Morning commute problem of urban rail transit: Modeling and equilibrium

Jiahua ZHANG[1], Kentaro WADA[2] and Takashi OGUCHI[3]

[1]Non-member of JSCE, Graduate Student, Dept. of Civil Eng., the University of Tokyo
(4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan)
E-mail:zhangjh@iis.u-tokyo.ac.jp
[2]Member of JSCE, Assistant Professor, Inst., Industrial Science, the University of Tokyo
(4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan)
E-mail:wadaken@iis.u-tokyo.ac.jp
[2]Fellow of JSCE, Professor, Inst., Industrial Science, the University of Tokyo
(4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan)
E-mail:takog@iis.u-tokyo.ac.jp

Urban rail transit plays a critical role in the public transportation system during the morning peak-hours due to its large capacity and high service frequency. Recently, frequent delays caused by unexpected incidents or passenger congestion are significantly deteriorating the level of service of rail transit system. In order to relieve the congestion and prevent the propagation of delays, a considerable number of models have been built so far to describe the operation of rail transit system. However, most of them separately considered the microscopic operation principles of trains and the passenger commuting behavior.

This paper formulates a macroscopic equilibrium model to describe both the train operation condition and passenger behavior during the morning commute. By introducing a dynamic in-vehicle congestion index, the equilibrium arrival pattern of commuters is derived, and simultaneously, the impact of passenger congestion on train operation is considered. Finally, the proposed model is examined by a numerical experiment.

Key Words : *Mass transit, Macroscopic modeling, Passenger congestion, User Equilibrium*

1. INTRODUCTION

Urban mass transit generally serves as the major solution to the morning commute travel demand in metropolises owing to its high capacity[1)]. However, the operation of mass transit system during the morning peak hours can easily be disrupted by accidents caused by passengers, equipment failures or extreme weathers. Even without such unexpected incidents, urban mass transit suffers from disturbances due to passenger congestion and consequent knock-on delay[2) - 4)]. To improve the reliability and level of service (LOS) of mass transit system, engineering approaches such as building quadruple tracks or operating trains with more cars are effective solutions, but limitations due to space and cost widely exist so that physically increasing the capacity turns out to be difficult in many cases.

On the other hand, ongoing efforts have also been made to optimize the timetable or obtain management strategies[5) - 9)]. Issues like minimizing travel time[10)], optimizing service frequency[5), 11), 12)] and interstation spacings[13)] for mass transit have widely been discussed by scholars. For instance, Mohring[11)] proposed a well-known “square root rule” to obtain optimal bus frequency which was quite effective for the transit service planning. Daganzo[12)] introduced a headway-based adaptive approach to eliminate bus bunching. However, these studies usually assume a steady or stochastic arrival pattern of passengers which is not appropriate for the problem of morning commute.

To tackle mass transit congestion during the morning peak hours, one critical issue is to understand the passenger departure time choice equilibrium which describes a temporal travel demand distribution. This issue started to be investigated from 1960s firstly by economists and transportation scientists such as Vickrey[14)], Henderson[15)], Hendrickson and Kocur[16)]. Even with simple bottleneck queuing model mainly used for highway, many profound insights were derived from their analysis. For example, no increase in capacity is capable of eliminating peak period congestion if many workers start work at the same time[16]. In general, congestion cannot be totally eliminated as long as there exists temporary excessive demand larger than the capacity. However, capacity increase can shorten the waiting or queuing time and therefore reduce the duration of morning commute.

Recently, some new efforts[19), 20)] are made to represent dynamics of mass transit system based on macroscopic fundamental diagram (MFD). Chiabaut[19] introduced the passenger-MFD concept and employed it to assess the performance of a multimodal transportation network. Seo et al.[20] proposed a macroscopic tractable model to describe the mass transit dynamics by employing a transit-specific fundamental diagram and they validated the macroscopic model by comparing the output with the microscopic model.

Although many previous studies[17) - 19), 21) - 23)] have proposed various models to derive and analyze the corresponding equilibria, they either assumed travel time determined by static model or neglected the influence of dynamic passenger congestion. Meanwhile, dynamics of rail transit system and passenger departure time choice equilibrium are generally treated as different issues in most studies[5) – 8), 20), 22)]. However, in order to obtain strategic implications (e.g., dynamic pricing, introduction of flexible working time) on relieving congestion and improving the LOS of mass transit system, a macroscopic and dynamic model that jointly considers the unique characteristics of rail transit system and passenger commuting behavior is needed.

This study formulates a macroscopic model to represent the dynamics of urban rail transit system and corresponding passengers' commute behavior. Specifically, a dynamic in-vehicle congestion index is introduced to describe the impact of passenger congestion on train operation. At the same time, this index is employed to obtain the arrivals and departures of passengers. In this sense, this model can be regarded as an effective tool to macroscopically evaluating the influence of management strategies on passengers' commute behavior.

The paper is structured as follows: Section 2 introduces the basic assumptions of rail transit operation and describes the dynamics of morning commute; Section 3 formulates the equilibrium condition of passengers' commute behavior; Section 4 validates the feasibility of the model by a numerical experiment and sensitivity analysis.

2. RAIL TRANSIT OPERATION AND PASSENGER BEHAVIOR

In this section, an analytical model based on the microscopic operation principles of rail transit system is formulated to represent the dynamics of rail transit operation and passengers' commute behavior.

(1) Assumptions

Firstly, consider a rail transit line that have multiple origins and one destination. Passengers arrive the stations nearest their homes to commute to their workplaces. Here we assume that all passengers using this rail transit line have the same destination and the total number of passengers is fixed as N_p. This situation is actually rather common considering radial rail transit lines connecting residential areas and the downtown of a city. Also, we assume that only local trains are operated so that first-in-first-out (FIFO) principle is satisfied for trains and passengers. Then, we introduce the assumptions of train cruising and dwelling behaviors based on a non-dimensional in-vehicle congestion index: congestion rate $\eta(t)$, which is determined by the passenger departure time choice equilibrium. The variable $\eta(t)$ lies in the interval of $[0, \eta_{max})$ where $\eta(t) = 0$ means no passenger is in the train and $\eta(t) = 1$ means passenger number reaches the "capacity" of the train car. Note, this capacity refers to the situation that all passengers are either seated or able to grab the handrails, and this value varies according to the type of train car. Thus, η_{max} is larger than 1 and it is generally assumed to be around 2.5 given the physical space limit of the train car.

a) Train dwelling behavior

In this study, we assume that the dwelling time $t_b(t)$ at stations is mainly affected by the congestion rate $\eta(t)$. In fact, most previous studies[20] used the boarding time calculated by the number of waiting passengers and a constant boarding rate to describe the train dwelling time. However, passengers' boarding rate will be reduced when the in-vehicle congestion is severe, which is the common situation during the morning commute. Therefore, we directly employed the congestion rate to represent the increase of dwelling time as Eq. (1):

$$t_b(t) = t_{b\min} + \mu \cdot \eta(t) \qquad (1)$$

where t_{min} is the minimum dwelling time needed for door opening/closing, and μ is the dwelling time sensitivity to the congestion rate.

b) Train cruising behavior

The cruising behavior of a train is modeled using the Newell's simplified car-following model[24]. In this model, a vehicle travels at the free-flow speed while maintaining the minimum safety speed clearance. Specifically, the headway of trains H should satisfy Eq. (2):

$$H \geq t_b + \frac{\sigma + v\tau}{v} \tag{2}$$

where σ is the minimum spacing when cruising speed is zero, and τ is the physical minimum headway of trains. Meanwhile, for simplicity, we assume that train bunching will not happen so that the average cruising speed of trains v can be kept as a constant. Therefore, the travel time of trains $c_t(t)$ and passengers $c_p(t)$ can be respectively expressed by Eq. (3) and Eq. (4):

$$c_t(t) = \frac{L}{l}\left(t_b(t) + l / v\right) \tag{3}$$

$$c_p(t) = \varepsilon \frac{L}{l}\left(t_b(t) + l / v\right) \tag{4}$$

Where ε is the proportional coefficient of average passenger travel time and $\varepsilon \approx 0.5$ if passengers homogeneously live along the railway line. L is the total length of the railway line and l is the average distance between stations.

(2) Dynamics of train operation and passenger behavior

In this sub-section, we formulate a model to describe the rail transit operation and passenger commute behavior where the demand (i.e., passenger flow) and supply (i.e., train-flow) change dynamically. Since the behavior of individual train or passenger is not explicitly described, this model is macroscopic. The proposed model is based on a delay-function-based link model. The delay-function-based link models refer to a class of link models that explicitly use delay functions or equivalently link performance functions to specify, in advance, the time-dependent link traversal time which otherwise becomes available only at the time when a trip is finished[25]. Delay-function (DF) models have been extensively used in the analytical formulation, analysis and computation in dynamic traffic assignment problems due to its tractability.

This study considers the railway system as an input-output system in the same spirit of Seo et al.[20]. Specifically, let $a(t)$ and $a_p(t)$ be the in-flow of trains and passengers at time t respectively, $d(t)$ and $d_p(t)$ be the corresponding out-flows. Then, let $A(t)$, $A_p(t)$, $D(t)$ and $D_p(t)$ be the cumulative numbers of $a(t)$, $a_p(t)$, $d(t)$ and $d_p(t)$. Here, $a(t)$ is considered to be determined by the rail transit operation plan (i.e., timetable) as the known input of the model, and $a_p(t)$ will be derived from passenger departure time choice equilibrium explained in the next section. The travel time of trains and passengers defined in Eq. (3) and Eq. (4) are also determined the equilibrium since dynamic congestion rate is solved from the departure time choice problem. Then, $d(t)$, $d_p(t)$ and their cumulative numbers can be endogenously obtained by employing the DF model. More specifically, we firstly derive cumulative train arrivals $A(t)$ by Eq. (5):

$$A(t) = \int a(t)dt \tag{5}$$

The cumulative train departures $D(t)$ can be calculated from Eq. (6) given the FIFO principle:

$$D\left(t + c_t(t)\right) = A(t) \tag{6}$$

Meanwhile, the out-flow of trains $d\left(t + c_t(t)\right)$ can be obtained by differentiate Eq. (6) as Eq. (7):

$$d\left(t + c_t(t)\right) = \frac{a(t)}{1 + dc_t(t)/dt} \tag{7}$$

Here, according to the car-following model explained in Eq. (2), the out-flow of trains should be bounded as shown in Eq. (8):

$$d\left(t + c_t(t)\right) \leq \frac{1}{t_b\left(t + c_t(t)\right) + \sigma / v + \tau} \tag{8}$$

So far, the dynamics of train operation is totally described by Eq. (5) to Eq. (8). In term of passengers' commute behavior, we use the similar idea to describe its dynamics by using the DF model. Firstly, we assume that the passenger in-flow rate is determined by Eq. (9) as

$$a_p(t) = N \cdot C \cdot \eta(t) \cdot a(t) \tag{9}$$

where N is the number of train cars, and C is the capacity of one train car. Eq. (9) can be understood as that passengers arrival pattern is affected by two main factors: one is the dynamic congestion rate determined by passenger departure time choice equilibrium; another is the information of train in-flow or timetable. At the same time, a constraint of total travel demand has to be introduced as Eq. (10)

$$\int_{t_0}^{t_e} a_p(t)dt = N_p \tag{10}$$

where t_0 and t_e is respectively the start and end time of morning commute. Since $a(t)$ is the given input, Eq. (10) actually determined the duration and peak of morning commute together with the user equilibrium. Therefore, the cumulative passenger arrivals can be obtained using Eq. (11):

$$A_p(t) = \int_{t_0} a_p(t)dt \tag{11}$$

Finally, again consider the FIFO principle, the passenger out-flow rate and its cumulative can be calculated from Eq. (12) and (13):

$$d_p\left(t + c_p(t)\right) = \frac{a_p(t)}{1 + dc_p(t)/dt} \tag{12}$$

$$D_p\left(t + c_p(t)\right) = A_p(t) \tag{13}$$

The notable feature of the proposed model is its high tractability, we expect it can be used to evaluate management strategies after empirical validation and calibration.

3. USER EQUILIBRIUM

In this section, we formulate the dynamic user equilibrium (DUE) of passenger departure time choice problem during the morning commute.

(1) Travel cost function

In this study, we assume that each commuter unilaterally attempts to minimize his own travel cost, expressed as a linear combination of travel time $c_p(t)$, schedule delay $s(t,t^*)$, and congestion rate $\eta(t)$. Note, we exclude the fare from cost function since it is assumed to be constant at this stage. The effect of dynamic pricing will be discussed in the following studies. Therefore, the generalized travel cost function can be written as:

$$TC(t,t^*) = \alpha c_p(t) + s(t,t^*) + \delta\eta(t) \tag{14}$$

where t is the passenger arrival time at rail transit system, t^* is the desired departure time from rail transit system or work start time, α is the user cost of travel time, and δ is the user cost of congestion. Here, the first term $c_p(t)$ on the right hand of Eq. (14) uses the definition in Eq. (4). The second term $s(t,t^*)$ is defined as:

$$s(t,t^*) = \begin{cases} \beta\cdot\left[t^* - \left(t + c_p(t)\right)\right] & \text{if } t + c_p(t) \le t^* \\ \gamma\cdot\left[\left(t + c_p(t)\right) - t^*\right] & \text{if } t + c_p(t) > t^* \end{cases} \tag{15}$$

where β is the user cost of early arrival and γ is the user cost of late arrival. In general, the relations between the three kinds of user cost should obey $\gamma > \alpha > \beta$. Besides, it can be understood from Eq. (15) that the schedule delay reaches the minimum of zero if a commuter departs the rail transit system at t^*. Now, by submitting Eq. (1), Eq. (4) and Eq. (15) into Eq. (14), the three terms of travel cost $TC(t,t^*)$ are essentially functions of $\eta(t)$.

(2) Equilibrium condition

Under DUE condition for arrival time t and desired departure time t^*, Eq. (16) holds which means all passengers have the same total travel cost no matter when he arrives the system:

$$\frac{\partial TC(t,t^*)}{\partial t} = \alpha\frac{dc_p(t)}{dt} + \frac{\partial s(t,t^*)}{\partial t} + \delta\frac{d\eta(t)}{dt} = 0 \tag{16}$$

By submitting Eq. (1), Eq. (4) and Eq. (15) into Eq. (16), the congestion rate $\eta(t)$ can be obtained as Eq. (17):

$$\eta(t) = \begin{cases} \eta_0 + M_1(t - t_0) & \text{if } t_0 \le t \le t_j \\ \eta_0 + M_1(t_j - t_0) - M_2(t - t_j) & \text{if } t_j < t \le t_e \end{cases} \tag{17}$$

Where η_0 is the initial congestion rate which describes congestion level of rail transit system during the off-peak hours. For simplicity, we assume $\eta(t \le t_0) = \eta(t \ge t_e) = \eta_0$ so that Eq. (18) holds:

$$M_1(t_j - t_0) = M_2(t_e - t_j) \tag{18}$$

where:

$$M_1 = \frac{\beta}{\mu\cdot\varepsilon\cdot L/l\cdot(\alpha-\beta)+\delta} \tag{19}$$

$$M_2 = \frac{\gamma}{\mu\cdot\varepsilon\cdot L/l\cdot(\alpha+\gamma)+\delta} \tag{20}$$

t_j is the arrival time that a passenger experiences zero schedule delay as expressed in Eq. (21):

$$t_j + c_p(t_j) = t^* \tag{21}$$

In this study, we only discuss the situation that all commuters have the same fixed work start time t^*. It can be observed that M_1 and M_2 are fixed if other parameters in Eq. (19) and Eq. (20) are given. Meanwhile, t_j and t_e can be expressed as functions of t_0 and t^* by using Eq. (18) and Eq. (21). Here, the work start time t^* is considered to be a given input. Finally, the dynamic congestion rate $\eta(t)$ can be analytically derived if the train in-flow $a(t)$ is given by jointly solving Eq. (9), Eq. (10) and Eq. (17) to Eq. (21). Fig. 1 shows the transition pattern of $\eta(t)$ based on Eq. (17).

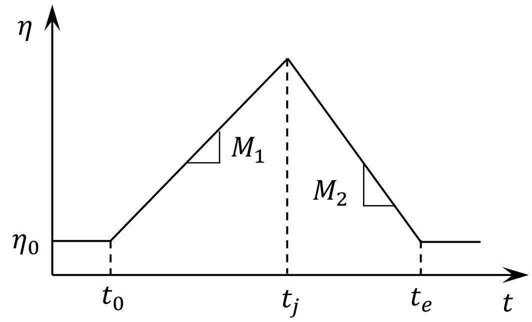

Fig. 1 Illustration of congestion rate $\eta(t)$.

So far, the equilibrium condition of passengers' behavior during morning commute has been analytically solved by Eq. (16). As a result, the equilibrium determines the dynamic congestion rate $\eta(t)$ and therefore the arrival and departure patterns of passengers can be calculated from Eq. (9), Eq. (11), Eq. (12) and Eq. (13). At the same time, passengers' congestion impact on train operation is also incorporated into the model since $\eta(t)$ affects the dwelling time and furthermore the travel time of trains. In the next section, we check the feasibility of the proposed model by a numerical experiment and sensitivity analysis.

4. NUMERICAL EXPERIMENT

(1) Simulation settings

Firstly, we explain the settings of train in-flow rate $a(t)$. In general, $a(t)$ can be understood as an equivalent of a timetable since the timetable gives the information of trains' operation headways, and headways are reciprocal values of in-flow rate $a(t)$. For this simulation experiment, we assume a trapezoid shape of $a(t)$ which is illustrated in Fig. 2. In fact, a short and fixed operation headway during the morning peak-hours is rather common if one observes the timetable of metro systems in metropolises.

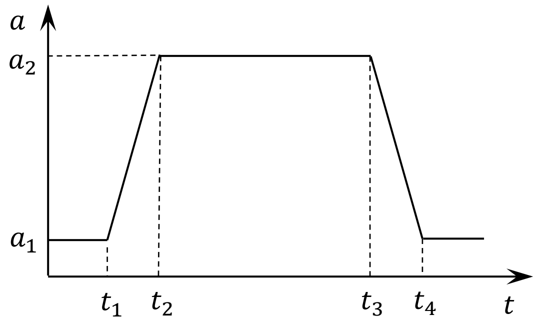

Fig. 2 Train in-flow settings

In Fig. 2, a_1 and a_2 respectively represents the off-peak and peak-hour in-flow rate. t_1, t_2, t_3 and t_4 control the duration of high-frequency supply and its transition rate. In this numerical simulation, all these values are considered to be the given inputs. But in the future we can utilize the operation data from railway companies to validate the model. The values of these parameters are shown in Table 1.

Table 1 Parameters of train in-flow information

Parameter	Value
a_1, a_2 (train/h)	10, 30
t_1, t_2, t_3, t_4 (min)	90, 120, 240, 270

Other parameters that needed to be initialized are summarized in Table 2. Here, *cr* refers to congestion rate and *pax* refers to passenger. For a quick reference, we list the definitions of all parameters in Table 3 in the appendix.

Table 2 Parameters of the numerical simulation

Parameter	Value
σ, τ, t_{bmin}	0.2 km, 1 min, 0.25 min
μ, ε, η_0,	0.5 min/cr, 0.6, 0.3
l, L, v	1.5 km, 45 km, 45 km/h
N_p, N, C	1×10^5 pax, 8 veh, 150 pax/veh
α, β, γ, δ	20 \$/h, 10 \$/h, 30 \$/h, 15 \$/cr
t^*	240 min

(2) Simulation results

Under the above-mentioned settings, the start and end of morning commute is solved to be $t_0 = 52\ min$ and $t_e = 245\ min$ which means the morning commute lasts for 192 min under a total travel demand of 100 thousands passengers. In addition, $t_j = 185\ min$ which indicates that the maximum travel time of passengers is 55 minutes, and passengers who arrive the rail transit stations at t_j can depart the system at their desired departure time t^*.

Then, we check the dynamics of rail transit operation by plotting the flow rates and cumulative curves of trains in Fig. 3 and Fig. 4. From Fig. 3 it can be observed that $d(t)$ drops a little at the beginning of the morning peak. This is due to the increase of travel time caused by a longer dwelling time. Then, $d(t)$ grows with the increase of $a(t)$ but keeps lower than $a(t)$. This implies that the train density continues increasing until $a(t)$ intersects with $d(t)$. Subsequently, a sudden rise of $d(t)$ larger than $a(t)$ appears. This is because travel time starts decreasing during this period so that the derivative of travel time becomes negative. From Eq. (7) it can be understood that when the derivative of travel is negative, $d(t)$ will be larger than $a(t)$. Finally, $d(t)$ declines with the decrease of $a(t)$.

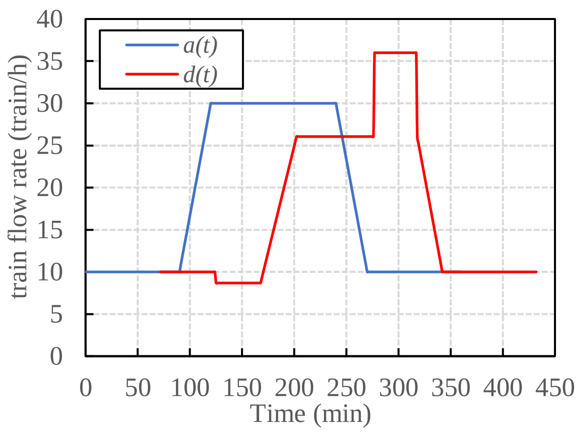

Fig. 3 Flow rate of trains

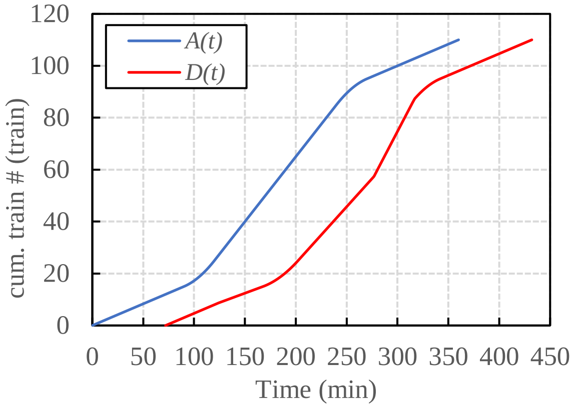

Fig. 4 Cumulative curves of trains

With regard to the cumulative curves in Fig. 4, it can be found that travel time starts to increase from $t_0 = 52\ min$ until $t_j = 185\ min$, and the maximum travel time of trains can be obtained as $c_t = 92\ min$. Meanwhile, the train density reaches the maximum at about $t = 248\ min$, and the maximum density can be calculated as $k_{max} \approx 0.98\ train/km$. Similarly, we plot the flow rates and cumulative curves of passenger in Fig. 5 and Fig. 6.

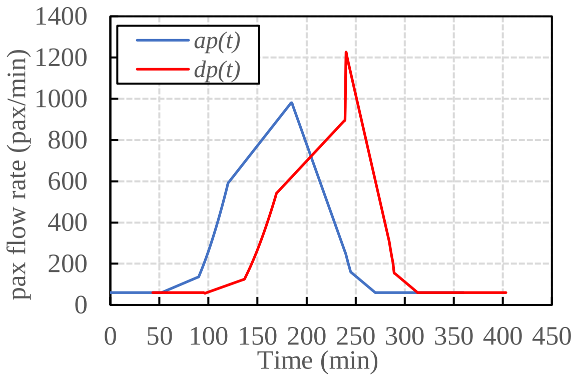

Fig. 5 Flow rate of passengers

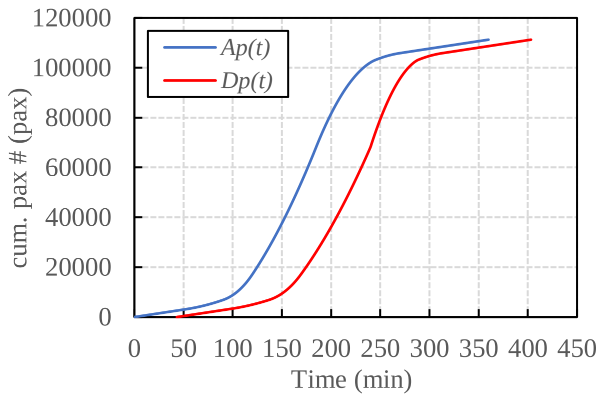

Fig. 6 Cumulative curves of passengers

From Fig. 5 it can be observed that both the arrival rate and departure rate of passengers are unimodal, $a_p(t)$ reaches the maximum at $t_j = 185\ min$, and $d_p(t)$ reaches the maximum at $t^* = 240\ min$. These results are consistent with our travel experience during the morning commute. In Fig. 6, the increase of passenger number is rather notable compared to the increase of travel time. By confirming the cross point of $a_p(t)$ and $d_p(t)$ in Fig. 5, it can be calculated that the maximum passenger number in the system is around 45000 pax reached at $t \approx 205\ min$.

By deriving the dynamic flow rates and cumulative curves of trains and passengers, we can monitor the congestion and delay of rail transit system. These information is important for the decision making of implementing control measures. Also, it can be used to evaluate the performance of the system during the planning stage of a new rail transit line. In the next sub-section, we check the feasibility of the proposed model by a sensitivity analysis.

(3) Sensitivity analysis

Since we don't have empirical data to validate and calibrate the proposed model at this stage, we firstly confirm whether the model can produce interpretable results by adjusting the values of several parameters. Here, we select total commute demand N_p and user cost of congestion δ as two examples to conduct the sensitivity analysis. Note, other parameters remains the same as shown in Table 1 and Table 2. For simplicity, we only compare $a_p(t)$ and $\eta(t)$ for the purpose of understanding how passengers' commute behavior and congestion pattern alter with the change of corresponding parameters.

Firstly, a comparison is made to confirm the impact of total commute demand. Fig. 7 and Fig. 8 respectively show the transition of $a_p(t)$ and $\eta(t)$ under three levels of demand. It can be observed from both figures that when the total commute demand increases, morning commute starts earlier, meanwhile both $a_p(t)$ and $\eta(t)$ ascends at any given time. For example, the congestion rate larger than 140% lasts for 33 minutes under a 100 thousands demand, but this time extends to 60 and 86 minutes when the demand separately increases 20% and 40%. Therefore, it can be recognized that severe congestion duration is sensitive to the growth of total commute demand.

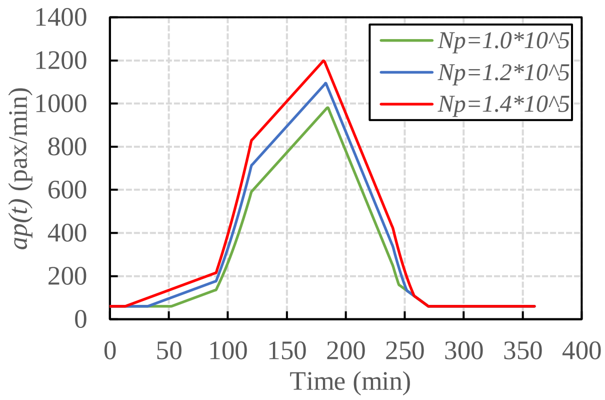

Fig. 7 Passenger arrival rate comparison under different demand levels

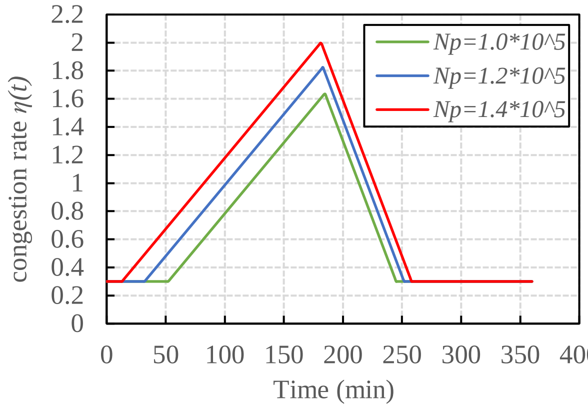

Fig. 8 Congestion rate comparison under different demand levels

Next, we compare $a_p(t)$ and $\eta(t)$ when user cost of congestion δ varies. A lower δ means passengers can tolerate higher level of congestion when they choose their departure time. Therefore, we expect a later start and early end of morning commute when δ is low. The comparisons are shown in Fig. 9 and Fig. 10. It can be observed that different from the impact of total commute demand, δ has a relatively small influence on both $a_p(t)$ and $\eta(t)$ when it varies from 80% to 120% of the original value. But still, smaller δ leads to a more concentrated and congested morning commute as we expected.

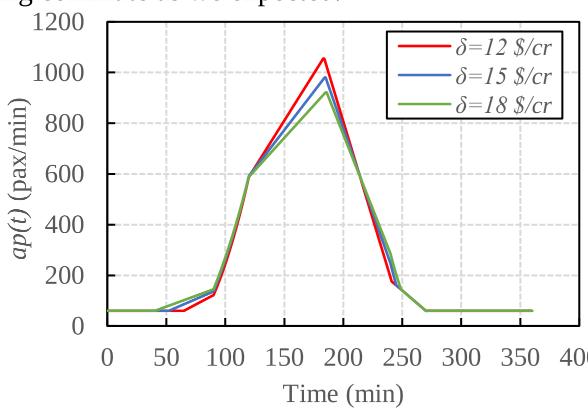

Fig. 9 Passenger arrival rate comparison for different user cost of congestion

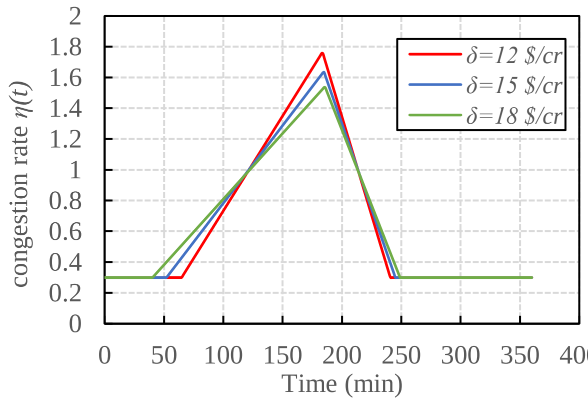

Fig. 10 Congestion rate comparison for different user cost of congestion

In summary, this section examined the performance of the proposed model by a numerical experiment, and through a further sensitivity analysis, the model was found to be able to produce reasonable interpretations.

5. CONCLUSION

This paper formulated a macroscopic tractable model to represent the dynamics of rail transit system and corresponding passengers' travel behavior during the morning commute. The proposed model was formulated based on the principles of train operation and dynamic user equilibrium of passengers' commute behavior. Through the numerical experiment, we preliminarily confirmed the feasibility of the model. A notable characteristic of this model is that it can predict the arrival pattern of passengers and further assess the passenger congestion's impact on rail transit operation. Based on the outputs of the model, the congestion level and delay in the system can be dynamically estimated. Moreover, this model can be used to evaluate the performance of a rail transit system under various settings of supply and demand. Due to its simplicity and well tractability, we expect that this model can contribute to obtaining policy implications on management strategies such as flexible working time or dynamic pricing.

Regarding the future works, we firstly need to validate the proposed model by empirical data from a case study on a specific rail transit line. Then, we plan to expand the fixed t^* to a general distribution of work start time and evaluate the congestion alleviation effect by introducing staggered work start time. Besides, different from the DF model used in this paper, we are also planning to employ exit-flow[26) – 27)] model or MFD-based approach[19) – 20)] to explore the dynamics of the rail transit system.

In addition, the proposed model has several limitations. First, the constant train cruising speed assumption do not hold when the dwelling time extends to be excessively long. This can be solved by relating the cruising speed to the density of trains. Second, the congestion rate calculated from DUE problem might be unrealistically high under an excessive passenger demand. Therefore, a constraint of maximum congestion rate η_{max} has to be introduced given the space limit of train cars. Meanwhile, discussions should be made on how to treat the situation when the equilibrium cannot be reached under extreme input environments. In other words, the robustness of the model is needed to be improved.

APPENDIX A.   NOTATION

Table 3 Notations used in the paper

Parameter	Definition
$\eta(t)$	Congestion rate at time t
$t_b(t)$	Train dwelling time at time t
t_{bmin}	Minimum train dwelling time needed for door opening/closing (min)
μ	Train dwelling time sensitivity to the congestion rate (min/cr)
H	Time headway of successive trains (min)
σ	Minimum spacing between trains when cruising speed is zero (km)
τ	Physical minimum headway time (min)
v	Cruising speed of trains (km/h)
$c_t(t)$, $c_p(t)$	Train travel time at time t, passenger travel time at time t
L	Total length of railway line (km)
l	Average distance between stations (km)
ε	Proportional coefficient of average passenger travel time
$a(t)$, $d(t)$	In-flow rate of trains, out-flow rate of trains (train/h)
$A(t)$, $D(t)$	Cumulative arrivals of trains, cumulative departures of trains (train)
$a_p(t)$, $d_p(t)$	Arrival rate of passengers, departure rate of passengers (pax/min)
$A_p(t)$, $D_p(t)$	Cumulative arrivals of passengers, cumulative departures of passengers (pax)
N	Number of train cars (veh)
C	Capacity of one train car (pax/veh)
N_p	Total travel demand of the morning commute (pax)
t_0, t_e	Start of the morning commute, end of the morning commute (min)
t_j	Passenger arrival time at stations when he/she can depart the system at the desired departure time (min)
t^*	Passenger desired departure time from the system/work start time (min)
α	User cost of travel time ($/h)
β	User cost of early arrival ($/h)
γ	User cost of late arrival ($/h)
δ	User cost of congestion ($/cr)
$s(t, t^*)$	Schedule delay when passengers arrive stations at time t with a desired departure time t^*
η_0	Initial congestion rate of the rail transit system
a_1, a_2	Train in-flow rate during off-peak hours, train in-flow rate during peak hours (train/h)

REFERENCES

1) Vuchic, V. R. : Urban Transit: Operations, Planning and Economics. John Wiley & Sons, 2005.
2) Tirachini, A., Hensher, D. A., Rose, J. M. : Crowding in public transport systems: effects on users, operation and implications for the estimation of demand. *Transportation Research Part A: Policy and Practice*, Vol. 53, pp. 36-52, 2013.
3) Keiji, K., Naohiko, H., Shigeru, M., : Simulation analysis of train operation to recover knock-on delay under high-frequency intervals. *Case Studies on Transport Policy*, Vol. 3(1), pp. 92-98, 2015.
4) Carey, M., and Kwieciński, A. : Stochastic approximation to the effects of headways on knock-on delays of trains. *Transportation Research Part B: Methodological*, Vol. 28(4), pp. 251-267, 1994.
5) Niu, H., and Zhou, X. : Optimizing urban rail timetable under time-dependent demand and oversaturated conditions. *Transportation Research Part C: Emerging Technologies*, Vol. 36, pp. 212-230, 2013.
6) De Palma, A., Kilani, M., and Proost, S. : Discomfort in mass transit and its implication for scheduling and pricing. *Transportation Research Part B: Methodological*, Vol. 71, pp. 1-18, 2015.
7) Wang, Y., Tang, T., Ning, B., van den Boom, T. J., and De Schutter, B. : Passenger-demands-oriented train scheduling for an urban rail transit network. *Transportation Research Part C: Emerging Technologies*, Vol. 60, pp. 1-23, 2015.
8) Robenek, T., Maknoon, Y., Azadeh, S. S., Chen, J., and Bierlaire, M. : Passenger centric train timetabling problem. *Transportation Research Part B: Methodological*, Vol. 89, pp. 107-126, 2016.
9) Wada, K., Akamatsu, T., and Osawa, M. : A control strategy to prevent propagating delays in high-frequency railway systems. In *The 1st European Symposium on Quantitative Methods in Transportation Systems*, 2012.
10) Vuchic, V. R., and Newell, G. F. : Rapid transit interstation spacings for minimum travel time. *Transportation Science*, Vol. 2(4), pp. 303-339, 1968.
11) Mohring, H. : Transportation economics. Cambridge, MA, Ballinger, 1976.
12) Daganzo, C. F. : A headway-based approach to eliminate bus bunching: Systematic analysis and comparisons. *Transportation Research Part B: Methodological*, Vol. 43(10), pp. 913-921, 2009.
13) Vuchic, V. R. : Rapid transit interstation spacings for maximum number of passengers. *Transportation Science*, Vol. 3(3), pp. 214-232, 1969.
14) Vickrey, W. S. : Congestion theory and transport investment. *The American Economic Review*, Vol. 59(2), pp. 251-260, 1969.
15) Henderson, J. V. : The economics of staggered work hours. *Journal of Urban Economics*, Vol. 9(3), pp. 349-364, 1981.
16) Hendrickson, C., and Kocur, G. : Schedule delay and departure time decisions in a deterministic model. *Transportation science*, Vol. 15(1), pp. 62-77, 1981.
17) Kraus, M., and Yoshida, Y. : The commuter's time-of-use decision and optimal pricing and service in urban mass transit. *Journal of Urban Economics*, Vol. 51(1), pp. 170-195, 2002.
18) Tian, Q., Huang, H. J., and Yang, H. : Equilibrium properties of the morning peak-period commuting in a many-to-one mass transit system. *Transportation Research Part B: Methodological*, Vol. 41(6), pp. 616-631, 2007.
19) Chiabaut, N. : Evaluation of a multimodal urban arterial: The passenger macroscopic fundamental diagram. *Transportation Research Part B: Methodological*, Vol. 81, pp. 410-420, 2015.
20) Seo, T., Wada, K., and Fukuda, D. : A Macroscopic and Dynamic Model of Urban Rail Transit with Delay and Congestion. In *Transportation Research Board 96th Annual Meeting*, 2017.
21) Takayama, Y. : Bottleneck congestion and distribution of work start times: The economics of staggered work hours revisited. *Transportation Research Part B: Methodological*, Vol. 81, pp. 830-847, 2015.
22) Gonzales, E. J., and Daganzo, C. F. : Morning commute with competing modes and distributed demand: user equilibrium, system optimum, and pricing. *Transportation Research Part B: Methodological*, Vol. 46(10), pp. 1519-1534, 2012.
23) Trozzi, V., Gentile, G., Bell, M. G., and Kaparias, I. : Dynamic user equilibrium in public transport networks with passenger congestion and hyperpaths. *Transportation Research Part B: Methodological*, Vol. 57, pp. 266-285, 2013.
24) Newell, G. F. : A simplified car-following theory: a lower order model. *Transportation Research Part B: Methodological*, Vol. 36(3), pp. 195-205, 2002.
25) Nie, X., and Zhang, H. M. : Delay-function-based link models: their properties and computational issues. *Transportation Research Part B: Methodological*, Vol. 39(8), pp. 729-751, 2005.
26) Merchant, D. K., and Nemhauser, G. L. : A model and an algorithm for the dynamic traffic assignment problems. *Transportation science*, Vol. 12(3), pp. 183-199, 1978.
27) Carey, M., and McCartney, M. : An exit-flow model used in dynamic traffic assignment. *Computers & Operations Research*, Vol. 31(10), pp. 1583-1602, 2004.