An Analysis on the Capacity of Permitted Right-turn Movement at Signalized Intersections with Autonomous Vehicle Mixed Traffic Flow

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Recently, the society is showing a great interest in autonomous vehicle, and the improvement in efficiency of road traffic system is being expected by spreading them. However, this expectation may not be easily realized with simply spreading autonomous vehicles. Particularly at signalized intersections, due to the complicated interaction between vehicles, the influence of autonomous vehicles on traffic flow is hard to evaluate. In this paper, some existing models are applied to describe the influence of autonomous vehicles by changing some paremeter settings. And the capacity of permitted right-turn movement with autonomous vehicle mixed traffic flow is estimated by using both of calculation and simulation methods. Finally it is concluded that the capacity of permitted right-turn movement is likely to decrease as the increase of percentage of autonomous vehicles.

Key Words : *Autonomous vehicle, signalized intersection, permitted right-turn movement, capacity*

1. INTRODUCTION

In last several years, the technical possibilities of autonomous driving systems have been remarkably enhanced, and some field experiments on real roads have also started. Although it seems to be reasonable that the traffic congestion problem can be relieved when the occupancy of autonomous vehicles (referred to as AV hereinafter) becomes 100% in a simple situation such as on basic segment, for the application in the real world there are two problems to be solved: (1) The movement of AV in the complicated situation such as at signalized intersections cannot be described definitely. (2) The efficiency and safety of AV mixed traffic flow and human driven vehicle (referred to as HDV hereinafter) are still unclear.

The treatment of right-turn behavior is one of the most important issues at signalized intersections for the left-hand traffic system. When the traffic demand is not so high, the permitted right-turn (referred to as PRT hereinafter) is usually applied. The capacity of PRT movement is one of the important concerns to evaluate the efficiency of the signalized intersections.

This paper develops a method to estimate the capacity of PRT movement with AV and HDV mixed traffic flow at signalized intersections.

2. LITERATURE REVIEW

(1) Mixed traffic flow of AV and HDV

Although more and more attention has been paid to the AV since a few years, research works regarding the mixed traffic flow of AV and HDV are very limited yet.

Patcharinee *et al.*[1)], Shi and Panos[2)] estimated that the highway capacity can be increased exponentially with the increase of percentage of AV due to the shortened average headway. Horiguchi and Oguchi[3)] evaluated the satuation flow rate under various logics of AV. Wei *et al.*[4] summarized the relationship between reaction time (or communication delay for AV) and the backwave speed. Wanibe *et al.*[5] studied the relationship between the occupancy of AV and the efficiency of the start-up behavior at signalized intersections.

This paper follows some settings of parameters in the previous studies when considering the AV's behavior in the opposing through traffic flow.

(2) Headway distribution

Research on the headway distribution on roadways with unrestricted overtaking has a long history.

Breiman[6] mentioned the headway distribution at some reasonably large distance follows Poisson distribution. Cowan[7] refined the distribution into M1-M4 distributions by dividing vehicles into two groups, tracking vehicles and free vehicles. Catbagan and Nakamura[8] gave some new definition of tracking vehicle considering every vehicle's desired speed. Konda and Nakamura[9] used rich real-world data to calibrate the distributions.

To estimate capacity of PRT movement, the gap distribution of opposing through vehicles is necessary. The headway distribution of opposing through vehicles at the conflict point is equal to the gap distribution of opposing through flow used in the gap acceptance model.

(3) Car-following behavior

Several car-following models have been proposed for more than 50 years. Some of the representative models are briefly summarized below.

Chandler *et al.*[10] found a phenomenon; when the relative speed or following distance between leading car and following car become larger, the change of acceleration also becomes larger. Gazis and Herman[11] proposed General Motors (GM) model which regards relative speed and following distance between leading vehicle and following vehicle as influence factors. Newell[12], Ceder[13], Aron[14] did several research works on parameter settings of car-following model. To add the desired speed as an influence factor, Treiber *et al.*[15] proposed the Intelligent Driver Model (IDM).

Since the distributions of reaction time and desired speed are supposed to be different between AV and HDV, this paper chooses the IDM model to describe the behavior of opposing through vehicles.

(4) Gap acceptance behavior of PRT vehicles in the opposing through traffic flow

The gap acceptance behavior of PRT vehicles in the opposing through vehicle is an important behavior at signalized intersections.

The Highway Capacity Manual (HCM)[16] provides a rough relationship between gap acceptance probability and the flow rate of opposing vehicles when estimating the capacity of PRT movement. Watanabe and Nakamura[17] established a regression model to show the relationship between gap acceptance probability and several influence factors including gap time and waiting time.

The sensitivity of waiting time which reflects the degree of patience is one of the most significant differences between AV and HDV. As a result, this paper chooses Watanabe and Nakamura[17]'s model to calculate the gap acceptance probability.

(5) Capacity of PRT movement at signalized intersections

To estimate capacity of PRT movement, the HCM[16] methodology is commonly used. It considers the saturation flow rate, cycle length, green time, gap acceptance probability and arrival traffic flow rate. Nikiforos *et al.*[18] simplified the HCM model and established a regression model with opposing volume, number of opposing lanes, cycle length and percent green time.

Although the HCM calculation of PRT is complicated to use in practice, its calculation concept is widely accepted. This paper mainly uses the HCM methodology to calculate the capacity of PRT.

3. METHODOLOGY

(1) Car-following behavior of AV mixed traffic flow on opposing through movement

a) Model analysis

It is quite important to describe the opposing through flow to estimate the capacity of PRT movement. To describe the behavior of opposing through vehicles, IDM model is used. IDM model assumes that the following vehicle is only influenced by the leading vehicle. If the difference between current speed and desired speed, or the difference between current distance and the safety distance is large, the vehicles tend to increase their speed. The main equation of IDM model is as Equations (1a)-(1d):

$$\alpha(t+\tau) = a[1-(\frac{v_{i+1}(t)}{V})^{\delta} - (\frac{s^*(v(t),\Delta v)}{s(t)})^2] \quad (1a)$$

$$s^*(v(t),\Delta v) = s_0 + \max(0, v_{i+1}(t)T + \frac{v_{i+1}(t)\Delta v}{2\sqrt{ab}}) \quad (1b)$$

$$\Delta v = v_{i+1}(t) - v_i(t) \quad (1c)$$

$$s(t) = x_i(t) - x_{i+1}(t) - l \tag{1d}$$

where, α: acceleration, a: maximum acceleration, b: maximum decelaration, $v_i(t)$: speed of vehicle i at time point t, V: desired speed, T: minimum time headway, s_0: minimum space headway, $x_i(t)$: position of vehicle i at time point t, l: vehicle length, τ: reaction time and δ: parameter.

If some initial status of the vehicles are set, speeds and positions of vehicles in the opposing through traffic flow at any moment t can be calculated from the acceleration by the IDM model. By using some simulation analysis, the time-space diagram can be drawn.

b)Parameter setting of AV and HDV

The AV and HDV's parameter settings in IDM model are shown in **Table 1**. In most cases, the communication delay is smaller than human driver's reaction time, so that the setting of reaction time of AV is obviously less than HDV. Desired speed of HDV is a little bit higher than AV, because all the AV can strictly follow the designed speed, while HDV usually tends to go with a distribution around the posted speed limit.

(2) Gap distribution

Generally, headway distribution is divided into two parts, tracking vehicles and free vehicles. Because the headway between tracking vehicles are much smaller than the critical gap, it is almost impossible for the right-turn vehicles to cross the gap between tracking vehicles. As a result, this paper only focuses on the headway distribution of free vehicles.

The headway distribution of opposing through vehivles on the conflict point means the gap distribution of the opposing through vehicles used in the gap acceptance model. The distribution can be described as Equations (2a) and (2b):

$$P(t) = \lambda e^{-\lambda t} \tag{2a}$$

$$\lambda = \frac{3600}{q} \tag{2b}$$

where, $P(t)$: probability density function of the headway t and q: traffic flow rate (vph).

(3) Gap acceptance model

a) Model analysis

This paper chooses the model established by Watanabe and Nakamura[17)] to calculate the gap acceptance probability of PRT movement as shown by Equation (3a):

Table 1 Parameter settings of HDV and AV in IDM model

Parameter	Unit	HDV	AV
Reaction time	sec	0.7	0.1
Desired speed	km/h	60	50
Min distance	m	2	2
Min headway	sec	1	1
Max acceleration	m/s2	2	2
Max deceleration	m/s2	3.6	3.6
δ	-	4	4
Vehicle length	m	4.5	4.5

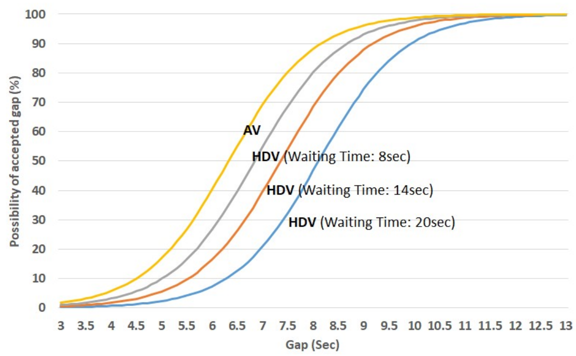


Fig.1 Relationship between gap acceptance probability and gap time

$$P_{Accepted} = \frac{e^{V_{Accepted}}}{1 + e^{V_{Accepted}}} \tag{3a}$$

where, $P_{Accepted}$: gap acceptance probability and $V_{Accepted}$: utility function of accepted gap.

To calculate the utility function of accepted gap, a regression model with real-world data from several signalized intersections was established. The utility function of accepted gap is as Equation (3b):

$$V_{Accepted} = 1.20 * t_{Gap} + 0.112 * t_{Wait} - 9.73 \tag{3b}$$

where, t_{Gap}: gap time and t_{Wait}: waiting time.

The regression result of gap model shows that as the increase of gap time or the waiting time, the higher probability the vehicle accepts the gap. With this model, if the gap time and the waiting time can be measured, the gap acceptance probability can be calculated.

b)Parameter settings of AV and HDV

For simplicity in this study, it is assumed that the vehicle performance of AV and HDV is very similar when they cross the opposing through vehicle, which means the gap time they need to cross does not have obvious difference. On the other hand, AV can be hypothesized not to care about how long time they wait for the accepted gap, and thus only the gap time is considered. Therefore in this paper, HDV is assumed to follow the original gap model, while AV's sensitivity of waiting time is set as 0.

Under such a setting, HDV tends to accept smaller gap time compared with AV when waiting time increases. As a result, the relationship between gap acceptance probability and gap time is shown as **Fig.1**.

(4) Capacity calculation

The HCM methodology mentions that generally, when the signal phase converses from red into green, the discharge flow of opposing direction is almost the saturated flow that cannot be crossed by right-turn vehicles. The right-turn vehicles can only cross after the saturated discharging flow. The end of the discharge flow and the passing time can be calculated with Equations (4a) and (4b):

$$t_0 = \frac{qR}{s-q} = \frac{q(C-G)}{s-q} \tag{4a}$$

$$t_r = G - t_0 = G - \frac{q(C-G)}{s-q} = \frac{sG-qC}{s-q} \tag{4b}$$

where, t_0: time of discharge flow, t_r: passing time, s: saturation flow rate of through flow, q: arrival flow rate, C: cycle length, G: effective green time and R: red time.

The capacity of right-turn flow is then calculated by the saturation flow rate of right-turn vehicle and the gap acceptance probability by Equations (4c) and (4d):

$$\frac{t_r}{C} = \frac{sG-qC}{(s-q)C} \tag{4c}$$

$$c_r = s_r * \frac{t_r}{C} * f_r = s_r * \frac{sG-qC}{(s-q)C} * f_r \tag{4d}$$

where, s_r: saturation flow rate of right-turn movement, f_r: gap acceptance probability and c_r: capacity of right-turn movement.

This model enables us to estimate the capacity of PRT movement with three groups of varibles: setting of signal timing, arrival traffic rate flow of opposing through vehicles, gap acceptance probability. The setting of first two groups of varibles will be carefully described in the following section 4 Case Study, where the probability of accepted gap can be calculated with gap acceptance model.

4. CASE STUDY

(1) Signalized intersection setting

This paper hypothesizes a signalized intersection with one right-turn lane and one opposing through lane. The geometric layout of the intersection is shown as **Fig.2**.

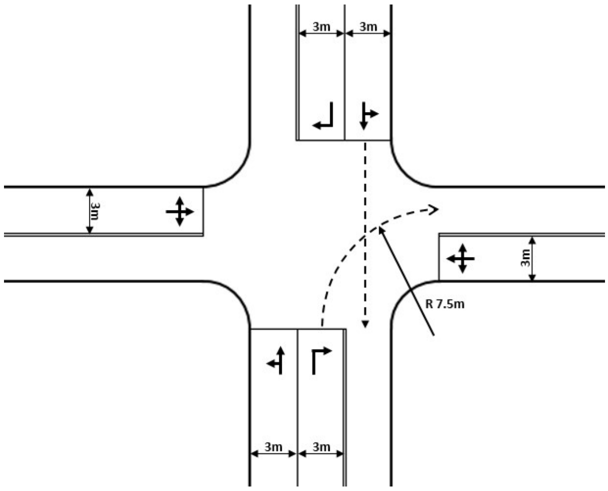
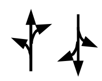
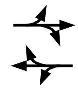


Fig.2 Geometric layout of the hypothesized signalized inter section

Table 2 Signal timing plan

Phase Structure		
Phase Length(sec)	60	60
Cycle Length(sec)	120	

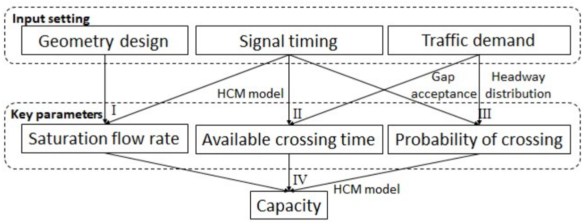


Fig.3 Method flow of calculation

Due to the symmetry of this intersection, the result of every opposite approach is similar. It is no need to repeat the calculation or simulation for the opposite approach and thus the objective crossing movement is marked in **Fig.2**.

The signal phasing of this intersection is assumed to be the simple 2-phase without protected right-turn phase. The signal timing plan of the intersection is shown in **Table 2**.

(2) Calculation method

a) Basic procedure

The calculation method is mainly based on the HCM methodology and its flow is briefly shown in **Fig.3**.

b) Traffic demand setting

The opposing through traffic flow rates Q_{opp} are set as 0, 200vph, 400vph, 600vph, 800vph, 1000vph. Within the objective hour, the traffic flow rate is assumed to be stable.

Because the gap distribution of opposing through mixed traffic flow of AV and HDV is difficult to estimate without using simulation, in this sub-section, only HDV is considered, whether in the opposing through flow or in the PRT flow.

The saturation flow rate of through vehicle s is set as 2000vph. The waiting time t_{Wait} is set as the average waiting time in Watanabe and Nakamura[17)]'s model as 15s.

c) Conputing process

Step I: Using HCM model, the saturation flow rate of right-turn "s_r" is set as 1800vph.

Step II: Input s, Q_{opp}, and green time G, cycle length C into Equations (4a) and (4b), then get the available passing time t_r.

Step III: Input Q_{opp} into Equations (2a) and (2b), then get the probability density function of headway. Input such function and waiting time t_{wait} into Equations (3a) and (3b), then get the probability of crossing f_r.

Step IV: Input s_r,t_r,C and f_r into Equation (4d), then get the capacity result of different opposing through traffic flow rate.

Following the above steps, the capacity results are shown in **Table 3**.

(3) Simulation method

a) Basic procedure

This paper assumes that the crossing behavior of right-turn vehicles has no impact on the opposing through traffic flow. As a result, the simulation process is divided into two parts: i) The time-space trajectory of opposing through flow is reproduced by iteration algorithm based on the IDM car-following model. ii) The first vehicle in right-turn flow makes judgement based on gap acceptance model. Then how many vehicles can cross the opposinig through flow is calculated as the capacity result.

The simulation in this paper is based on the software developed by the author. The development platform is Visual Studio, C# language.

The procedure is briefly shown in **Fig.4**.

b) Traffic demand setting

The opposing through traffic flow rate are set as 200vph, 400vph, 600vph, 800vph, 1000vph. The right-turn vehicles are set to be completely over-saturated.

The occupancy of AV is set from 0 to 100%, and its increment is 10%.

The simulation section is from 300m upstream from the stop line to 50m downstream from the stop line. The simulation time is from 0 to 3600s. The time granularity is 0.1 second. The space granularity

Table 3 Capacity estimation result by calculation method

Q_{opp} (vph)	0	200	400	600	800	1000
C_r (vph)	900.0	586.5	354.3	197.9	102.0	45.0

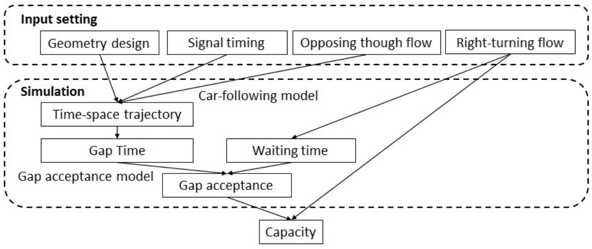


Fig.4 Method flow of simulation

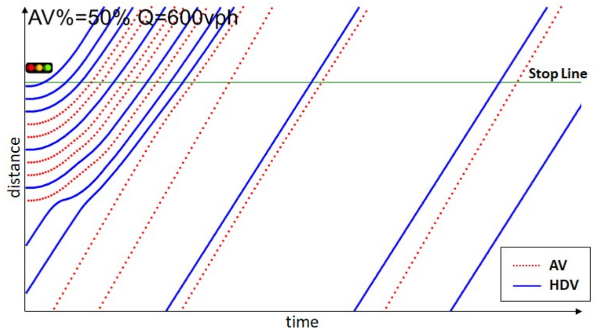


Fig.5 Example of time-space trajectories

is 0.1m. Every case will be repeated for 100 times.

c) Simulation process

For the opposing through vehicles, the simulation is based on the following steps:

Step I: Input. The vehicles can be input to the starting point at any time from 0 to 3600s unless the time difference to the next vehicle is less than the minimum time headway.

Step II: Iteration. All the vehicles follow Equations (1a)-(1d) in IDM model with the parameter setting of HDV and AV shown in **Table 1**.

IDM model has a shortage in the simulation process: if this model is used to describe the case that the following vehicle is changed from free driving condition into tracking condition, the following vehicle will miss the critical moment to decrease speed. To overcome this shortage, a emergency brake process is added to IDM model in the program used in this paper. If the following distance is equal to or smaller than the safety distance which can exactly avoid the crash, the following vehicle will ignore the IDM model, but choose to decrease its speed at the maximum deceleration, until the following distance is larger than the safety distance.

Step III: When all opposing through vehicles have passed the conflict point, all the gap will be recorded, and the time-space trajectory will be drawn. **Fig.5** is an example of a part of the time-space trajectories.

For the right-turn vehicles, the waiting time for the vehicle is calculated from when the previous vehicle crosses. The moment the first vehicle meets a gap, it will judge whether it can cross by using Equations (3a) and (3b) in gap acceptance model.

Because the right-turn traffic flow is set as completely over-saturated, how many right-turn vehicles can cross within one hour will be regarded as the capacity of PRT flow.

After all the cases have been simulated for 100 times, this paper calculates the average result of each case. Results of capacity estimated by the simulation method is summarized in **Table 4**.

(4) Capacity analysis result

a) Comprison between two methods

To calibrate the simulation model, this paper compares the results between calculation method and simulation method. Since the calculation mehod can be only used for the cases without AV, the cases in simulation mehod where AV percentage is 0 are chosen. The result is shown in **Fig.6**.

The comparison result shows that in most cases the difference of capacity result by calculation method and simulation mehod is not so large.

When the opposing through flow rate is low, the capacity result by calculation method is higher than the result by simulation method. And when the opposing through flow rate is high, the capacity result by calculation method is lower than the result by simulation method. That is because the randomness of simulation method will increase the the deviation of gap. The gap acceptance model in this paper is based on logit model, so that the increase of deviation of gap will lead to lower probability of crossing when opposing through flow is high, and higher probability of crossing when opposing through flow is low.

b) Analysis on capacity by percentage of AV

The simulation method shows different capacity in different opposing through flow rate and different percentage of AV shown in **Fig.7**.

There is an obvious trend that as the increase in the AV percentage, the capacity of PRT flow decreases. That is mainly because the AV is more “patient” than HDV, they don’t tend to try a short but dangerous gap even if the waiting time is very long.

Although as the percentage of AV increases in the opposing through flow, the effiency will increase, but such improvement happens mostly in the start-up behavior, which cannot be used for right-turn vehicles to cross. For the vehicles that does not track the leading vehicle, there is no clear improvement in effiency from HDV to AV.

Table 4 Capacity estimation result by simulation method

i) \ ii)	200	400	600	800	1000
0%	687.7	416.8	158.6	27.6	0.0
10%	683.7	410.4	144.9	34.0	5.0
20%	659.9	403.4	143.9	33.5	5.1
30%	654.9	380.6	117.2	40.3	5.5
40%	646.8	365.3	114.2	36.2	11.9
50%	637.5	355.7	106.3	30.9	6.1
60%	621.4	347.8	99.2	25.3	6.2
70%	621.4	325.4	102.8	19.4	6.4
80%	613.3	323.4	81.2	19.7	0.0
90%	605.3	329.3	76.6	13.2	0.0
100%	605.3	336.2	71.7	6.3	0.0

Where, i) Percentage of AV (%), ii) Q_{opp} (vph)

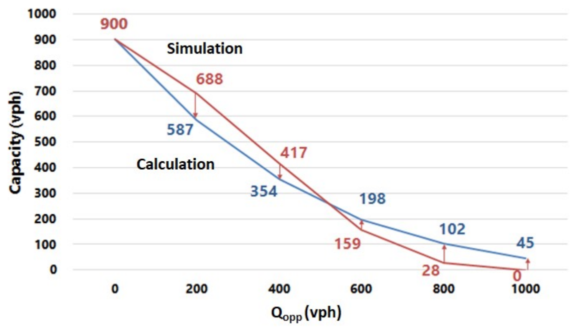


Fig.6 Comparison of calculation method and simulation method

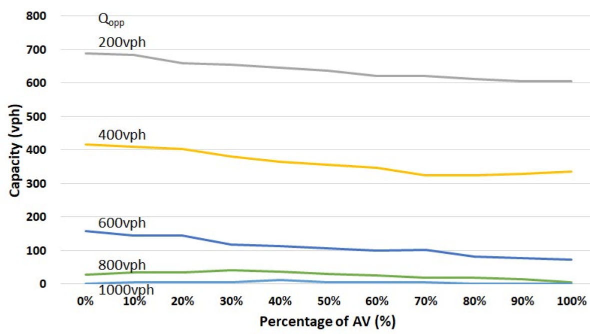


Fig.7 Capacity results of different AV occupancy by simulation method

5. CONCLUSIONS

This paper investigated the capacity of PRT movement at signalized intersections with AV mixed traffic flow.

It was demonstrated that the capacity of PRT movement at signalized intersections can decrease as the increase of percentage of AV, contrary to the common public awareness. In the case of gap acceptance, the vehicle performance of AV and HDV is similar, but AV is not affected by a long waiting time. AV's improvement of reaction time can not make up such decresement of capacity of PRT.

There are still many limitations in this paper. The reaction by human drivers should be considered, particularly for the complicated movement such as PRT movement. When human drivers see some strange behavior of AV, some current conclusions that are suitable for the HDV cases may not be useful for the AV mixted flow.

Some hypothesises in this paper should be reconsidered. The desired speed of AV and HDV should follow some distribution instead of a constant value. The headway distribution of AV mixed traffic flow may change a lot from the case of HDV. The AV's sensitivity of waiting time may not be simply 0. Some vehicle performance of AV such as turning may be different from that of HDV. Besides these limitations, the interactions between AV and pedestrians is not considered in this paper.

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