2）Effective braking distance

The braking distance calculated using the common method is $5172.28 \mathrm{~m} .{ }^{[4]}$

While using the online optimized method，when fully utilizing train braking force，the deceleration $\beta=\frac{\left(c_{i}+x\right) \cdot v+\left(d_{i}+y\right)}{M(1+\gamma)},|\beta| \in[0,0.8]$ ，then get the maximum deceleration that satisfies the constraints． After the calculation，the CRH2－300 brakes from the initial speed of $300 \mathrm{~km} / \mathrm{h}$ to stop，the braking distance $S=5148.61 \mathrm{~m}$ ，the braking time $\mathrm{T}=113.87 \mathrm{~s}$ ． According to the calculation method above，the braking mode curve of the CRH2－300 train can be finally obtained，as shown in Figure 3.

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