# Network Assignment with Free Floating Bicycles 

Ziang YAO ${ }^{1}$ and Jan－Dirk SCHMÖCKER ${ }^{2}$<br>${ }^{1}$ Master Student，Department of Urban Management，Kyoto University （C1－2－438 katsura Nishikyo－ku，Kyoto 615－8540，Japan）<br>E－mail：yao．ziang．63r＠st．kyoto－u．ac．jp<br>${ }^{2}$ Associate Professor，ITS laboratory，Department of Urban Management，Kyoto University<br>（C1－2－436 katsura Nishikyo－ku，Kyoto 615－8540，Japan）<br>E－mail：schmoecker＠trans．kuciv．kyoto－u．ac．jp


#### Abstract

Registered users of free floating bicycle service（FFBS）have exceeded 0.4 billion around the world， which makes FFBS an unneglectable emerging new travel mode．Since FFBS require no docking stations the distance to an available bicycle become difficult to predict and with it mode choice and route choice．In this research，we propose a quasi－dynamic bicycle location forecast model considering travellers＇optimal choice strategies in the FFBS integrated network．The traveller chooses a strategy which allows him to reach his destination at minimum expected cost．We consider the closest－bicycle seeking process，traffic assignment and bicycle density update process in each time interval．The approach is illustrated with a network consisting of three zones and a single public transport line serving the centre of these zones．


Key Words：free floating，bikesharing，traffic assignment，order statistic

## 1．INTRODUCTION

Although the idea of＇free floating＇is rather new， free floating bicycle services（FFBS，also called stationless，dockless or station－free），have boomed on the street since 2015，thanks to the rapid expansion of dozens of private start－up companies in China and around the world．The new free floating mode provides users more flexible choices because it allows users to start and end their trip much closer to their true origin and destination，compared to the conventional station based bikesharing．The hardware on the free floating bicycles has evolved into second generation，which now integrated with global positioning systems（GPS）modules and Bluetooth communications modules．These characteristics have enabled the service providers to gather the time and GPS location information of each travel，and even may be able to record the GPS path of each travel．This system have the potential to become a rich mine of robust travel data．

The user needs a smartphone installed with corresponding applications of FFBS providers to use their service．A common approach is presented as below：

Firstly，the user needs to open the corresponding application to check the surrounding available bicycles，whose locations are reported through the GPS module on the bike．After deciding his
favourable bicycle，the user will have to walk to the bicycle，and use the QR scanner in the application to scan the unique QR code on each bicycle．Currently， some service providers have also abled the user to make an up to 15 minutes reservation on the decided bicycle，preventing from other user＇s using．The scanned bicycle will receive an＇unlock＇signal from the cloud service and enter charging state．The user can cycle within the service area once the bicycle is correctly unlocked．During the cycling，the user＇s smartphone will report its location every few seconds．When the user arrives at his destination，he should park the bicycle in areas allowed by the local law．Once the bicycle is manually locked，the bicycle will report to the cloud service，and the user＇s app will end the charging state，and present the total trip fare．Currently，a common trip fare is 100 yen every half hour．

## 2．LITERATURE REVIEW

Since FFBS is a relatively new service mode obtaining the characteristic of station free，there is limited relevant research and focus mostly on station based bikesharing．

Shen et al ${ }^{11}$ gathered over 14 million GPS records of dockless bike－sharing services for nine consecutive days from one of the largest bike sharing operators in Singapore．They adopted spatial
autoregressive models to analyse the spatiotemporal patterns of bike usage during the study period．

Wu et $\mathrm{al}^{2)}$ proposed a continuous model for minimizing the transit cost in a bike－sharing integrated transit network．However，they assumed a uniformly distributed transit demand，and modelled by using Manhattan distance in bicycle seeking process．

In our quasi－dynamic model，with given initial bicycle density and OD demand in this time interval， we firstly proposed a closest－bicycle seeking process universally feasible for any distribution under Euclidean distance，to generate the travel cost function for each mode．Based on the travel cost function，incremental assignment is used to assign OD demand among different modes．The volume information on each mode in time N can be used to update the initial bicycle distribution for time interval $N+1$ ．For the building of a quasi－dynamic distribution forecast model，we conduct the research processes in each individual time interval．We firstly define：
$\varphi_{A}(t)$ ：Distribution of bicycles at time $t$ in subzone A
$g_{i j}(t)$ ：Travel demand from demand point $i$ to $j$ in time interval $t$
$U_{i j}^{m}(t)$ ：Utility of mode $m$ from demand point $i$ to $j$ in time interval $t$
$P_{i j}^{m}(t)$ ：The probability of travel from demand point $i$ to $j$ by mode $m$ at time interval $t$

Within each time interval $t$ ，our expected process is shown in Figure 1.


Figure 1 the expected process within each time interval
Section 3 will be the explanation of different modes and network settings．Section 4 will give an introduction on methodology，focusing on the closest－bicycle seeking process．Section 5 will use a 3－node network as example．

## 3．NETWORK LAYOUT AND FEASIBLE MODES

## 3．1 Network Layout

We first describe the network layout as shown below in Figure 2.


Figure 2 An example PT network integrated with walking and cycling

We can see that the research network is composed of several subzones（dashed boxes）indexed as $i$ ，and each subzone $i$ has a FFBS density $\rho_{i}^{t}\left(\rho_{1} \sim \rho_{6}\right.$ in Figure 2），which stand for the sub－zonal average number of bicycles per unit area in time interval $t$ ． The exact location of bicycles are unknown，and we assume travellers always walk to the closest available bicycle when trying to use FFBS．The red points （ $s_{1} \sim s_{3}$ in Figure 2）and lines in Figure 2 stand for the public transport（PT）stations and lines in Figure 2．The exact location of PT stations are known，and travellers can only board or alight at PT stations． Black hollow circles $\left(d_{1} \sim d_{6}\right.$ in Figure 2）stand for demand point，which are the true origin／destination of traveller．The exact location of demand point is also unknown．

## 3．2 Feasible Modes

We assume there are 6 feasible modes between demand points as shown in Figure 3．We assume PT stations have the known location，but the location of demand points and bicycles are randomly distributed．


Figure 3 The 6 feasible modes when travelling from $\boldsymbol{d}_{\mathbf{1}}$ to $\boldsymbol{d}_{\mathbf{6}}$
Walk：Walk represents traveller walking from a demand point $\left(d_{1}\right)$ to another demand point $\left(d_{6}\right)$
directly．The trip distance is the Euclidean distance between OD．

PT：PT represents travellers walking from a demand point $\left(d_{1}\right)$ to the closest PT station $\left(s_{1}\right)$ and boards．Alighting at the closest PT station $\left(s_{3}\right)$ and walk to another demand point $\left(d_{6}\right)$ ，which is the destination of trip．

Cycle：Cycle represents traveller walks from a demand point $\left(d_{1}\right)$ to the closest bicycle（location unknown），and cycle directly to another demand point $\left(d_{6}\right)$ ，which is the destination of trip．

C－PT－W：C－PT－W represents travellers walking from a demand point $\left(d_{1}\right)$ to the closest bicycle （location unknown），cycle to the closest PT station $\left(s_{1}\right)$ and boards．Alighting at the closest PT station $\left(s_{3}\right)$ and walk to another demand point $\left(d_{6}\right)$ ，which is the destination of trip．

W－PT－C：W－PT－C represents travellers walking from a demand point $\left(d_{1}\right)$ to the closest PT station $\left(s_{1}\right)$ and boards．Alighting at the closest PT station $\left(s_{3}\right)$ and walk to the closest bicycle（location unknown），and cycle to another demand point（ $d_{6}$ ）， which is the destination of trip．

C－PT－C：C－PT－C represents travellers walking from a demand point $\left(d_{1}\right)$ to the closest bicycle （location unknown），cycle to the closest PT station $\left(s_{1}\right)$ and boards．Alighting at the closest PT station （ $s_{3}$ ）and walk to the closest bicycle（location unknown），and cycle to another demand point $\left(d_{6}\right)$ ， which is the destination of trip．

Because the bicycles are free floating in the subzones，it require FFBS users to walk an unfixed distance to use bicycle．This distance is modelled in the closest－bicycle seeking process to be introduced in next section．

## 4．CLOSEST－BICYCLE PROCESS

We firstly give the following assumptions．We assume a unit square with side length $a$ as the research area，and assume the demand point and the bicycle are distributed within the research area．We assume the FFBS user always choose the closest available bicycle to walk to．Under these assumptions，we hope to obtain the PDF of walking distance between demand point and bicycle under these three progressive scenario

1－Uniform：In 1－Uniform scenario，we assume the fleet size of FFBS is one in the research area，and both bicycles and demand points are uniformly distributed．

N－Uniform：In N－Uniform scenario，we assume the fleet size of FFBS is $N \geq 2$ in the research area， and both bicycles and demand points are uniformly distributed．

N－NonUniform：In N－NonUniform scenario，we assume the fleet size of FFBS is $N \geq 2$ in the research area，and bicycles and demand points obey non－uniform distributions．．

## 4．1 Solving for 1－Uniform Scenario

The 1－Uniform scenario has been well discussed in the field of geometric probability in mathematics．We follow the approach provided by Philip ${ }^{3)}$ ，and omit the deductions．

Assume that $X_{1}, X_{2}$ are independent and evenly distributed in the interval $(0, a)$ ．The same is assumed for $Y_{1}, Y_{2}$ in $(0, \mathrm{~b})$

We start by calculating the distribution function $F_{a}(t)=\operatorname{Prob}\left(\left(X_{1}-X_{2}\right)^{2} \leq t\right)$ ，and the corresponding density function $f_{a}(t)=\mathrm{d} F_{a}(t) / d t$ ． Then，the density $g(s)$ corresponding to $G(s)=$ $\operatorname{Prob}\left(\left(X_{1}-X_{2}\right)^{2}+\left(Y_{1}-Y_{2}\right)^{2} \leq s\right)$ is obtained by convolving $f_{a}$ and $f_{b}$ ．The wanted distribution function for the distance is $K(v)=G\left(v^{2}\right)$ with the density $k(v)=2 v g\left(v^{2}\right)$ ．

## 4．2 Solving for $\mathbf{N}$－Uniform Scenario

The N－Uniform scenario can be seen as the PDF of distance to walk from demand point to the $1^{\text {st }}$ closest bicycle when there are $N$ independent－identically distributed bicycles available．This problems can be solved with the help from order statistic，in which the $\mathrm{k}^{\text {th }}$ order statistic $Y_{k}$ of a statistical sample is defined as the $\mathrm{k}^{\text {th }}$ smallest value of this sample．

Let $X_{1}, \ldots, X_{n}$ be random variables，and order statistic $\quad Y_{k}=h_{k}\left(X_{1}, \ldots, X_{n}\right)$ be a 1－1 transformation．

The general form of the joint PDF of order statistic $Y_{1}$ to $Y_{k}$ is：

$$
\begin{equation*}
f_{Y_{1, \ldots}, \ldots} Y_{k}\left(y_{1}, \ldots, y_{k}\right)=n!\prod_{k=1}^{k=n} f\left(y_{k}\right) \tag{1}
\end{equation*}
$$

Thus，the marginal PDF of $Y_{j}$ is：

$$
\begin{equation*}
f_{Y_{j}}\left(y_{j}\right)=\frac{n!}{(j-1)!(n-j)!}\left[F\left(y_{j}\right)\right]^{j-1}\left[1-F\left(y_{j}\right)\right]^{n-j} f\left(y_{j}\right) \tag{2}
\end{equation*}
$$

In which $f_{Y_{j}}\left(y_{j}\right)$ is the PDF of walking distance from a random demand point to the $j^{t h}$ closest bicycle． Combining with $k(v)$ and $K(v)$ obtained from 1－ Uniform scenario，the general function of the PDF and CDF to the $1^{\text {st }}$ closest bicycle is：

$$
\begin{gather*}
f_{Y_{1}}(v)=n[1-K(v)]^{n-1} k(v)  \tag{3}\\
F_{Y_{1}}(v)=1-[1-K(v)]^{n} \tag{4}
\end{gather*}
$$



Figure 4 CDF of N－Uniform scenario with different available bicycles $(N)$ and side length of research area（a）．

A comparison of empirical and analytical CDF in N－Uniform scenario can be seen in Figure 4．The upper 4 empirical CDFs are generated by using Monte Carlo method，and the lower 4 analytical CDFs are generated by using function（4）．Different available bicycles $(N)$ and side length of research area （a）are tested to see model validity．

## 4．3 Solving for $\mathbf{N}$－NonUniform Scenario

Function（1）and（2）deducted in section 4.2 hold universally in generating the joint PDF and marginal PDFs of order statistics $Y_{j}$ ，for any PDFs in there corresponding continuum intervals．The only difficulty exists when the initial PDF of $X_{i}$ is a piecewise function．Station－centered two－ dimensional normal distributions will be expanded following the previous discussions in the future research．

## 5．EXAMPLE TEST NETWORK

In this section，we propose a three－zone example network with given OD matrix as example．


Figure 5 The 3－node example network layout

The network layout can be shown as above in Figure 5．This network is composed of 3 subzones， each contains one demand point（black hollow circle），one PT station（red hollow circle）and a sub－ zonal bicycle density（ $\rho_{A}, \rho_{B}, \rho_{C}$ ）．The demand points and bicycles are randomly distributed in each subzone，and PT stations are assumed at the centroid of each zone．There is no direct PT links between subzone $A$ and $C$ ，travelers need to follow the path PT－AB and PT－BC to travel between A and C when using PT．The OD matrix is shown in Table 1.

Table 1 OD matrix among A，B and C

| From | To | $\mathbf{B}$ | $\mathbf{C}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{A}$ |  | 100 | 300 |
| $\mathbf{B}$ | 200 |  | 200 |
| $\mathbf{C}$ | 300 | 100 |  |



Figure 6 The 3－zone network structure and its links
The network structure of the example network is shown in Figure 6．In order for less line intersection， the whole network is separated into two layers connected with zero cost transit links．

Incremental all－or－nothing assignment is adapted to approximate the user－equilibrium solution．We follow the modification recommended by Sheffi ${ }^{4}$ ： the OD pairs are selected in random order during incremental loading．

We conduct traffic assignment on the following scenarios for different provided bicycle number percentage constraints as shown in Table 2．The diagram of the relation between total bicycles and total travel time is shown in Figure 7.

Table 2 Scenarios of different constraints combination

| A | B | C | Total Bicycle | Total Travel Time |
| :---: | :---: | :---: | :---: | :---: |
| $100 \%$ | $100 \%$ | $100 \%$ | 2400 | 929.5 |
| $80 \%$ | $100 \%$ | $100 \%$ | 2220 | 944.7 |
| $50 \%$ | $100 \%$ | $100 \%$ | 1950 | 974.6 |
| $20 \%$ | $100 \%$ | $100 \%$ | 1680 | 1007.0 |
| $80 \%$ | $80 \%$ | $100 \%$ | 2100 | 963.0 |
| $80 \%$ | $50 \%$ | $100 \%$ | 1920 | 985.2 |
| $80 \%$ | $20 \%$ | $100 \%$ | 1740 | 1005.4 |
| $50 \%$ | $20 \%$ | $100 \%$ | 1470 | 1042.2 |
| $80 \%$ | $80 \%$ | $80 \%$ | 1920 | 990.8 |
| $80 \%$ | $80 \%$ | $50 \%$ | 1650 | 1019.1 |
| $80 \%$ | $50 \%$ | $20 \%$ | 1200 | 1081.0 |
| $50 \%$ | $50 \%$ | $50 \%$ | 1200 | 1084.3 |
| $50 \%$ | $20 \%$ | $20 \%$ | 750 | 1151.6 |



Figure 7 The relation between total bicycles and total travel time

We can observe that with the same total amount of bicycles，a better allocation strategy can effectively reduce the total travel time when the provided bicycles are between $70 \% \sim 90 \%$ of total bicycle demand．

Upon these results，we will extend this approach to a larger network and evaluate different allocation strategies．

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