# Non－Uniform Operation Time Optimization of Urban Rail Transit Line 

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#### Abstract

Urban rail transit or called Metro plays an essential role for transporting daily commuters．Adjusting scheduled train departure time from the terminal may reduce waiting time．A mathematic model is developed，and the optimal train departure time which minimizes the total wait time was found by the genetic algorithm（GA）．The model adapts to spatio－temporal origin－destination（OD）demand that was estimated based on data provided by an automatic fare collection system（AFCS）．In a case study of a real－world metro line，the proposed model has demonstrated the effectiveness to reduce waiting time．


Key Words ：Metro，Departure Time，Spatio－temporal Demand，Wait Time，Optimization，genetic algorithm

## 1．INTRODUCTION

Urban rail transit or called Metro plays an essential role for transporting daily commuters due to its high capacity，frequent services and reliability． With the evolution of electronic ticketing system， it＇s possible to obtain accurate spatial and temporal origin－destination（OD）demand，which can be utilized to optimize service planning（i．e．，service frequency，timetable，etc．）to increase system＇s attractiveness ${ }^{[1]}$ ．However，it is very challenging to deal with some operational and safety issues，such as trains bunched up at bottlenecks caused by the variation of demand at stops and travel time between stops，especially during peak periods． Optimizing train departure time at the terminal may reduce the user and operator costs．

## 2．METHODOLOGY

The aims of this study is to optimize train departure times from a terminal which minimize the passenger waiting time at stations while maintaining
a fixed service frequency within in a given time period subjected to minimum headway．The core decision variables include train departure times from the terminal．

## （1）A General Metro Line

This study considers train service on a one－directional urban metro line with N stations，as shown in Figure 1．Each station is given a unique station ID．The hourly service frequency is denoted as F ，and the index of trains is denoted as $\mathrm{m}(\mathrm{m}=1$ ， $2, \ldots, F)$ ．


Fig． 1 Configuration of train service on one－directional．
The assumptions are discussed below：
（1）Train running times between stations and dwell times at stations are deterministic within a time period，but may vary over different periods．
（2）Passenger arrivals at origin station $i$ to destination station $j$ are determined by the distribution function by time，which can be obtained by automatic fare collection systems（AFCS）．
（3）The passenger access／egress times are constant with given stations，and in－vehicle time is not affected by the timetable adjustment，which are omitted in the model．
（4）At a station，all passengers boarding a train obey the Service－in－random－order（SIRO）principle． Passenger waiting to board a train in an order which bears no relation to the order in which they arrive．

## （2）Objective Function

In this study，the operation cost is assumed to be constant，only a shorter headway or an increased coach size would lead to increase operation cost．In order to provide an efficient and effective solution to improve the congested metro network operations during the peak hour，this study aims at minimizing the total passenger waiting time without increasing operation cost by adjusting the departure time at start station．

In this model，the train service capacity is sufficient to meet demand and the dispatching headways at the terminals are fixed for a line within a time period，but may vary over time periods，the objective function of this model is to minimize the total passenger waiting time at the origin stations， which is formulated as

$$
\begin{equation*}
T W=\sum^{F} \sum^{N-1} P_{m i}\left(d_{m i}-t\right) \tag{1}
\end{equation*}
$$

where $T W$ is tot ${ }^{2=1}{ }^{1}$ wditing time of passengers；$P_{m i}$ is number of passengers arriving at station $i$ from outside of station；$d_{m i}$ is departure time of train $m$ at station $i ; F$ is hourly train service frequency on line； $t$ is index of time．

## （3）Model Constraints

Some constraints are incorporated into the model for train operation of real world，which are formulated and discussed below．Note that，the frequency of metro line is not changed，while the departure time would be optimized．Hence，the fleet size is not considered in the constraints．

The minimum and maximum headway for is bounded between the first train of studied time period and the adjacent previous train，as well as between the last train of studied time period and the adjacent latter train．Thus，

$$
\begin{equation*}
h_{\max } \geq d_{F+1,1}-d_{F 1} \geq h_{\min } \tag{2}
\end{equation*}
$$

Where $d_{l l}$ and $d_{F I}$ represent the departure time from a terminal for the first and the last train of studied time period，respectively．$d_{01}$ indicates the departure time from a terminal for the adjacent previous train of the first train of studied time
period，and $d_{F+l, l}$ states the departure time from a terminal for the adjacent latter train of the last train of studied time period．

## 3．SOLUTION ALGORITHMS

In addition to developing a model，the key component of this paper is to develop an effective procedure to reach an optimal solution，namely the minimum total waiting time．For a metro line spans tens of kilometers containing at many stations，the passenger flow volume is tremendous．The hourly frequency of the line is denoted as $F$ ．Therefore，the optimization of timetable for a crowded metro line contains large solution spaces，becoming a large combinatorial optimization problem．

In previous studies，the Heuristic Algorithm（HA） and Simulated Annealing（SA）had been widely applied to search for an optimal solution．Compared with HA and SA，Genetic Algorithm（GA）is easier to find the global optimum ${ }^{[2]}$ ．Genetic Algorithm （GA）method has been regarded as an effective algorithm to produce a global optimal solution within a reasonable computing time ${ }^{[3]}$ ．It has been used in urban rail transit feeder bus route generation and train timetable optimization for a subway system．

An efficient solution algorithm based on GA is developed．A step－by－step procedure for optimizing coordinated operation over a metro line is discussed below．It is as follows：
Step 1：Input the original timetable of the line and the initial dispatching time of train $m$ at start terminal，which is defined as $d^{(0)}{ }_{m l}$ and collected in the matrix $X^{(0)}$ ．

$$
\begin{equation*}
X^{(0)}=\left(d_{11}, d_{21}, \ldots, d_{F 1}\right) \tag{4}
\end{equation*}
$$

Step 2：Adjusting $d_{m 1}$ with $h$ ，so the adjusted dispatching time can be derived as

$$
\begin{gather*}
d_{11}=d_{01}+h_{1}  \tag{5}\\
d_{m 1}=d_{m-1,1}+h_{m} \quad(m \geq 2) \tag{6}
\end{gather*}
$$

where，$d_{01}$ refers to the departure time of the last train during the period before the object time range．$d_{11}$ is the departure time of the first train at start terminal．
$h_{m}$ ，defined as $\left\{h_{1}, h_{2}, h_{3}, \ldots, h_{F}\right\}$ representing
allowable range for schedule justification，is

$$
\begin{equation*}
h_{\min }<h<h_{\max } \tag{7}
\end{equation*}
$$

Step 3：Calculating the variable of the departure time then adjust the timetable．

$$
\begin{align*}
& \Delta v=d_{m l}-d_{m l}^{(0)}  \tag{8}\\
& d_{m i}=d_{m i}^{(0)}+\Delta v \tag{9}
\end{align*}
$$

Step 4 ：Apply GA to search for the optimal $h^{*}{ }_{m}$ ． The steps of GA include：
Step 4－1：Initialize the parameters for the GA such
as population size，number of total generations， selection ratio（rS），crossover ratio（rC），mutation ratio（rM），termination rule，etc and code the variables and parameters by using double vector．
Step 4－2：Generate N initial string structure data as the initial population randomly．Let the generation $g=0$ ．
Step 4－3：Calculate the objective function $f_{j}$ as the fitness．
Step 4－4：Implement the stochastic uniform in the selection process，and use the crossover and mutation operations to reproduce the new solutions． Let $g=g+1$ ．
Step 4－5：Update the next generation and best solution．
Step 4－6：Check if the stop criteria（i．e．，maximum iterations，maximum function tolerance ）is satisfied． Otherwise，go to Step 4－3．
Step 5：Terminate GA search and output the best solutions $v^{*}{ }_{m}, d^{*}{ }_{m l}$ ．Update the arrival and departure time at each station of timetable according to the calculation based on Eqs．10，11．

$$
\begin{gather*}
d_{m i}^{*}=d_{m 1}^{*}+\Delta v  \tag{10}\\
A_{m i}^{*}=A_{m 1}^{*}+\Delta v \tag{11}
\end{gather*}
$$

## 4．NUMERICAL EXAMPLE

The algorithm of the model above applies to the Shenzhen Metro Line 3 for inspection． There are 16 variates，the population size is 100 ，the crossover probability is 0.8 ，the mutation probability is 0.2 ，and the number of iterations is set to 100 ．The minimum departure interval is 90 seconds，and the maximum departure interval is 360 seconds． After 6 hours calculation，the results have been put out．

Table 1．Departure time before and after optimization during 7－8 pm

| long <br> routing <br> before <br> optimizing | long <br> routing <br> after <br> optimizing | short <br> routing <br> before <br> optimizing | short <br> routing <br> after <br> optimizing |
| :---: | :---: | :---: | :---: |
| 7：08：05 | $7: 09: 33$ | $7: 24: 34$ | $7: 25: 45$ |
| 7：14：55 | $7: 17: 40$ | $7: 31: 44$ | $7: 35: 01$ |
| $7: 20: 30$ | $7: 26: 31$ | $7: 38: 22$ | $7: 45: 08$ |
| $7: 26: 30$ | $7: 32: 48$ | $7: 44: 38$ | $7: 51: 00$ |
| $7: 32: 30$ | $7: 39: 13$ | $7: 50: 54$ | $7: 57: 28$ |
| $7: 38: 00$ | $7: 46: 15$ | $7: 57: 10$ | $8: 04: 08$ |
| $7: 41: 00$ | $7: 48: 48$ | $8: 04: 06$ | $8: 12: 15$ |
| $7: 47: 00$ | $7: 52: 44$ | $8: 10: 22$ | $8: 16: 22$ |


| $7: 53: 00$ | $7: 57: 34$ | $8: 16: 20$ | $8: 21: 05$ |
| :---: | :---: | :---: | :---: |
| $7: 59: 00$ | $8: 01: 21$ | -- | -- |

Table 2．Departure interval before and after optimization during $7-8 \mathrm{pm}$

| long routing before optimizing <br> （s） | long routing after optimizing （s） | short routing before optimizing （s） | short routing after optimizing （s） |
| :---: | :---: | :---: | :---: |
| 426 | 514 | 430 | 432 |
| 410 | 487 | 430 | 556 |
| 355 | 551 | 398 | 607 |
| 360 | 377 | 376 | 352 |
| 360 | 385 | 376 | 388 |
| 340 | 432 | 376 | 400 |
| 180 | 153 | 446 | 517 |
| 360 | 236 | 376 | 247 |
| 360 | 290 | 380 | 283 |
| 360 | 227 | －－ | －－ |



Fig 2．Number of passengers entering the station from 7－8 am．

The total waiting time before optimization is 5564403 seconds，the optimized total waiting time is 4559192 seconds，and the passenger＇s total waiting time is optimized for 279 hours．Since the first half of 7 o＇clock to 8 o＇clock，the passenger flow is low， so after the optimization，the train departure interval has obviously increased，but the passenger flow in the second half begins to increase，and the train departure interval has a significant reduction．This uneven starting time reduces the waiting time of passengers and improves the operating efficiency of the train．

## 5．CONCLUSION

Through the above optimization，it can be seen that
the optimization scheme proposed in this paper can reduce the waiting time of passengers to a certain extent，and the model can determine the departure time and timetable for adapting the passenger flow according to the distribution of passenger flow．On the basis of this article，there are still some directions to research： 1 considering crowding situation． 2 considering change of train stop time． 3 considering robustness of timetable．

## REFERENCE

［1］Niu H ，Zhou X ．Optimizing urban rail timetable under time－dependent demand and oversaturated conditions． Transportation Research Part C：Emerging Technologies， 36：212－230， 2013.
［2］Liu，X．，Qu，H．，Chien，S．，Ran，B．，Huang，M．Optimizing Passenger Transfer Coordination in a Large Scale Rapid Rail Network，Transportation Research Board：The 94th Annual Meeting，Washington，D．C．，No．15－4225， 2015
［3］Zhen，W．，Wang，W．Genetic algorithm and its application． Microcomputer system，23（6）：9－10， 2000.
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