Link as an aggregate alternative: a continuous recursive logit representation of pedestrian behavior

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The objective of this study is to propose a new model of mode, route, and path choice behavior of pedestrians under the recursive logit modeling framework which follows the random utility maximization theory. A main issue tackled in this paper is about how to handle the fact that paths for pedestrians are essentially continuous in space and thus the path enumeration is almost impossible, making difficult to develop a consistent measure of economic welfare for both "route" choice behavior in a network and "path" choice behavior in a continuous space. To overcome this issue, we consider each pedestrian link as an aggregate alternative of infinite pedestrian paths, where the link utility is defined as the expected maximum utility of all possible pedestrian paths. Since the choice set generation problem in the discrete choice context becomes the problem of specifying an appropriate probability density function in the continuous choice context, we introduce the path density function which is constructed from a *primitive* free-flow pedestrian behavior model in a continuous space, where geometric conditions of the link are taken into account. We show an empirical strategy to estimate the parameters in the proposed model, and confirm the feasibility of the proposed model through a numerical study.

Key Words : *pedestrian behavior, aggregate alternative, recursive logit, continuous space, path density function*

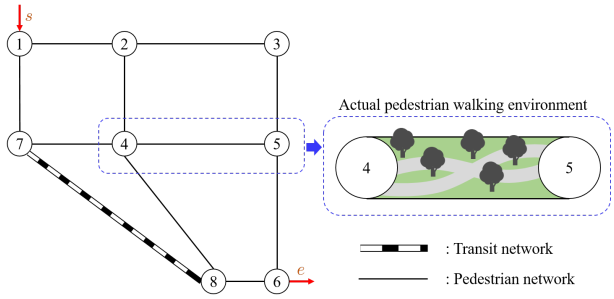
1. Introduction

Understanding pedestrian behavior is essential in the planning of transit stations and city centers. Among others, evaluating the impacts of the walking environment on route choice and mode choice decisions for short-distance trips would particularly be important, considering the health benefits and vitality of the city. In modeling pedestrian behavior, the conventional "shortest travel time rule" may not be dominant. Instead of travel time, the walking environment such as shade from trees would be a major influential factor for pedestrians [1)], while its evaluation is challenging technically because paths for pedestrians are essentially continuous in space. Although the network representation could be used as an approximation of the actual space [2)], this could oversimplify the actual walking environment. Given such a situation, a number of pedestrian models have been proposed, including a continuous Markovian equilibrium model [3)], a discrete choice model of speed and direction [4)], a social forces model [5)], and a cellular automata microsimulation model [6)]. These models mainly focus on movements in a particular pedestrian space to create optimal space designs and to meet safety concerns, and thus it is essential to have a better representation of pedestrian behavior in a crowd, where behavioral constraints are induced by interaction with other individuals nearby. On the other hand, these models lack a microeconomic foundation in the sense that consumer surplus would not be able to be derived directly from the models, preventing the use of the models for urban and transport project evaluations where the expected benefits need to be estimated and compared across different policy options. In particular, to the authors' knowledge, there is no model that can be consistently used to evaluate (1) the benefits obtained from transport network improvements which is measured mainly by travel time reduction, and (2) the benefits obtained from the improvement of walking environment such as planting trees. This study attempts to fill in such a gap.

The discrete choice model has been widely used for calculating the consumer surplus from transport investments [7]. Recently, Fosgerau et al. [8] further proposes the recursive logit model which can model route choice behavior in a network without route enumeration, but it is still consistent with the utility maximization framework and thus the consumer surplus can be directly calculated from the model [9]. The objective of this study is to extend the recursive logit model in the way that the impacts of walking environment on pedestrian behavior in a continuous space are taken into account in the model, while it is still consistent with the utility maximization framework. To achieve it, we consider a pedestrian link as an aggregate alternative of infinite paths in a continuous space, which can be seen as one type of continuous logit [10,11,12]. To operationalize the model, we utilize the idea that the continuous version of choice set generation problem is about the specification of the probability density function. We also provide an empirical framework to estimate the parameters in the model with observed data, and show the practical applicability of the proposed model through a numerical study. We also confirm that the proposed model is the generalization of an existing pedestrian route choice model which consider walking environment variables as attributes of the link. Note that, in this paper, to avoid the confusion, the term *path* will be used to indicate a choice alternative within a certain link, and the term *route* will be used to indicate a choice alternative in the network (i.e., a sequence of links will be a route).

2. The proposed model

An example of the multi-modal network considered in this study is shown in **図–1**. In this network, given origin o and destination d, mode and route choices are simultaneously represented. A sequence of nodes {1,7,8,6} indicates a traveler chooses transit mode, while other sequences of nodes, such as {1,2,4,8,6} and {1,2,3,5,6}, indicate the choice of walking with different routes.



図–1 An example of the multi-modal network for short-distance trip (o: origin link; d: destination link)

A major difference between transit and pedestrian links is that each pedestrian link contains multiple paths in a continuous space with different attractiveness factors. For example, arcade street (**図–2**) protects pedestrians from rain and traffic accident. Riverside pedestrian path (**図–3**) with trees would protect pedestrians from sun while enjoying the scenery. Those will bring certain utilities for pedestrians, affecting their route/path choice decisions. Path utility can vary across paths on the link. For example, paths passing in front of shops could provide the higher utility since pedestrians can enjoy window shopping, compared to other paths on the same link. In a similar way, a path with more shade would provide the higher utility during the summer time compared to other paths.

The above mentioned benefits come largely from public investments, while such benefits are not well evaluated compared to other well-known benefits such as travel time reduction. We believe that this is mainly due to the lack of appropriate methodologies that allow policy makers, for example, to directly compare the benefits from planting trees in pedestrian paths with the benefits from public transit travel time reduction. This study attempts to fill in this gap by developing a model of path, route and mode choice behavior under the utility maximization framework that provides a consistent measure of the change in consumer surplus across different transport investments at different scale.

In Subsection (1), we introduce a route and mode choice model in a network which follows the conventional recursive logit model proposed by Fosgerau et al [8]. Subsection (2) introduces a path choice model on a pedestrian link which has a continuous space. We show that the conventional network-based pedes-

図–2 Arcade street (Hiroshima city, Japan)

図–3 Riverside pedestrian path (Hiroshima city, Japan)

trian route choice model is a special case of the path choice model in a continuous space. We then show that, while the path choice model can be considered as one type of continuous logit model, it brings a new challenging issue that is the integration of all possible paths on a link. To overcome this issue, Subsection (3) introduces the concepts of *path density function* and *attractiveness density function*. The path density function represents the number of paths going through a particular point on a pedestrian link. It reflects the geometric features of the pedestrian space, e.g., walkers cannot go through the points where trees are planted. The attractiveness density function gives the attractiveness level of each point, e.g., the shaded area made by tree will increase the attractiveness of passing through there. Assumption involved here is that point attractiveness is irrelevant to the trajectory of path. Note that It could also reflect social norms under the proposed framework. For example, left-hand side of the path may have higher attractiveness if "keep to the left" is considered acceptable in a society as an unwritten rule of behavior or social norm. We then show that such density-based model specification allows for empirical estimation of parameters. Since both route choice behavior and path choice behavior are consistently modeled under the utility maximization framework, the consumer surplus can be directly calculated through logsum as shown in Subsection (5).

(1) Route choice model in a network

We adopt the recursive logit to model route and mode choice behavior in a multi-modal network [8),13),14)]. The recursive logit model is one type of dynamic discrete choice model where the route choice problem is formulated as a sequence of link choices, which follows the random utility maximization framework. Suppose that an individual n is traveling from origin o to destination d. In the recursive logit model, at each link k, a traveler is assumed to choose a next link a from the set of outgoing links from the current state k, $A(k)$. The random utility choosing link a, $u_n^d(a|k)$, is defined as

$$u_n^d(a|k) = \tilde{u}_n(a|k) + V_n^d(a) \tag{1}$$

where $\tilde{u}_n(a|k)$ is instantaneous random utility which is defined as $v_n(a|k) + \mu\epsilon_n(a)$. Here $v_n(a|k)$ is the deterministic utility of a link a conditional on being in state k, $\epsilon_n(a)$ is random component of utility (i.i.d. extreme value type 1), and μ is scale parameter for $\epsilon_n(a)$. $V_n^d(a)$ is value function (the expected maximum utility from a link a to destination d) which will be obtained via Bellman equation as follows:

$$V_n^d(k) = E\left[\max_{a\in A(k)}\Big(v_n(a|k) + V_n^d(a) + \mu\epsilon_n(a)\Big)\right] \tag{2}$$

Since $\epsilon_n(a)$ follows the Gumbel distribution, $V_n^d(k)$ is the logsum defined as $\mu \ln \sum_{a\in A(k)} \exp((v_n(a|k) + V_n^d(a))/\mu)$. Note that $V_n^d(d)$=0 since there is no outgoing link from d. The link choice probability corresponding to the link utility given in eq. (1) can be written as

$$P_n(a|k) = \frac{\exp\left(\frac{1}{\mu}\Big(v_n(a|k) + V_n^d(a)\Big)\right)}{\sum_{a'\in A(k)} \exp\left(\frac{1}{\mu}\Big(v_n(a'|k) + V_n^d(a')\Big)\right)} \tag{3}$$

Since a path σ_n is the sequence of links, i.e., $\sigma_n = \{k_i\}_{i=0}^{I_n}$, the path choice probability can be defined as

$$P_n(\sigma_n) = \prod_{i=0}^{I_n-1} P_n(k_{i+1}|k_i) = \frac{\prod_{i=0}^{I_n-1} \exp\left(\frac{1}{\mu}v_n(k_{i+1}|k_i)\right)}{\exp\left(\frac{1}{\mu}V_n^d(k_0)\right)} \tag{4}$$

where k_0 is the origin link. Note that $V_n^d(k_0)$ can be efficiently derived through solving a system of linear equations $\boldsymbol{z} = \boldsymbol{M}\boldsymbol{z} + \boldsymbol{b}$, where $\boldsymbol{z}_k = \exp(V_n^d(k)/\mu)$,

$\boldsymbol{M}_{ka} = \delta(a|k)\exp(v_n(a|k)/\mu)$ where $\delta(a|k)$ is equal to one if link k and link a are directly connected and 0 otherwise, and $b_k = 0$ if $k \neq d$ and $b_d = 1$. See Fosgerau et al. [8] for details.

In terms of model structure, the recursive logit model is equivalent to the Markovian assignment model [15),16)]. Fosgerau et al [8] shows that the Markovian model can be defined as a dynamic discrete choice model and the parameters can be empirically estimated by using structural estimation techniques [17]. Furthermore, recent studies show that a large-scale network can also be handled under the recursive logit modeling framework within a feasible computation time [18),19)], indicating that increasing network size caused by the extension to the multi-modal network would not be a problem.

(2) Path choice model in a continuous space

In order to construct the pedestrian link from a set of paths in a manner consistent with the utility maximization framework, this section develops the path choice model under the continuous logit framework. A random utility obtained from a path r on a link a is defined as $\tilde{u}(r|a) = v_r + \mu'\epsilon_r$, where v_r is the deterministic utility of path r on a link a, and $\epsilon_n(a)$ is random component of utility (i.i.d. extreme value type 1) with the scale parameter μ'. Note that we omit the notations of n and d for simplicity. Assume also that a traveler will choose a path r from a set of all possible paths R_a, which maximizes his/her utility. Then, the choice probability of path r on a link a is defined as

$$P(r|a) = \frac{\exp(\frac{1}{\mu'}v_r)}{\int_{r\in R_a}\exp(\frac{1}{\mu'}v_r)dr} \tag{5}$$

where dr indicates integration over all paths. The exact solution of the integration may not be able to be obtained, but approximate estimation can be made as we will discussed in Subsection (3).

Since link is an aggregate alternative of paths, now the instantaneous utility of the link appeared in eq. (1) can be rewritten as

$$\tilde{u}(a|k) = E\big[\max_{r\in R_a}(v_r + \mu'\epsilon_r)\big] + \mu\epsilon(a) \tag{6}$$

Depending on the specification of $E\big[\max_{r\in R_a}(v_r + \mu'\epsilon_r)\big]$, the path choice model can be either deterministic or stochastic as shown below.

a) Deterministic path choice

When $\mu' \to 0$, path choice becomes deterministic. In this case, a path producing the maximum utility will represent the utility of the link, i.e.,

$$\tilde{u}(a|k) = v_{r^*} + \mu\epsilon(a) \tag{7}$$

where v_{r^*} is the largest systematic path utility for $r \in R_a$. While such deterministic path choice assumption is convenient since the network-based modeling approach can be directly used, the impacts of walking environment on pedestrian behavior may not be well captured since the link utility depends solely on the attributes of a particular path and the attributes of the rest of paths do not affect the link utility.

b) Stochastic path choice

When $\mu' \gg 0$, the path choice becomes stochastic. The instantaneous utility can be defined as:

$$\tilde{u}(a|k) = \tilde{v}(a|k) + \mu'\ln\int_{r\in R_a}\exp\left(\frac{v'_r}{\mu'}\right)dr + \mu\epsilon(a) \tag{8}$$

where $v'_r = v_r - \tilde{v}(a|k)$. The first term represents the link-level attributes, and the second term reflects the utility gained form the walking environment on link a through v'_r.

In the case that no path attribute is available, the instantaneous utility of link a can be written as:

$$\tilde{u}(a|k) = \tilde{v}(a|k) + \mu'\ln M_a + \mu\epsilon(a) \tag{9}$$

where $M_a = \int_{r\in R_a} 1dr$, since v_r for all r will be represented by $\tilde{v}(a|k)$ and thus $v'_r = 0$. M_a is a measure of the size of aggregate alternative (i.e., link) [7].

(3) Empirical strategy

The path choice model defined in eq. (5) can be seen as one type of continuous logit model [10),11),12)]. The continuous logit model has been applied for location choice behavior and departure time choice behavior, but, to the author's knowledge, no application has been made for path choice behavior. The application to path choice brings a new challenging issue, which is analogous to the choice set generation problem in the discrete route choice model. Concretely speaking, the continuous version of choice set generation problem is about the specification of the probability density function (PDF), but the plausible PDF for path choice is not easy to construct compared to that for location choice and departure time choice (the density estimated from the actual facility distribution can be used for location choice [10], and a uniform distribution [density estimated from timetable] can be applied for departure time choice for car [public transit] users [12]). Related to this, the interval of integration corresponds to the spatial boundary and temporal boundary for location and departure time choices respectively and thus the choice set can be clearly defined, but such boundary for path choice is not clear. In modeling the discrete route choice model, the size of the choice set is inherently infinity when we allow cyclic paths/routes. Similarly, the PDF for path choice would also depend on how we assume *possible* pedestrian paths. For example, if we assume that pedestrians can go round and round the same place, the corresponding PDF can be quite large.

Our strategy to solve this problem is similar with the one used in the recursive logit model. In the recursive logit model, as we mentioned above, the choice generation problem is solved by using a system of linear equations which is analogous to the spatial autoregressive process appeared in spatial econometrics to find the steady state [20]. In a similar manner, for the continuous case, instead of enumerating all possible paths, a stationary distribution can be used as a PDF. We generate such a PDF from a *primitive* free-flow pedestrian behavior model where only geometric conditions are taken into account. All other factors affecting pedestrian behavior (such as shade from trees and window shopping) are reflected in the attractiveness density function defined below.

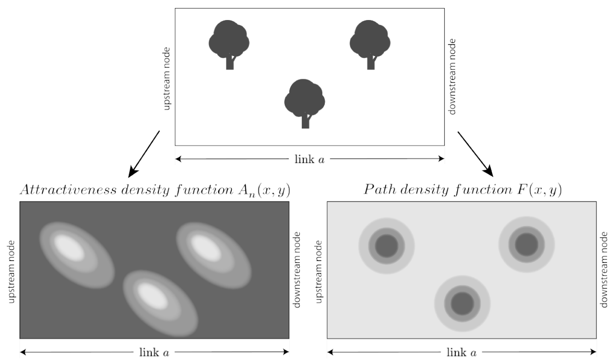
To introduce the PDF, we first consider each path is a sequence of dots, i.e., $r = \{(x_l, y_l)\}_{l=0}^{L}$, where x_l specifies the point in the direction of forward movement, y_l specifies the vertical direction from x_l, (x_0, y_0) represents a point at the upstream node, and (x_L, y_L) represents a point at the downstream node. A sequence of dots will be a path when $L \to \infty$. We assume that path utility can be decomposed into a set of point (or dot) utilities where a point utility is assumed to be independent from others, i.e., $v'_r = \sum_{(x_l,y_l)\in r} v'(x_l, y_l)$. Given the above decomposition, eq. (5) can be rewritten as

$$P(r|a) = \frac{\int_{x\in r}\int_{y\in r} A(x,y)dydx}{\int_{x\in R_a}\int_{y\in R_a} A(x,y)F(x,y)dydx} \tag{10}$$

where $F(x, y)$ is called *path density function* and $A(x, y) = \exp(v'(x, y)/\mu')$ is called *attractiveness density function*. The former indicates the number of paths going through a point (x, y), while the latter indicates the attractiveness at a point (x, y). 図–**4** illustrates an example of these two density functions. The advantage of this decomposition is that parameters in the model can be empirically estimated when pedestrian path choice results are observed for example by video recordings [4]. Specifically, we first discretize the space in eq. (10) into sufficiently small grid cells as follows:

$$P(r|a) = \frac{\sum_{(x_l,y_l)\in r} A(x_l, y_l)}{\sum_{x_l\in R_a}\sum_{y_l\in R_a} A(x_l, y_l)F(x_l, y_l)} \tag{11}$$

and then parameterize $v'(x_l, y_l)$ as $\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{z}(x_l, y_l)$, where $\boldsymbol{\beta}$ is a vector of parameters to be estimated, and $\boldsymbol{z}(x_l, y_l)$ is a vector of observed attractiveness factors at point (x_l, y_l). For the path density function, we use a stationary distribution obtained under a certain primitive free-flow pedestrian behavior rule, for example, giving simple transition probabilities in the 2-dimensional grid space which sorely depend on the existence of obstacles in the pedestrian space. Note that this is the continuous version of (implicit) choice set generation. In principle, any kind of paths can be in the choice set, including cyclic paths. The possible specifications of path density function will be explained in details in the next section.



図–**4** Decomposition of walking environment into attractiveness density function $A(x, y)$ and path density function $F(x, y)$ [lighter color indicates higher density]

(4) Likelihood functions

Since the computational burden is relatively high for estimating the recursive logit model, here we consider the sequential model estimation process. First, we estimate the path choice model. The corresponding log-likelihood function for the path choice on a link a

is

$$LL_{path}(\boldsymbol{\beta'}|r) = \ln \prod_{n=1}^{N} P(r|a) =$$

$$\ln \prod_{n=1}^{N} \frac{\sum_{(x_l,y_l)\in r_n} \exp(\boldsymbol{\beta'}^{\mathrm{T}} \boldsymbol{z}(x_l, y_l))}{\sum_{x_l \in R_a} \sum_{y_l \in R_a} \left\{ \exp(\boldsymbol{\beta'}^{\mathrm{T}} \boldsymbol{z}(x_l, y_l)) F(x_l, y_l) \right\}} \tag{12}$$

where $\boldsymbol{\beta'} = \boldsymbol{\beta}/\mu'$ for normalization. Since this log-likelihood function is a natural extention of the log-likelihood function of the conventional logit model, we can use the conventional estimation procedure such as quasi-Newton method. We denote the estimated parameters as $\hat{\boldsymbol{\beta}}'$.

For the route choice model, we first parameterize the link-level systematic utility $\tilde{v}_n(k_{i+1}|k_i) = \boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{w}(k_{i+1}|k_i)$, where $\boldsymbol{\alpha}$ is a vector of parameters to be estimated, and $\boldsymbol{w}(k_{i+1}|k_i)$ is a vector of explanatory variables representing link level attributes such as average travel time. Then, the corresponding log-likelihood function for the route choice model is

$$LL_{route}(\boldsymbol{\alpha}|\sigma) = \ln \prod_{n=1}^{N} P_n(\sigma_n)$$

$$= \ln \prod_{n=1}^{N} \frac{\prod_{i=0}^{I_n-1} \exp\left(\frac{1}{\mu} v_n(k_{i+1}|k_i)\right)}{\exp\left(\frac{1}{\mu} V_n^d(k_0)\right)} \tag{13}$$

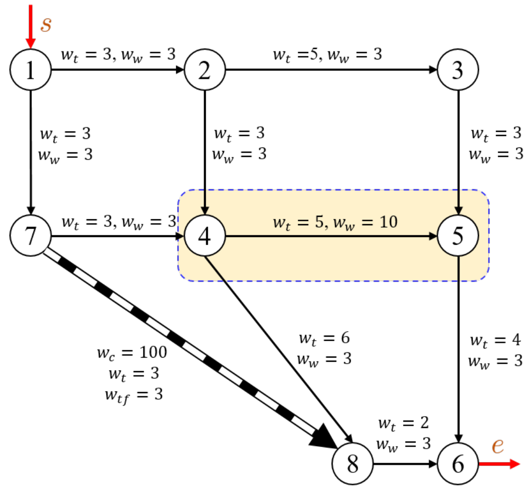
where $v_n(k_{i+1}|k_i) = \boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{w}(k_{i+1}|k_i) + \zeta_{k+1}\mu' \ln \sum_{x \in R_{k_{i+1}}} \sum_{y \in R_{k_{i+1}}} \left\{ \exp(\hat{\boldsymbol{\beta}}'^{\mathrm{T}} \boldsymbol{z}(x_l, y_l)) F(x_l, y_l) \right\}$. Note that $\zeta_{k+1} = 1$ if link $k+1$ is a pedestrian rink, 0 otherwise. As shown in Fosgerau et al. [8)], we can use the nested fixed point algorithm [17)] for the model estimation.

(5) Consumer surplus

One important characteristic of the proposed model is that we can construct a consumer surplus measure that can be used for evaluating both path-level and link-level transport investments. Concretely, the *log-sum* can be defined as

$$V^d(k_0) = \mu \ln \sum_{\sigma \in U} \exp(v(\sigma)/\mu) \tag{14}$$

where U is the set of all routes from origin k_0 to destination d, and $v(\sigma)$ is the systematic utility of route σ that is sum of systematic link utilities on the route given by $v(a|k) = \boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{w}(a|k) + \zeta_{k+1}\mu' \ln \sum_{x_l \in R_a} \sum_{y_l \in R_a} \left\{ \exp(\boldsymbol{\beta'}^{\mathrm{T}} \boldsymbol{z}(x_l, y_l)) F(x_l, y_l) \right\}$. Thus, the logsum depends not only on link-attributes



図–5 A simulated network with link attributes

such as the average link travel time, but also on path attributes such as planting trees. The difference in individual's consumer surplus ΔCS between two situations $v^{wo}(\sigma)$ and $v^{w}(\sigma)$ can be defined as

$$\Delta CS = \mu \ln \sum_{\sigma \in U} \exp(v^w(\sigma)/\mu) - \mu \ln \sum_{\sigma \in U} \exp(v^{wo}(\sigma)/\mu) \tag{15}$$

Note that this is the expressed in utility termed, but it can be directly transformed into monetary units for example by dividing ΔCS by travel cost coefficient.

3. Numerical study

We conduct a numerical study to illustrate the behavior of the proposed model. Subsection (1) briefly explain the basic settings for the numerical study. Subsection (2) focuses on the behavior of path choice model. We first show the empirical specifications of path density function and attractiveness density function, and then test the parameter estimation. Subsection (3) focuses on the behavior of route choice model. We first illustrate how the walking environment influences the route choice behavior.

(1) Basic settings

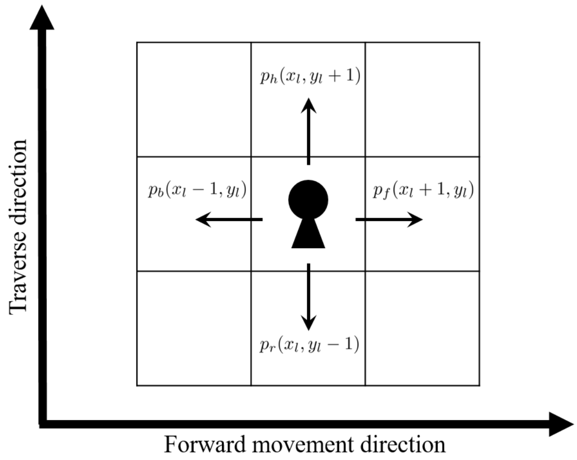
The simulated network used in the numerical study is shown in **図–5**. For the pedestrian link attributes, we only consider travel time, i.e., $\tilde{v}(a|k) = \alpha_t w_t$ where w_t is travel time (min). For public transit link, we assume $\tilde{v}(a|k) = \alpha_c w_c + \alpha_t w_t + \alpha_w w_{tf}$, where w_c is travel cost (yen), w_{tf} is waiting/transfer time (min) at the transit station. The true parameter values are assumed to be $\alpha_c = -0.03$, $\alpha_t = -1.0$, and $\alpha_w = -1.0$.

For the path characteristics, we first give the width of each pedestrian link as w_w (m) shown in 図–**5** . As we can confirm from the figure, the widths of all links are 3, except the link connecting nodes 4 and 5 where $w_w = 10$. The width is an important information for the proposed model, since it implicitly determines the size of choice (path) set on the link. Our main focus is to illustrate how different path attributes of the link connecting nodes 4 and 5 influences the route choice behavior. We assume that other links do not have any path attributes, i.e., eq. (9) is applied for the rest of pedestrian links.

In this simulation analysis, we consider the situation that 1,000 persons are traveling from node 1 to node 6. we also assume that pedestrian path choice behavior on the link connecting nodes 4 and 5 were observed for example through video recordings.

(2) Path choice model

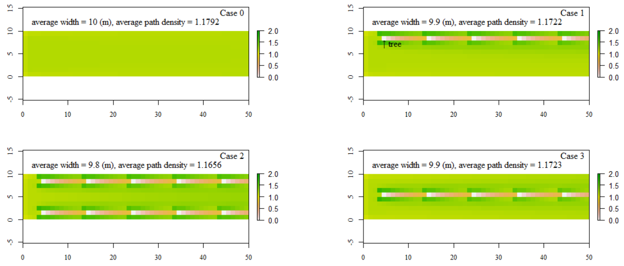
As we discussed in the previous section, the path density function needs to be specified to develop a path choice model. To derive the path density function, we introduce a primitive free-flow pedestrian behavior model where only geometric conditions are taken into account. We use the term *primitive* because all attractiveness factors in the walking environment are not taken into account in the determination of path density function (the attractiveness factors are reflected in the path choice model through the attractiveness density function). The basic concept of the primitive model is adapted from the 2-dimensional cellular automaton model [21], but interactions among pedestrians are not considered here. Suppose that a pedestrian (also called as *particle* in the cellular automaton model) is currently at a location (x_l, y_l) and has a preferred walking direction given by transition probabilities as shown in 図–**6** , where p_f, p_b, p_h, and p_r are transition probabilities of the forward, backward, left-turn, and right-turn movements, respectively. Note also that $p_f(x_l+1, y_l) + p_b(x_l-1, y_l) + p_h(x_l, y_l+1) + p_r(x_l, y_l-1)$ should be equal to one. Depending on the specification of the transition probabilities, we can model different primitive free-flow pedestrian behavior, resulting in different path densities. The simplest case would be $p_f = 1$ and 0



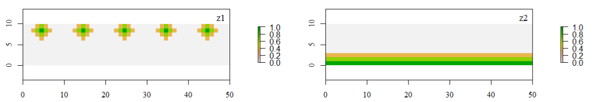
図–**6** Pedestrian's preferred walking direction represented by transition probabilities

for the rest of directions. In this case, the pedestrian directly move from the upstream node to the downstream node without fluctuations. We can also reflects the existence of obstacles. For example, when a tree is planted on (x_l, y_l), then the transition probabilities to that cell can be set as zero.

The transition probabilities and cell size can flexibly be chosen depending on research objective, and it can be decided based on empirical observations such as actual movement in an uncrowded situation and typical space occupied by a pedestrian [21], but, in this numerical study, we arbitrarily set these values due to the lack of such observations. Concretely, we assume that the cell size is $1 \times 1\ m^2$, and transition probabilities are set as $p_f = 0.88$, $p_b = 0.04$, $p_h = 0.04$, and $p_r = 0.04$. When an obstacle exists on either the right or left hand side of pedestrian or backward direction, we reallocate the transition probability to the forward direction. For example, when an obstacle exists on the right hand side, transition probabilities will be $p_f = 0.92$, $p_b = 0.04$, $p_h = 0.04$, and $p_r = 0.0$. Similarly, when an obstacle exists on the forward movement direction, reallocate the transition probability to left-turn and right-turn movement evenly, i.e., $p_f = 0.0$, $p_b = 0.04$, $p_h = 0.48$, and $p_r = 0.48$. Given the above mentioned rules, we obtain a stationary distribution, which is used as path densities of a given link. To get the path density, we first set particles on the entrance of the link which are uniformly distributed. Note that the density at the initial position is given by the the proportion of the average width of the link (walkable area



図–7 Examples of calculated path density (x: forward movement direction, y: traverse direction)



図–8 Examples of attractiveness density for Case 1 (z_1: trees; z_2: window shopping)

size divided by the length of link) to the width of the entrance of the link.

図–7 shows an examples of the calculated path density. Case 0 shows the path density without any obstacles, and thus the distribution is almost uniform. In Case 1, trees are planted at $(x_l, y_l) = (i, 9)$ where $i = 5, 15, 25, ..., 295$. Case 2 and Case 3 show the different planting patterns, $(i, 2)$ and $(i, 9)$ for Case 2 and $(i, 5)$ for Case 3 ($i = 5, 15, 25, ..., 295$). As we can confirm from the figure, the path density becomes larger on the right/left hand side of the tree since pedestrians should avoid the tree. Note that, though the model used here is very simple, it can generate all possible paths on the link including cyclic paths.

Examples of observed attractiveness factors are shown in **図–8**. The left hand side of the figure indicates the attractiveness generated from trees (shade) which corresponds to Case 1 in **図–7**. The right hand side figure demonstrates the attractiveness generated from shops (window shopping), where we assume that shops are on the right hand side of the street. Note that scenic attractiveness factors may not be easy to be prepared, since senary factors are not exactly attached to the location (while attractiveness variables representing shade and rain protection can be easily defined in the space).

We estimate the model with 1,000 sampled paths where the attractiveness defined as $\beta_1 z_1 + \beta_2 z_2$ where

表–1 Estimation results of the path choice model

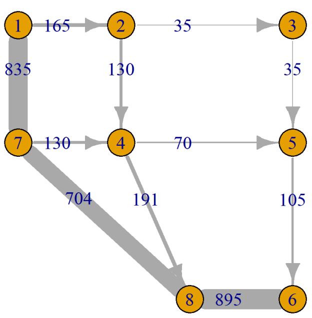
	estimate	t-value
β_1 (tree)	5.160	12.75
β_2 (window shopping)	2.137	9.15
Initial log-likelihood	-2304.71	
Final log-likelihood	-2237.98	
Sample size (paths)	1000	

z_1 represents the shade by tree and z_2 represents utilities obtained from window shopping as we shown in **図–8**. We set the true values of β_1 and β_2 are 5.0 and 2.0, respectively. The estimation results are shown in **表–1**. We can confirm that the estimated values are close to the true values.

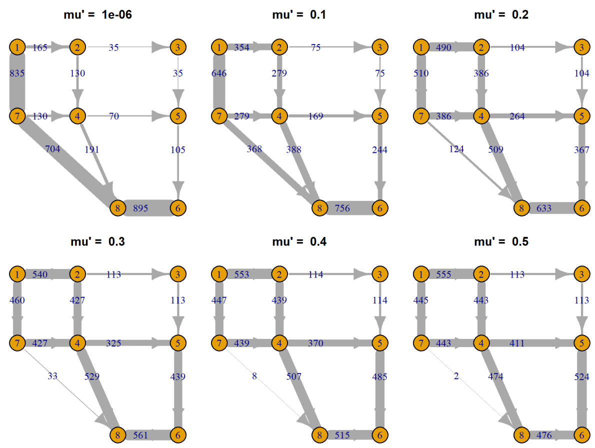
(3) Route choice model

Once we obtain the parameters of the path choice model, we can straightforwardly use the recursive logit model to model the route and mode choice behavior in a multi-modal network as shown in the previous section. To empirically estimate the parameters in the model, mode and route choice behavior in a network is need to be observed for example through GPS trajectories. Since the estimation of recursive logit model with empirical data can be found in literature [8),13),9)], we will not demonstrate whether we can estimate the parameters precisely. Rather, this section demonstrates the impacts of walking environment on link flows.

We start from the route choice model with $\mu' = 0$ with no path attribute and no obstacles on pedestrian links as a baseline. **図–9** shows the simulated link flows, indicating that 70.4% of travelers choose public transit. When $\mu' \gg 0$, the probability of choosing pedestrian links increase since different paths on a link are taken into account in route choice decisions (**図–10**). Interestingly, the flows first shift from public transit link to the link connecting node 4 with node 8 with the increase in μ', but the link flows then shift to the link connecting node 4 with node 5. This is because the link connecting nodes 4 and 5 has a greater width (10 m) and thus pedestrian flows are shifting to this pedestrian link from other pedestrian links. Note that this shift happens because (implicit) choice set size are larger for that link, and not because each path



図–9 The baseline results of simulated link flows under $\alpha_c = -0.03,\ \alpha_t = -1.0,\ \alpha_w = -1.0$ and $\mu' = 0$

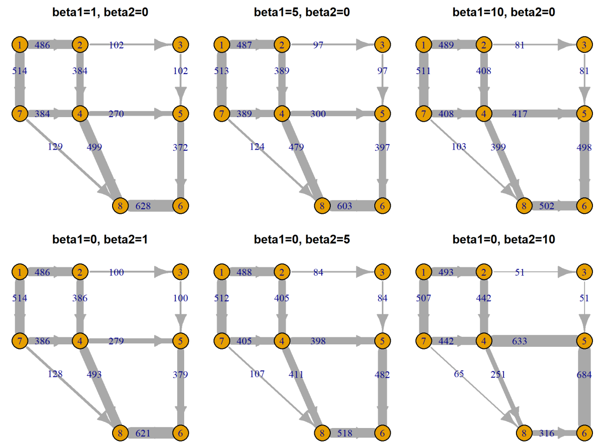


図–10 Simulated link flows under under $\alpha_c = -0.03,\ \alpha_t = -1.0$ and $\alpha_w = -1.0$ with different μ' (no path attribute, i.e., $\beta_1 = 0$ and $\beta_2 = 0$)

on the link is more attractive.

図–11 illustrates the impacts of path attributes on route choice behavior. The upper three figures show the impacts of planting trees (z_1), while the lower three figures illustrate the impacts of window shopping (z_2). The distribution of z_1 and z_2 are given in 図–8 . The results indicate that the impact of z_2 is rather than that of z_1 because of the following two reasons. First, clearly, the average value of z_2 across cells are higher than that of z_1. Second, trees are obstacles which prevent pedestrians from passing through that area (i.e., path density becomes lower), though the overall benefit is positive, since increase in z_1 increase the choice of the link connecting nodes 4 and 5.

By comparing 10 with 11, we can confirm that, while μ' mainly influences the modal share between public transit and walk, changes in β_1 and β_2 change



図–11 Simulated link flows under $\alpha_c = -0.03,\ \alpha_t = -1.0,\ \alpha_w = -1.0$ and $\mu' = 0.2$ with different impacts of path attributes

pedestrian route choice behavior.

4. Conclusions

The objective of this study is to propose a new model of mode, route, and path choice behavior of pedestrians under the utility maximization framework. While the base model is the recursive logit model [8] representing mode and route choice behavior in a network, we have introduced a path choice model component as a lower problem where the path choice model is utilized to construct the utility of each pedestrian link. More specifically, the link utility is expressed as an aggregate alternative of infinite paths in a continuous space under the logit framework, i.e., the link utility is defined as the expected maximum utility of all possible pedestrian paths on the link. The major advantage of the proposed model is that, since route choice behavior in a discrete network space and path choice behavior in a continuous pedestrian space are consistently modeled under the utility maximization framework, we can compare the benefits obtained from transport network improvements with the benefits obtained from the improvements of walking environment by using a consumer surplus (logsum) measure.

The major methodological challenge is on solving the choice set generation problem in a continuous space. Although there are some studies applying continuous logit model for location choice behavior and departure time choice behavior, presumably because of the difficulty of putting plausible assumptions on generating *possible* paths, no application has been made for path choice behavior. Since the continuous version of choice set generation problem is the specification of the probability density function (PDF), establishing a plausible PDF for path choice is essential. We have proposed a solution to this problem where the path density function is constructed from a *primitive* free-flow pedestrian behavior model, in which the basic concept is adapted from the 2-dimensional cellular automaton model. We have also showed an empirical strategy to estimate the parameters in the model, and confirmed the feasibility of the proposed model through a numerical study.

The choice probability and likelihood function of the proposed model have closed-form expressions, and thus it would be easy to use the model in practice. Although GPS trajectory data for route choice modeling and video recording data for path choice modeling may be required for the empirical use of the model, collecting these data becomes easier than before. Providing empirical results is an important future task. From the theoretical viewpoint, the path density demonstrated in this paper is an example, there could be a better way of generating the path density function. Also, defining the attractiveness density function is also not very easy particularly for scenic attractiveness. Even though there are a number of remaining issues, we believe that the model proposed in this paper could be a core for the integration of transportation investments and other urban policies such as the planning of transit stations and city centers.

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集計化された選択肢としてのリンク：連続空間上の歩行者行動を考慮した離散選択モデル

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