Optimal Pricing Policies on Railways and Roads Considering their Interdependent Congestion in a City

Joji KONAGANE[1], Tatsuhito KONO[2], Masao KUWAHARA[3] and Keigo JOTO[4]

[1] Non-Member of JSCE, Graduate Student, Dept. of Information Sciencies, University of Tohoku
(6-06, Aza-aoba 6, Aramaki, Aoba-ku, Sendai-shi, Miyagi 980-8579, Japan)
E-mail:joji.konagane.r2@dc.tohoku.ac.jp
[2] Member of JSCE, Professor, Dept. of Information Sciencies, University of Tohoku
(6-06, Aza-aoba 6, Aramaki, Aoba-ku, Sendai-shi, Miyagi 980-8579, Japan)
E-mail:kono@plan.civil.tohoku.ac.jp
[3] Member of JSCE, Professor, Dept. of Information Sciencies, University of Tohoku
(6-06, Aza-aoba 6, Aramaki, Aoba-ku, Sendai-shi, Miyagi 980-8579, Japan)
E-mail:kuwahara@plan.civil.tohoku.ac.jp
[4] Non-Member of JSCE, Graduate Student, Dept. of Information Sciencies, University of Tohoku
(6-06, Aza-aoba 6, Aramaki, Aoba-ku, Sendai-shi, Miyagi 980-8579, Japan)
E-mail:joto@plan.civil.tohoku.ac.jp

Road congestion and railway car congestion are important problems in urban city. Several cities introduce the uniform car toll or extra rail fares during rush hours in order to mitigate the congestion. Because they depend on the modal share of cars and railways at every location, we must consider both congestion in order to conduct pricing policies. This paper explores the optimal car toll and rail fares, and the efficacy of optimal uniform pricing policies which considers both congestion and urban spaces simultaneously.

Numerical results show as follows. When the location of bottleneck is at the edge of the CBD, the optimal uniform car toll and rail fares achieve approximately 75(%) to the time-dependent policy. When the location of bottleneck is some distance from the CBD, they achieve approximately 37(%).

Key Words: *Bottleneck congestion, Congestion toll, Public transportation, Railway fare, Railway car congestion,*

1. Introduction

Today, road congestion is one of the most problems in urban areas. Several cities such as London and Singapore introduce area or cordon pricing, which are easier than the first-best tolls.

Congestion in trains is also an important problem in urban areas. As one of the countermeasures, during rush hours, congestion fares have been introduced in Washington and Singapore subways.

However, both pricing policies have price distortions which are interdependent on each other. Population density and the modal share at each point change according to pricing policies. Therefore, we must consider both road and railway congestion and urban space simultaneously. Joto (2018) theoretically explores this situation.

This paper quantitatively explores the optimal tolls and fares, and the efficacy of pricing policies which considers both congestion and urban spaces simultaneously.

2. The model

The model is a congested monocentric city of a rectangular. The width of the residential area is c(km). The residential area expands from $x=\underline{x}$ at the edge of the CBD to $x=\overline{x}$ at the UGB. The

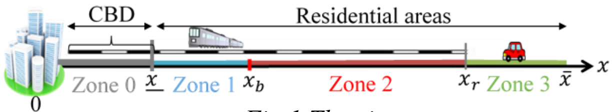

Fig.1 The city

bottleneck point is only one at $x=x_b$ in the city. The end point of railway is $x=x_r$. The city has two transport modes with congestion: cars and railways. When car commuters pass through a bottleneck, they incur dynamic traffic congestion. Railway commuters are affected by railway car congestion depending on the number of railway commuters. The city structure is shown in Fig.1.

We consider a closed city. The city land is public ownership. The road operator introduces uniform toll τ, which is levied constantly throughout the day at the bottleneck to mitigate traffic congestion. The railway operator optimizes railway fare $e(x)$, which depends on location but is levied constantly throughout the day at location x to decrease railway car congestion.

We define Situations1 and 2 as,

Situation1: $x_b = \underline{x}$,

Situation2: $x_b > \underline{x}$.

N identical households reside in the city and inevitably commute to the CBD. They have a utility function:

$$\nu = \upsilon(k + \sum_{j=1}^{J} \zeta_j \varepsilon_j^{car}, q), \quad (1)$$

where k is numeraire composite goods, ε_j^{car} is the difference in utility using a car from using a railway. q is housing square footage. ε_j^{car} depends on day J. The distribution of ε_j^{car} follows a certain probability function. ζ_j is a dummy variable which equals 1 if a person uses his car and equals 0 if he uses a railway. J is a certain term and j is one day of J.

A probability of using a car is expressed as

$$\Phi(\kappa(x)) = \int_{\kappa(x)}^{\infty} \chi(\varepsilon^{car}) d\varepsilon^{car}, \quad (2)$$

where $\chi(\varepsilon^{car})$ is an example of the probability function. $\kappa(x)$ is defined as

$$\kappa(x) = t^{car}(x) - t^{rail}(x), \quad (3)$$

where $t^{car}(x)$ and $t^{rail}(x)$ are the commuting cost by car and railway, respectively.

Each household earns income y. A household rents floor space from developers at price r per unit floor space. The income constraint is given by

$$k + rq = y - \left\{ \sum_{j=1}^{J} \zeta_j t^{car}(x) + \sum_{j=1}^{J} \left[1 - \zeta_j \right] t^{rail}(x) \right\} + \frac{G}{N}, \quad (4)$$

where G is non-labor income, which is explained in eq. (9). We define $n(x)$ as the number of commuters passing location x.

$$n(x) = \int_{x}^{\bar{x}} \left[1/q(m) \right] dm, \; x \in [\underline{x}, \bar{x}]. \quad (5)$$

Since $x = \underline{x}$ is the inner edge of the residential area, $n(\underline{x})$ represents the total number of commuters (i.e., $n(\underline{x}) = N$). $n^{car}(x)$ and $n^{rail}(x)$ are the number of commuters who use a car and railway at location x, respectively.

All car commuters incur the same private cost, $\delta n^{car}/s$ due to the bottleneck congestion as obtained in Arnott et al. (1993) where s and δ are capacity and parameter of bottleneck, respectively. Car commuting cost is expressed as

$t^{car}(x) = bx$ at $x < x_b$ (6),

$t^{car}(x) = \delta n^{car}(x_b)/s + bx + \tau$ at $x \geq x_b$ (7)

where b is generalized cost per distance of car and τ is congestion toll.

Railway commuting cost is expressed as

$$t^{rail}(x) = ax + e(x) + \left[\underline{x} \cdot f\left(n^{rail}(\underline{x})\right) + \int_{\underline{x}}^{x} g\left(n^{rail}(s)\right) ds \right] \text{at } x^{\forall} \in [\underline{x}, \bar{x}] \quad (8)$$

where $f(n^{rail}(\underline{x}))$ and $g(n^{rail}(x))$ are railway car congestion costs in the CBD area and at $x \in (\underline{x}, x_r]$ respectively, a is travel time cost per distance and $e(x)$ is railway fare.

All households equally gain the net revenues of land rent, car toll and railway fare, and equally incur the total maintenance and construction cost of railway. The non-labor income is expressed as

$$G = R + T + E - Z - X \quad (9)$$

Then, G/N is distributed to each household.

3. Theoretical results

Joto (2018) obtains optimal car toll and rail fares in the following.

Proposition 1(optimal toll and fares in Situation1)

$$\tau - e(x) = \frac{\delta_c n^{car}(\underline{x})}{s_c} - \left\{ \begin{array}{l} zx + \underline{x} n^{rail}(\underline{x}) f'(n^{rail}(\underline{x})) \\ + \int_{\underline{x}}^{x} n^{rail}(s) g'(n^{rail}(s)) ds \end{array} \right\}. \quad (10)$$

When the location of bottleneck is at the edge of CBD, the differences between optimal car toll and rail fares equal the differences of each price distortions.

Proposition 2(optimal fares in Situation2)

$$e(x) = \begin{bmatrix} zx + \underline{x}\left(n^{rail}(\underline{x})\right) f'(n^{rail}(\underline{x})) \\ + \int_{\underline{x}}^{x} \left(n^{rail}(s) - n^{rail}(x_b)\right) g'(n^{rail}(s)) ds \end{bmatrix} \text{at } x^{\forall} < x_b \quad (11)$$

Proposition 3(optimal fares in Situation2)

$$e(x) = \begin{bmatrix} zx + \underline{x}\left(n^{rail}(\underline{x})\right) f'(n^{rail}(\underline{x})) + \int_{\underline{x}}^{x_b} n^{rail}(s) g'(n^{rail}(s)) ds \\ + n^{rail}(x_b) g'(n^{rail}(x_b)) + \int_{x_b}^{x} n^{rail}(s) g'(n^{rail}(s)) ds \end{bmatrix} \text{at } x^{\forall} \geq x_b \quad (12)$$

Proposition 4(optimal toll in Situation2)

$$\tau = \delta_c n^{car}(x_b)/s_c. \quad (13)$$

When the location of bottleneck is in residential areas, optimal car toll and rail fares equal their respective price distortions.

4. Numerical setup

We divide residential areas into narrow, discrete 6 rings with an equal width c. The length of each band is 2km. CBD edge, the UGB and end-point of railway are set at 1km, 13km and 10km, respectively. The utility function is set as

$$v(k + \sum_{j=1}^{J} \zeta_j \varepsilon_j^{car}, q) = k + \sum_{j=1}^{J} \zeta_j \varepsilon_j^{car} + \alpha \ln q + 31000 \quad (14)$$

The income is $42,628.555. The housing parameter α is 8,000. J is 230 day.

We quantitatively compare second-best welfare gains W with the first-best.

5. Numerical results

When bottleneck is at the edge of CBD, the optimal uniform tolls and fares achieves 75% of the first-best policy. If the bottleneck is far from the CBD edge, they achieve approximately 40%. In Situation1, total road congestion cost decreases by $1,060,000 and railway car congestion cost increases by $840,000. In Situation2, road congestion cost decreases by $500,000 and railway car congestion increases by $283,000.

6. Concluding remarks

Numerical results show that when the bottleneck is at the edge of CBD, the optimal uniform pricing policy is efficient. In contrast, when the bottleneck is distant from the CBD edge (i.e., Situation2), this pricing policy is not so efficient because utilities of residents inside the bottleneck decrease with an increase in railway congestion whereas utilities of residents outside the bottleneck increase with a decrease in road congestion.

Table1 Numerical Results

	W	Road conge-stion	Railway conge-stion	τ	e_1	e_2	e_3	e_4	e_5
	(%)	(10^7\$)				($/day)			
Situation.1									
A	－	1.84	7.76	0	0.44	0.88	1.3	1.8	2.2
B	75	1.73	8.60	7.0	0.73	1.2	1.6	2.1	2.5
Situation.2									
A	－	1.18	7.47	0	0.44	0.88	1.3	1.8	2.2
B	37	1.13	7.75	9.1	4.8	5.5	5.9	6.4	6.8

Note: A, laissez-faire; B, second-best