

OPERATIONAL PERFORMANCE EVALUATION OF FOUR-LEG SIGNALIZED MULTILANE ROUNABOUT

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The installation of signal control at roundabout is considered as one of the most effective ways to improve the capacity and safety performance of roundabout. It has been shown that signalization can manage high traffic flow more efficiently than geometric layout improvements. Thus, researchers have been working on the signalization of roundabout internationally. However, few previous studies focused on the signal controlled roundabout design combined with influence of pedestrians. This study aims to develop a feasible signal phase sequence for both vehicles and pedestrians in roundabouts and compared its operational performance with typical signalized intersection through a case study based on a hypothesized layout and traffic demands. It was found that signalized roundabout outperforms signalized intersections with the phase sequence designed in this paper under traffic conditions with large ratio of turning vehicles or high traffic volumes.

Key Words: signalized roundabout, phase plan, pedestrian, cycle length, capacity, delay

1. INTRODUCTION

In recent years, single lane roundabouts have been built in Japan at intersections with low to medium traffic demands after roundabouts were defined by Road Traffic Law of Japan in 2014 because of its safety performance and little delay. More and more Japanese traffic engineers have begun to focus on the study of roundabout, mainly including the geometric layout improvements of roundabout, capacity and delay of roundabout and so on. However, currently roundabouts can only be installed at rural intersections with low flows of both vehicles and pedestrians in Japan. The main drawback of roundabout is its unbalanced entry flow, which will result in long delays and blocked road under high traffic volume. To solve the congestion of roundabouts, some other countries installed signal control devices at roundabouts to balance entry volume. Ma et al.¹⁾ have shown that signalization improves the capacity of roundabouts, as well as decreases the average delay. It was found that signalized roundabout (SRAB) has better

performance than unsignalized RAB, but few studies compared the SRAB with signalized intersection (SIG) with similar geometric layout of approaches. Besides, although Yang et al.²⁾ have already put forward a relative complete design scheme for signal control of RAB, the effects of pedestrians were ignored.

The objective of this paper is to compare the operational performance of SRAB and SIG with the influence of pedestrians under the same hypothesized geometric layout of approaches and traffic flow rates and turning ratios. However, the size of occupied area is neglected in this paper. The operational performance is evaluated based on the cycle length, capacity and delay of SRAB and SIG.

The remaining paper is structured as follows: Chapter 2 introduces background of SRAB, including reasons for signalization of RAB, geometric features and basic logic of signalization of RAB; Chapter 3 explains methodology of signal timing design and evaluation of its operational performance; Chapter 4 summarizes a case study for typical four-leg multiplane SRAB, which compares

its performance with SIG under various types of traffic conditions, and finally Chapter 5 presents conclusions and relevant future works.

2. BACKGROUND

The United Kingdom has begun to study on signalization of roundabout since 1959³⁾. Signal controlled roundabout has been valued in recent years to be one of possible solutions when traffic conditions have changed after the roundabout's implementation³⁾. There are three main reasons to conduct signal control to RAB. First, signalization can improve the safety of pedestrians and bicyclists. If there are multilane in RAB, the walk distance of pedestrians will be very long and more time is required to clear pedestrians. Under high traffic volume conditions, it is difficult for pedestrians to find acceptable gap to walk across the road and the conflict between vehicles and pedestrians increase dramatically. Second, *Signal Controlled Roundabout manual*⁴⁾ shows that signals decrease the accidents happened at the entrance of roundabout. Without signal control, drivers need to observe an acceptable gap to enter the roundabout, while keeping reasonable spacing with the leading vehicle. Therefore, the risk of rear-end accidents increases and signalization can solve that problem. What's more, after implementation of signal control device to RAB, the capacity of RAB increases significantly, which results in decrease of average delay of vehicles.

Because SRAB is a new research topic in Japan, some basic signal control methods are introduced as follows. The current commonly-used control method is to stop right-turn traffic twice, which eliminates the conflict between right-turn and through movements. **Fig.1** shows a typical layout of SRAB. The first stopline is located at the entry of each approach, while the second stopline is located on the circulatory lanes. The eight stoplines are represented by bold black lines in **Fig.1**. There is a signal control device corresponding to each stopline. The basic control logic is given by two elements. First, different traffic movements use different lanes and are controlled by different signal heads. The inner circulatory lane is only utilized by right-turn vehicles, while the outer circulatory lane is used by through vehicles. Second, right-turn vehicles stop twice when driving through a SRAB. For example, the first and second stoplines for right-turn vehicles coming from east approach are labeled on **Fig.1**.

To give the highest priority to pedestrians and to ensure their safety, in this paper, an exclusive green phase is given to pedestrians on four crosswalks. The

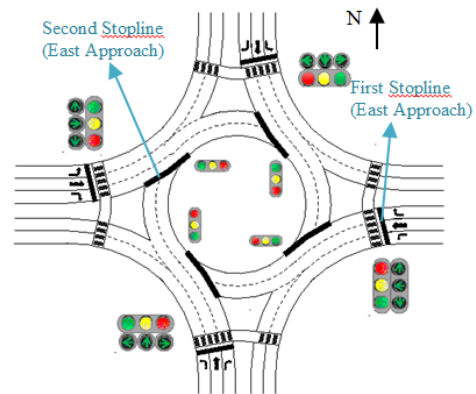


Fig.1 Typical geometric layout of roundabout

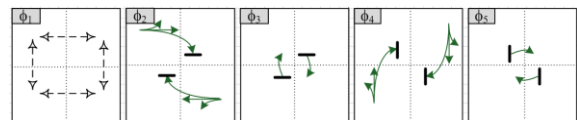


Fig.2 Phase sequence of traffic signal control

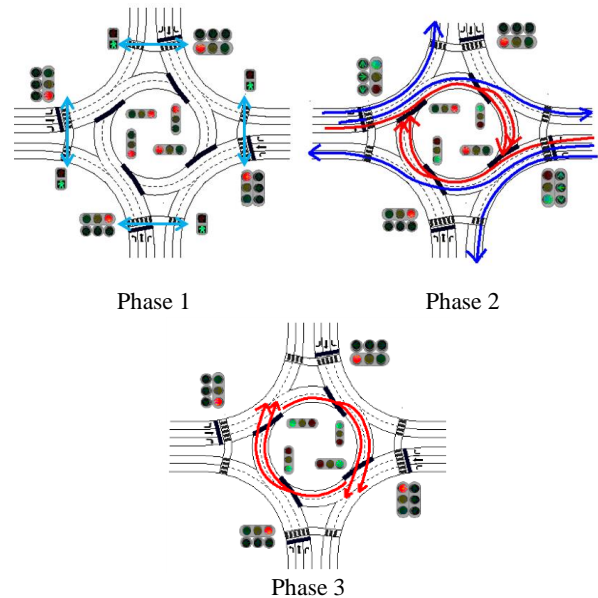


Fig.3 Example of user movements

signal phase plan is shown in **Fig.2**. During phase 1, pedestrians coming from all approaches are released and all vehicles are forced to stop at the first stopline. After a short clearance time for pedestrians, all vehicles queued in east and west approaches are released in phase 2. All right turning vehicles are forced to stop at the second stopline, which not only gives the priority to through vehicles but also eliminates the conflict with opposing right-turn vehicles. Before the queuing area for right-turn vehicles are fully occupied, the green time for right-turn should be ended to avoid the block of roundabout. Phase 3 is applied for clearing the queuing right-turn vehicles before the second stopline. **Fig.3** further explains the phase diagram by drawing the trajectory of each movement in a roundabout. Another thing worth to mention is that there is no clearance time between phase 3 and phase

4. After phase 3, phase 4 and phase 5 are applied for vehicles coming from south and north directions, which are the same with phase 2 and phase 3.

3. METHODOLOGY

(1) Signal timing

Because of the influence of geometric features, the signal timing design for RAB is different with the design for SIG. The followings explain several basic indexes for signalized roundabout.

a) Lost Time

Based on Japan's *Manual on Traffic Signal Control*⁵⁾, when the speed of vehicles is 40km/h, the suggested yellow time is 3 seconds.

All red (AR) time is applied to allow the vehicles that has entered the intersection in the yellow time interval to pass the clearance distance and will not influence the movements of next phase. AR time is calculated by clearance distance dividing average circulatory speed of vehicles.

Based on the *Highway Capacity Manual (HCM)*⁶⁾, total lost time includes start-up lost time l_1 and clearance time l_2 . The default value for l_1 is 2 seconds and l_2 is calculated by Equation (1).

$$l_2 = Y + AR - e \quad (1)$$

Where,

Y : duration of yellow time (s),

AR : duration of all red time (s) and

e : encroachment of vehicles into Y and AR , which is usually taken as 2s.

Therefore, total lost time $L (=l_1+l_2)$ is equal to the sum of length of Y and AR time.

b) Cycle Length

Based on Japan's *Manual on Traffic Signal Control*⁵⁾, minimum cycle length is calculated by Equation (2).

$$C_{min} = \frac{L}{1 - (\lambda/0.9)} \quad (2)$$

Where,

λ : the sum of critical flow ratio for phases 2 and 4.

This means when the phase sequence and total lost time of SRAB are determined, the minimum cycle length is also fixed, since the exclusive pedestrian phase is applied in this study.

The storage area for right-turn vehicles on the circulatory lanes is one influence factor for the design of RAB. In Fig.4, a trajectory of right-turn vehicles coming from east approach is shown and separated in three parts. Part I and part II are the storage area on circulatory lanes for right-turn vehicles in one cycle. In part I (blue), both through lanes and right-turn lanes can be utilized to store

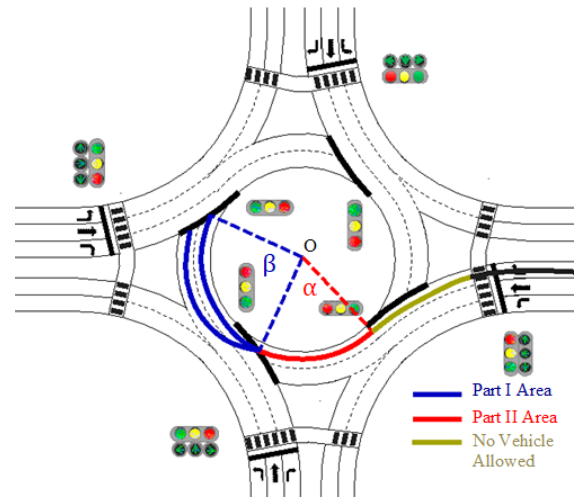


Fig.4 Storage area for right-turn vehicles

right-turn vehicles during phase 2. The average length of all available circulatory lanes is measured to be the length of part I area. In part II (red), only inner circulatory lane can be utilized by right-turn vehicles, while the outer circulatory lane is used by through vehicles. However, vehicles are not allowed to stand on the light green part III area. Vehicles standing in part III will block the queuing right-turn vehicles coming from opposite direction at the beginning of phase 3, which results in extra lost time of vehicles. Based on Fig.4, total length of storage area is calculated by Equation (3).

$$L_Q = \frac{\pi\alpha}{180}(r + 0.5w) + 2 * \frac{\pi\beta}{180}\left(r + \frac{n}{2}w\right) \quad (3)$$

Where,

L_Q : maximum queue length on circulatory lanes (m),

α, β : corresponding angle of part I and II (deg),

r : radius of central island (m),

w : width of one lane (m) and

n : total number of circulatory lanes.

Therefore, capacity of right-turn vehicles at the second stopline in one cycle c_r is calculated by maximum queue length L_Q divided by the average spacing of vehicles at standstill $l (=1/\text{jam density})$, which is shown in Equation (4).

$$c_r = L_Q/l \quad (4)$$

It is important to note that to prevent oversaturated condition, the number of coming right-turn vehicles during phase 2 cannot be larger than the capacity of storage area, which means $c_r \geq q_r * C/3600$, where q_r is the flow rate of right-turn vehicles. From this constraint, the maximum acceptable cycle length is $C_{max} \leq 3600c_r/q_r$.

c) Effective green time

Effective Green time for right-turn vehicles at second stopline is determined by the number of queuing vehicles in the storage area. Here,

minimum green time for queue clearance is considered.

As mentioned above, the number of right-turn vehicles coming in one cycle is $q_r * C/3600$. *Signal Timing Manual*⁷⁾ suggests that by assuming 3 seconds for start-up lost time and 2 seconds for each vehicle crossing intersection in average, the minimum green time based on queue clearance theory is calculated by Equation (5).

$$G_q = 3 + 2n \quad (5)$$

Where,

G_q : minimum green time for queue clearance (s) and
 n : number of vehicles in queue.

By adding the effective green time for phase 3 and phase 5, a new value of cycle length will be obtained. Therefore, number of right-turn vehicles coming in a cycle will be increased and minimum green time for phase 3 and phase 5 will also be changed. The iteration can be ended until the number of coming right-turn vehicles is not changed, which means the coordination between cycle length and green time G_q has been achieved.

Then, effective green time for phase 2 and phase 4 can be calculated by Equation (6).

$$G = (C - L) \frac{\max(V_l/S_l, V_t/S_t, V_r/S_r)}{\lambda} \quad (6)$$

Where,

C : cycle length (s),

V_l, V_t, V_r : traffic volume of left-turn, through and right-turn vehicles (pcu/hr) and

S_l, S_t, S_r : saturation flow rate of left-turn, through and right-turn lanes (pcu/hr).

Besides, an exclusive green time for pedestrians should be determined based on the length of crosswalk. Assumed the average speed of pedestrians is 1.0m/s, minimum green time for pedestrians is calculated by the maximum length of crosswalk of four approaches dividing the average walking speed. In Japan, a minimum flashing green time should be given to pedestrians, which is time required by pedestrians to walk a half-length crosswalk. In this paper, 20 seconds are given to pedestrians and half of it is designed as flashing green time. Besides, extra 3 seconds are given for clearing pedestrians.

(2) Evaluation of operational performance

To compare the operational performance of SRAB with SIG, capacity and delay are calculated as evaluated indexes. The calculation of capacity and delay are summarized as follows.

a) Capacity

The operational mechanism of SRAB is similar with SIG, thus equations of capacity and delay for SIG can also be used for SRAB. The capacity for

vehicles of each movement can be calculated in HCM 6th Edition⁶⁾ by Equation (7).

$$c_i = S_i \frac{G_i}{C} \quad (7)$$

Where,

c_i : capacity of through vehicles for approach i .

For right-turn vehicles, the capacity is minimum value between the limited capacity for storage area and the capacity calculated based on HCM shown in Equation (8).

$$c_i^r = \min\left(S_i^r \frac{G_i^r}{C}, c_r \frac{3600}{C}\right) \quad (8)$$

b) Delay

For average delay at the first stopline, this paper only considers uniform delay d_1 by assuming uniform arrival, which is calculated by Equation (9) in HCM.

$$d_1 = \frac{0.5C \left(1 - \frac{G_i}{C}\right)^2}{1 - [\min(1, X) G_i/C]} \quad (9)$$

Where,

d_1 : average delay at the first stop line (s) and

X : degree of saturation.

For average delay at the second stop line, only right-turn vehicles suffer an extra delay. The delay can be calculated by using offset between first and second stopline minus average travel time of right-turn vehicles between first and second stopline, which is shown as Equation (10).

$$d_2 = offset - \frac{l_{12}}{v} \quad (10)$$

Where,

d_2 : average delay at the second stop line (s),

l_{12} : length of trajectory between first and second stopline (m) and

v : average circulatory speed of vehicles in roundabout (m/s).

Besides, geometric delay should also be considered for roundabouts. Geometric delay is caused by the layout of RAB. It is defined as the time difference between time used by vehicle driving through the points where deceleration of vehicle begins and acceleration ends, and time used by vehicle driving through those points in the absence of the junction. In this paper, to simplify process of calculation, geometric delay of roundabout is calculated by vehicle's average travel time for driving through roundabout minus travel time for same movement in SIG.

4. CASE STUDY

(1) Basic components of case study

To make a relative fairly comparison between the operational performance of SRAB and SIG, similar geometric layout of approaches and phase sequence for SRAB and SIG are hypothesized. The geometric layout of SRAB and SIG designed for this case study are illustrated in Fig.5 and Fig.6, which have the same lane configuration of approaches. Both SRAB and SIG have three entry lanes and two exit lanes at each approach. The width of lane is 3.25m and the margin is designed as 3.75m. Each entry lane is utilized by different traffic movements. Besides, there are two circulatory lanes in SRAB, inner one is set for right-turn vehicles and outer one is set for through vehicles. The design for SRAB is based on the *Roundabouts: An Informational Guide*⁸⁾.

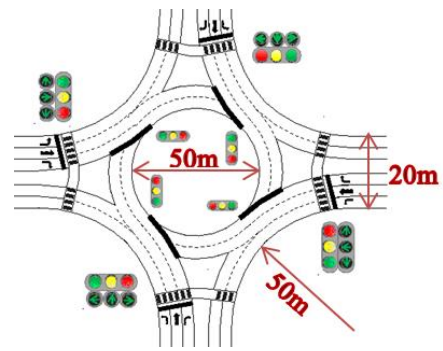


Fig.5 Geometric layout of SRAB

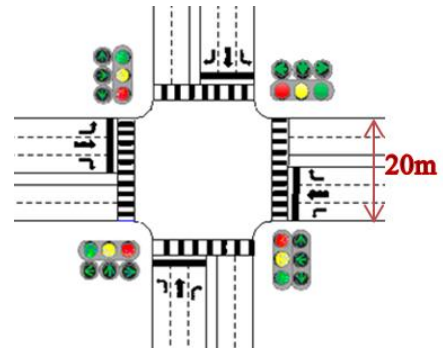


Fig.6 Geometric layout of SIG

(2) Signal setting

Four scenarios of traffic flow rates are hypothesized and analyzed in this paper. To simplify the process of calculation, the same values of traffic flow rate are given to left-turn, through and right-turn vehicles of each approach. Table 1 summarizes saturation flow rates and hypothesized traffic flow rates. Based on the lane configuration, saturation flow rates of each lane are determined and same values are used for both SRAB and SIG. The signal timing diagram designed for SRAB and SIG are shown in Tables 2 and 3.

SRAB and SIG have the same value of exclusive pedestrian phase, since they have the same length of crosswalks. As shown in Table 2, total lost time in one cycle is 39 (=20+3+3+4+3+4+2) seconds for SRAB. Where, 20 seconds of exclusive pedestrian phase and 3 seconds of AR for clearance of pedestrians are applied. 3 seconds of yellow and 4 seconds of AR are required in phase 2, because of the conflict between right-turn vehicles and through vehicles that coming from the opposite directions. There is no clearance time between phase 3 and phase 4, because there is no conflict of movements in those two phases. Besides, 2 seconds of AR is set at the end of cycle to clear all vehicles in the roundabout, which ensures safety of pedestrians. In the case of SIG in Table 3, total lost time is 47 (=20+3+4*(3+3)) seconds which includes 20 seconds of exclusive pedestrian phase and 3 seconds of yellow and 3 seconds of AR are set after each phase for clearance of vehicles.

Therefore, based on Equation (2) for estimating cycle length, SIG has a longer cycle length than SRAB under same traffic conditions due to longer lost time. The longer cycle length, the longer effective green time allocated to each phase, but also

Table 1 Base settings of case study

		Left-turn	Through	Right-turn	Ratio
Saturation flow rate (pcu/hr)		1800	2000	1800	-
Traffic flow rate (pcu/hr)	Scenario 1	200	200	200	1:1:1
	Scenario 2	200	200	400	1:1:2
	Scenario 3	400	200	200	2:1:1
	Scenario 4	400	400	400	2:2:2

Table 2 Signal timing for SRAB

Movement	Signal phasing (sec)												Cycle length (sec)
	φ ₁			φ ₂			φ ₃		φ ₄		φ ₅		
	1	2	3	4	5	6	7	8	9	10	11	12	
φ ₁ Pedestrian	[Green bar]												96
φ ₂ Vehicle (1st stopline)	[Green bar]												
φ ₃ Right-turn (2nd stopline)	[Green bar]												
φ ₄ Vehicle (1st stopline)	[Green bar]												
φ ₅ Right-turn (2nd stopline)	[Green bar]												
Scenario 1 Vehicle movements	-	-	-	17	3	4	8	17	3	4	8	2	160
Scenario 1 Pedestrians	15	15	3	-	-	-	-	-	-	-	-	-	
Scenario 2 Vehicle movements	-	-	-	36	3	4	21	36	3	4	21	2	110
Scenario 2 Pedestrians	15	15	3	-	-	-	-	-	-	-	-	-	
Scenario 3 Vehicle movements	-	-	-	23	3	4	9	23	3	4	9	2	160
Scenario 3 Pedestrians	15	15	3	-	-	-	-	-	-	-	-	-	
Scenario 4 Vehicle movements	-	-	-	36	3	4	21	36	3	4	21	2	160
Scenario 4 Pedestrians	15	15	3	-	-	-	-	-	-	-	-	-	
Signal phase sequence	[Diagram showing phase sequence]												

— Green ⚡ Amber — Red ||||| Pedestrian flashing green

the longer the waiting time for each vehicle. For instance, in scenario 2, when traffic flow of right-turn vehicles is higher, more green time is given to phases 3 and 5 of SIG, which results in a longer delay for through and left-turn vehicles in phases 2 and 4. While in SRAB, right-turn vehicles can move with through and left-turn vehicles during phases 2 and 4,

and can arrive at the second stopline to wait for departure, which reduces the clearance time for right-turn vehicles. Therefore, the average delays of through and left-turn vehicles in SRAB will be smaller than SIG.

(3) Results

The results of capacity and average delay for both SRAB and SIG under each scenario of traffic flow rate are shown in **Table 4**. To have a clear tendency of delay change, bar graphs of capacity and average delays are drawn in **Fig.7 and Fig.8**, respectively.

When the traffic flow rates for each movement are equal and at medium level, SIG has better performance than SRAB with larger capacity and smaller delay. Since the traffic demand is not high, right-turn vehicles can easily find a gap to cross the intersection between opposing through vehicles. While in SRAB, right-turn vehicles are stopped twice, and exclusive green phase is given to right-turn vehicles in phase 3, which results in increasing of delay of all movements.

In scenario 2, when flow rate of right-turn increased to double, SRAB works better than SIG with larger capacity of left-turn and through vehicles and smaller average delay. In SIG, if traffic demand of right-turn vehicles is very high, more green time should be given to right-turn vehicles, so that capacity of through and left-turn is smaller.

In scenario 3, flow ratio of left-turn vehicles is increased twice and SRAB still works better than SIG. Similar for SIG, more green time are given to through and left-turn vehicles, which increases the delay of right-turn vehicles. While in SRAB, during phase 2 and phase 4, through and turning vehicles can move at the same time, which decreases the total average delay.

In scenario 4, when traffic flow rates of all movements increase to a high level, SIG cannot afford the demand and will breakdown under this phase sequence. For SRAB, although the degree of saturation is very high for all movements, it is still under saturation and can serve the demand.

In conclusion, SRAB is outperformed SIG under the phase sequence designed in this paper under the conditions with large ratio of turning vehicles and high traffic volume.

5. CONCLUSION

This paper develops a method for designing one type of signal phase sequence with exclusive pedestrian phase and for determining the length of each signal phase. The evaluated factors, including cycle length, capacity and delay were calculated. A

Table 3. Signal timing for SIG

Movement	Signal phasing (sec)															Cycle length (sec)	
	φ ₁	1	2	3	4	5	6	7	8	9	10	11	12	13	14		15
φ ₁ Pedestrian	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	95
φ ₂ Through & Left-turn	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	185
φ ₃ Right-turn	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	185
φ ₄ Through & Left-turn	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	185
φ ₅ Right-turn	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	185
Scenario 1	Vehicle movements	-	12	3	3	12	3	3	12	3	3	12	3	3	12	3	3
Scenario 1	Pedestrians	10	10	3	-	-	-	-	-	-	-	-	-	-	-	-	-
Scenario 2	Vehicle movements	-	23	3	3	46	3	3	23	3	3	46	3	3	46	3	3
Scenario 2	Pedestrians	10	10	3	-	-	-	-	-	-	-	-	-	-	-	-	-
Scenario 3	Vehicle movements	-	46	3	3	23	3	3	46	3	3	23	3	3	46	3	3
Scenario 3	Pedestrians	10	10	3	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 4. Results of capacity and delay for SRAB and SIG

Scenario	Ratio of LT:TH:RT	Movement	SRAB		SIG	
			Capacity (vph)	Delay (s)	Capacity (vph)	Delay (s)
1	1:1:1	Left	260	36.9	237	39.4
		Thr	289	42.4	263	38.9
		Right	260	53.0	237	39.4
		Mean	-	44.2	-	39.2
2	1:1:2	Left	432	48.9	227	77.9
		Thr	480	54.3	252	76.9
		Right	432	63.4	454	65.1
		Mean	-	57.5	-	73.3
3	2:1:1	Left	365	43.6	454	65.1
		Thr	405	44.3	505	56.3
		Right	365	61.2	227	77.9
		Mean	-	48.2	-	66.4
4	2:2:2	Left	432	56.09	-	-
		Thr	480	60.53	-	-
		Right	432	63.59	-	-
		Mean	-	60.1	-	-

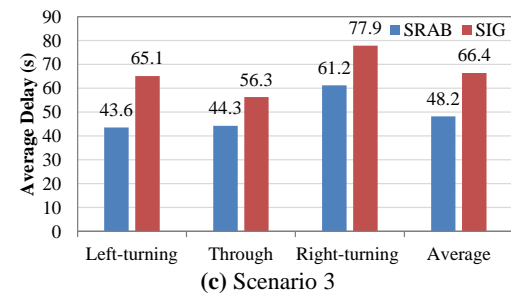
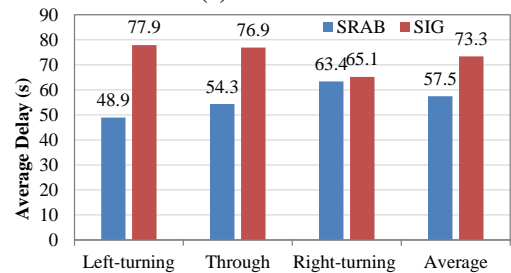
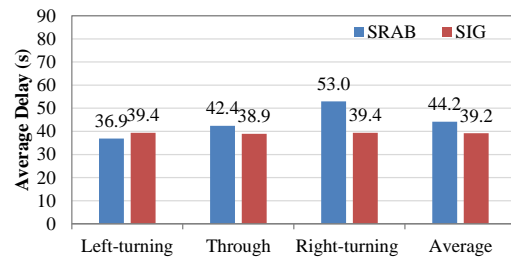


Fig. 7 Bar graph of average delays

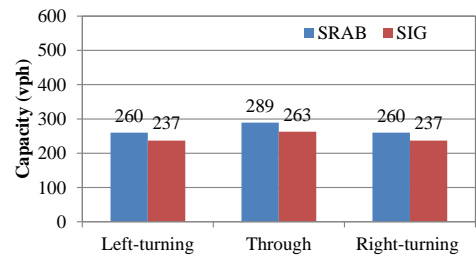
case study for typical four-leg multilane roundabout and intersection was conducted.

By designing an intersection as SRAB and compare its operational performance with traditional SIG without considering the influence of geometric factors, signalized roundabout has been proved as a feasible choice for design of road junction. It has better performance than SIG for traffic demand with large ratio of right-turn vehicles, large ratio of left-turn vehicles, or large traffic flow rates for all approaches. Because of geometric features of SRAB, it will eliminate some clearance time between phases, which increases the percentage of effective green time utilized by vehicles. With smaller lost time, cycle length of SRAB designed under the same traffic demand will be smaller than SIG. Furthermore, capacity of SRAB will be increased and average delay of both vehicles and pedestrians will be decreased. Besides, by separating the pedestrians and vehicles, right-turn vehicles and through vehicles, the safety of intersection is improved significantly.

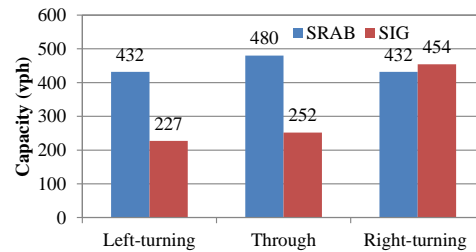
Future work should focus on different feasible signal phase sequences and their advantages and disadvantages compared with SIG. More schemes designed for reducing the green time of pedestrians and decreasing the average delay of both pedestrians and vehicles should be studied, for example, treating the spare space on splitter island as waiting area for pedestrians and utilizing two stage signal phases.

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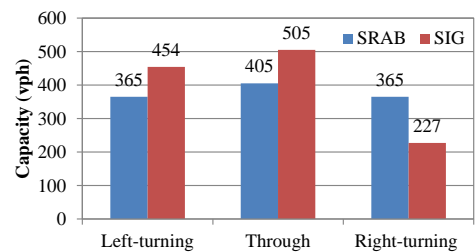
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(a) Scenario 1



(b) Scenario 2



(c) Scenario 3

Fig. 8 Bar graph of capacity

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