EVALUATION OF SAFETY WITH MIXED TRAFFIC OF CONNECTED AUTONOMOUS VEHICLES AND CONVENTIONAL VEHICLES

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This study aims to analyze the impact of connected and autonomous vehicles (CAVs) on traffic safety under various penetration rates. Based on a recently proposed heterogeneous flow model, the mixed traffic flow with both conventional vehicles and CAVs was simulated and studied. The frequency of aggressive stops and value of time-to-collision in the mixed flow under different CAV penetration rates was calculated and used as indicators of CAV impact on traffic safety. Acceleration rate and velocity distribution of the mixed traffic flow was presented to show the evolution of mixed traffic flow dynamics with the increase in CAV penetration rates within the mixed flow. Results show that the condition of traffic safety is greatly improved with the increase in the CAV penetration rate. More cautious following strategy of the CAV would contribute to a greater benefit on traffic safety, though less gain in capacity. With the increase in CAV penetration rate, the portion of smooth driving is increased. Velocity difference between vehicles are decreased and traffic flow is greatly smoothed. Stop-and-go traffic will be significantly eased. This work provides some insights into the impact of the CAV on traffic safety and sheds light on how would the mixed traffic flow dynamics evolve with the gradual adoption of CAV under current traffic system.

Key Words : *connected autonomous vehicle, heterogeneous flow, traffic safety, CAV penetration rate*

1. INTRODUCTION

Recent advances in automotive technology are about to change the traffic system fundamentally. In particular, the development of connected and autonomous vehicles (CAVs) has attracted amounts of attention from both the public and the research field. People are expecting that with the deployment of this emerging technology, problems such as traffic congestion, accidents would be greatly eased [1]. Other merits such as fuel saving and pollution reduction are also regularly expected. However, to which extent the current transportation system can be improved through the deployment of this new technology is unknown. The gradual adoption of CAV in the vehicle composition indicates that the state of a mixed traffic flow including both conventional vehicles and the CAVs on the road simultaneously will last a long time period. Meanwhile, CAV technology is still evolving with time. Varying levels of vehicle automation ranging from partial automation to full automation would exist during this time period. The impact of CAVs on traffic flow during this transition period has not yet been studied thoroughly.

There are many predictions concerning the impact of CAVs on traffic safety. Some researchers argue that CAVs would reduce crashes 90% because more than 90% of traffic accidents are caused by human drivers' error, autonomous vehicles are able to avoid such driving errors [1, 2]. Such prediction may seem too optimistic since it is based on a simple assumption, and it solely concerns about the utopic future and without paying any attention to the transition period. Other researchers indicate that the introduction of CAVs would smooth the traffic flow, avoid stop-and-go driving and thus result in a significant reduction in fuel consumption and air pollution [3, 4, 5, 6, 7, 8]. However, some researchers hold a quite different point of view over this problem. Their studies demonstrate that low-level automated vehicle in the mixed traffic flow would rather have a negative effect on traffic flow and road capacity. Improvement in traffic flow can only be attained when CAVs reached a high penetration rate in the mixed flow [9, 10]. There is a lot of existing literature address the potential impact of autonomous vehicles on traffic flow, most of the work use stability analysis and simulation approach to assess to which extent the mixed traffic flow can be smoothed. Talebpour and Mahmassani studied the potential impact of CAVs on traffic flow using a proposed simulation framework, results show that the introduction of CAV would increase the throughput of highway facilities and improve the stability of the traffic flow [11, 12]. In our previous work, a heterogeneous-flow model was proposed to model the CAVs in heterogeneous traffic flow: the results show that the increase in capacity is strongly related to the market penetration rate and CAV parameter in the car-following process [13]. Existing literature also indicates that the implementation of the connected vehicle would be beneficial for a safer traffic system [14]. In the experimental approach, Stern et al. conducted a car-following experiment on a circuit track: results demonstrate that intelligent control of a single autonomous vehicle is able to dampen the stop-and-go traffic flow [15].

Existing researches have already recognized the positive effect of CAVs would bring to the current traffic system. But to which degree of potential benefits can be attained when only a portion of vehicles being CAVs is yet to be studied. Due to a lack of real consequence, it is relatively difficult to find a proper way to estimate the impact of CAVs on traffic safety, especially under varying levels of penetration rate. However, a fully understanding of the heterogeneous flow dynamics is vital for the making and deployment of future traffic control and management policies. In this regard, this work intends to provide some insights into the heterogeneous traffic flow dynamics during the transition period and to analyze the impact of connected and autonomous vehicles on traffic safety under various CAV penetration rates. This contribution is a successive study of our previous work on modeling CAVs in heterogeneous traffic flow, which aims to provide a better understanding of the heterogeneous flow dynamics [13]. The heterogeneous traffic flow is simulated using the model proposed in the aforementioned work, which is a two-lane cellular automaton model. This study mainly focuses on the impact of the CAV on traffic safety under various CAV penetration rate in the mixed traffic flow.

The rest part of this work is organized as follows. The heterogeneous flow model is first reviewed in Section 2. Section 3 introduced the indicators for evaluating the safety impact. Section 4 presents the results of this study and followed with discussions. Finally, this work is ended with conclusions in Section 5.

2. MODEL

The methodology for modeling CAVs in heterogeneous flow is identical with our previous work [13]. A cellular automaton (CA) model was developed, wherein both the CAVs and conventional vehicles were incorporated in the heterogeneous traffic flow. The established model considered both autonomous driving through the adaptive cruise control and inter-vehicle connection via short-range communication. For the sake of completeness, the heterogeneous flow model is first reviewed. For modeling of regular vehicles, the two-state safe-speed model is applied, which is able to reproduce the metastable state, traffic oscillations, phase transitions, and other real traffic flow dynamics [16, 17]. For modeling the CAVs, new rules were established in the heterogeneous-flow model. The steps involved in the model are as follows [13].

(1) Deterministic speed update

$$v'_{\mathrm{det}} = \min(v+a,\ v_{\max},\ d_{\mathrm{anti}},\ v_{\mathrm{safe}}) \tag{1}$$

Here, v and v' denote the speed at the current and subsequent time steps, respectively. a and $v_{\max}$ are the acceleration rate and maximum velocity of the vehicle, respectively. d_{anti} denotes the anticipated space gap, v_{safe} denotes the safe speed.

For regular vehicles, d_{anti} and v_{safe} are defined as follows.

$$d_{\mathrm{anti}} = d + \max(v_{\mathrm{anti}} - g_{\mathrm{safety}},\ 0) \tag{2}$$

$d = x_l - x - L_{\mathrm{veh}}$ is the real space gap. x and x_l denote the position of the object vehicle and its preceding vehicle. L_{veh} is the length of the vehicle. $v_{\mathrm{anti}} = \min(d_l,\ v_l + a,\ v_{\max})$ denotes the expected velocity of the preceding vehicle. d_l, and v_l denote the real space gap and speed of the preceding vehicle, respectively. g_{safety} is a safety parameter that helps in avoiding accidents considering the limitation of human perception.

$$v_{\mathrm{safe}} = \left[-b_{\max} + \sqrt{b_{\max}^2 + v_l^2 + 2 b_{\max} d} \right] \tag{3}$$

$b_{\max}$ is the maximum deceleration rate. The round function $[x]$ helps return the integer nearest to x. This equation assumes (i) a reaction time of 1 s (which is presumably the time step of the CA model), (ii) no acceleration at the present time.

For CAVs, corresponding $d_{\mathrm{anti}}^{\mathrm{cav}}$ and $v_{\mathrm{safe}}^{\mathrm{cav}}$ are defined as follows.

Based on the capability of obtaining an exact value of the space gap, the anticipation distance for CAVs can be transformed to the following function.

$$d_{\mathrm{anti}}^{\mathrm{cav}} = \begin{cases} d + v_{\mathrm{anti}}^{\mathrm{cav}} & \text{if } v_l \text{ is a CAV} \\ d + v_{\mathrm{anti}} - b_{\mathrm{defense}} & \text{otherwise} \end{cases} \tag{4}$$

$$v_{\mathrm{anti}}^{\mathrm{cav}} = \min(d_l,\ v_l + a,\ v_{\max},\ v_{li}) \tag{5}$$

Connectivity is incorporated in Equation (5), where v_{li} denotes the average velocity of the preceding connected vehicles within the connected range (*CR*). If there is no CAV within the *CR*, a default value of $v_{\max}$ is applied for v_{li}. The CAVs are able to obtain the driving condition within the *CR* via dedicated short-range commutation (DSRC) technology. *CR* is larger than *DR*. Connectivity of the CAVs is another approach of obtaining additional road condition from a wider connected range (*CR*) compared to its sensor-detection range. b_{defense} is the randomization-deceleration rate under the defensive state. Here, a worst case is assumed to ensure the safety during the operation of the CAVs when following a conventional vehicle. Because the driving behavior of humans is unpredictable, a conventional vehicle is always assumed to stay in the defensive state in the operation of a CAV.

In determining safe speed v_{safe} for the regular vehicles, a reaction time of 1 s is incorporated in Equation (3) for human driving. For the CAV, this reaction time is eliminated. Compared to conventional vehicles, CAVs are only able to detect vehicles located within the detection range of the sensors. Based on this characteristic, the maximum velocity of a CAV is limited to the detection range (*DR*) of the sensors.

$$v_{\text{safe}}^{\text{cav}} = \left\lfloor \sqrt{v_l^2 + 2b_{\max} \min\left(d_{\text{anti}}^{\text{cav}}, DR\right)} \right\rfloor \quad (6)$$

Here, the velocity of a CAV is assumed to be sufficiently low such that the vehicle can be completely stopped within the *DR*, i.e., the maximum velocity of the CAVs $v_{\max}^{\text{cav}} = \left\lfloor \sqrt{2b_{\max} DR} \right\rfloor$.

For regular vehicles, acceleration rate a is a constant value. While for CAVs, a classical ACC model is employed to determine the acceleration rate a_{ACC} for the autonomous driving [10], which is defined as follows.

$$a_1 = K_1(d - vT_{\text{ACC}}) + K_2(v_l - v) \quad (7)$$

$$a_{\text{ACC}} = \lfloor \max(\min(a_1, a_{\max}), b_{\max}) \rfloor \quad (8)$$

Here, K_1 and K_2 are coefficients with respect to the ACC, and T_{ACC} is a desired net time gap of a CAV with respect to the preceding vehicle. $\lfloor x \rfloor$ is the floor function used to return the maximum integer no greater than x.

(2) Stochastic deceleration for regular vehicles

$$v' = \begin{cases} \max(v'_{\text{det}} - b_{\text{rand}}, 0) & \text{with probability } p \\ v'_{\text{det}} & \text{otherwise} \end{cases} \quad (9)$$

The randomization deceleration b_{rand} and stochastic deceleration probability p are specifically defined as follows:

$$b_{\text{rand}} = \begin{cases} a \text{ if } v < b_{\text{defense}} + \lfloor d_{\text{anti}}/T \rfloor \\ b_{\text{defense}} & \text{otherwise} \end{cases} \quad (10)$$

$$p = \begin{cases} p_b & \text{if } v = 0 \\ p_c & \text{e if } v \le d_{\text{anti}}/T \\ p_{\text{defense}} & \text{otherwise} \end{cases} \quad (11)$$

Where b_{rand} denotes the randomization-deceleration rate. $p_{\text{defense}} = p_c + \frac{p\text{a}}{1+e^{\alpha(vc-v)}}$ is a logistic function used to define the randomization probability p_{defense}. In the function b_{rand}, two different randomization-deceleration values are employed to describe the difference in the driving behaviors under two different states, i.e., the defensive and normal states. b_{defense} is the randomization-deceleration rate under the defensive state, which equals to 1 m/s^2. Under the normal state, the randomization-deceleration rate equals to a.

For CAVs, no randomization-deceleration is applied.

(3) Position update

$$x' = x + v' \quad (12)$$

x' denotes position at subsequent time step. The time step of the model is 1 s and the vehicle will move forward at a distance of its updated velocity.

(4) Lane-changing rules

A classical lane-changing model is applied to extend the TSM to a two-lane traffic-flow model [18]. It is defined as follows.

Incentive criteria: $d(i, t) < \min\{v + \text{a}, v_{\max}\}$ and $d(i, t)_{\text{other}} > \min\{v + \text{a}, v_{\max}\}$ indicate space ahead of the object vehicle i is not enough for traveling with a higher velocity, and the driving condition in the target lane is better than that in the current lane.

The safety criteria $d(i, t)_{\text{back}} > v_{\max}$ indicates that, when changing the lanes, the vehicle immediately behind the object vehicle moving on the target lane will not crash the object vehicle after changing lanes. When the two conditions are fulfilled simultaneously, the object vehicle will move onto the target lane with a lane-changing probability P_{lc}.

Tables 1 and 2 list the parameters of the model for modeling the mixed traffic flow.

Table 1 Parameters for modeling regular vehicles [14]

Param.	L_{cell}	L_{veh}	$v_{\max}$	T	a	$b_{\max}$
Units	m	L_{cell}	m/s	s	m/s^2	m/s^2
Value	0.5	15	27	1.8	0.5	−3
Param.	P_{a}	P_{b}	P_{c}	g_{safety}	v_{c}	α
Units	-	-	-	L_{cell}	L_{cell}/s	s/L_{cell}
Value	0.85	0.52	0.1	20	30	10

Table 2 Parameters for modeling CAV [10, 13]

Param.	*DR*	*CR*	P_{lc}	K_1	K_2	$a_{\max}$
Units	m	m	-	s^{-2}	s^{-1}	m/s^2
Value	120	300	0.2	0.14	0.9	3

3. SAFETY ASSESSMENT

In the simulation, it is a crash-free environment. Thus, the model can not be used to measure crashes or traffic safety directly. This work adopted three rules to measure the number of aggressive stops that occurred during the simulation as an indicator for traffic safety evaluation [19]. The time step of the simulation is 1 s. $d(i, t)$ indicate the space gap ahead of vehicle i at time step t and $v(i, t)$ indicates its velocity.

1) $d(i, t)<v(i, t)$, indicating that the space between vehicle i and its predecessor vehicle i+1 is smaller than the current velocity, which means vehicle i could reach the position of its predecessor by the next time step.

2) $v(i+1, t)>0$, indicating that vehicle i+1still moving at time step t.

3) $v(i+1, t+1)=0$, indicating that vehicle i+1 will stop abruptly at time t+1.

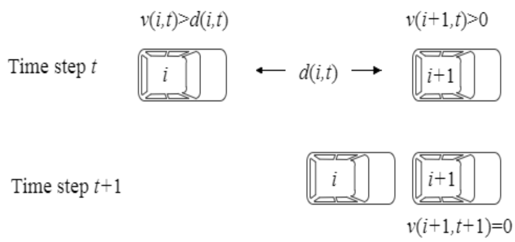


Fig.1 schematic illustration of the rules for detecting aggressive stops

Fig.1 shows the schematic illustration of an occasion described by the aforementioned three rules. The following vehicle is stopped due to a sudden stop made by its preceding vehicle. The phenomenon of sudden stops is very common in traffic oscillations and stop-and-go traffic flow. Traffic breakdown can be induced by individual vehicle's sudden brakes or stops. By measuring the number of aggressive stops in the traffic flow, possible insight could be shed into the mixed traffic flow dynamics and foster a deeper understanding of the impact of CAV on traffic safety at varying degree of penetration rates. Note that these rules are applied when the corresponding simulation is finished, and recorded time-space information of all individual vehicles during the simulation period is available. If the three conditions are met simultaneously, a rear-end accident may occur. The frequency of such occasion is calculated and considered as a negative sign of safety where a potential crash may occur, denoted as N (times/km/h).

The second indicator for evaluating safety impact is the time-to-collision (TTC) [20]. The time-to-collision is defined as the time that remains until a collision could occur if two successive vehicles maintain a speed difference, which has been applied in numerous studies for identifying safety impacts. The time-to-collision of vehicle i with respect to a leading vehicle i+1 at time step t can be calculated with:

$$TTC(i,t) = \frac{d(i,t)}{v(i,t)-v(i+1,t)} \quad \forall v(i,t) > v(i+1,t) \tag{12}$$

Where $d(i, t)$ and $v(i, t)$ denote the real space gap and the speed of vehicle i at time step t, respectively. A time-to-collision can only be calculated when a positive speed difference exists between two successive vehicles.

4. SIMULATION RESULTS

The simulation was conducted on a 10-km two-lane road segment under the periodic boundary condition. P_{av} denotes the percentage of the CAVs with respect to the total number of vehicles in the traffic flow. T_{ACC} denotes the desired net time gap in the adaptive cruise control (ACC) process of CAVs. A smaller value for T_{ACC} indicates the CAV can keep a closer distance when following its preceding vehicle.

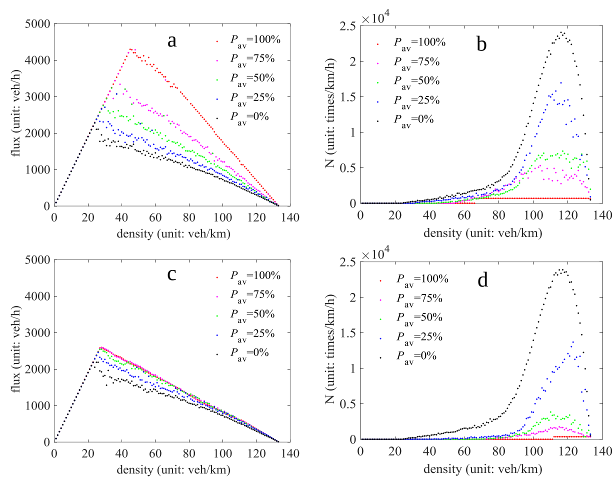
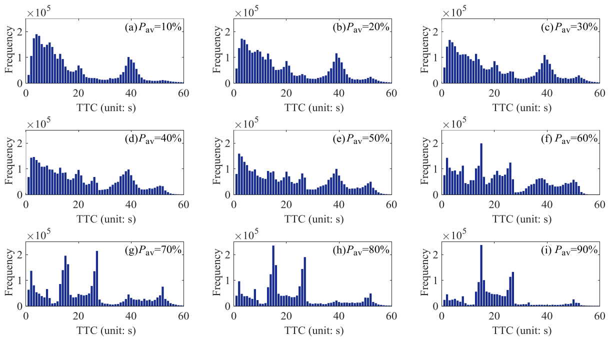
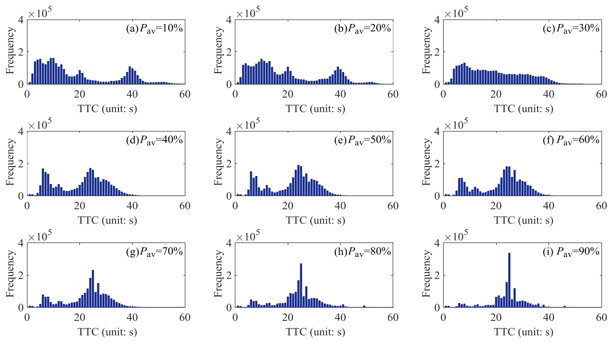


Fig.2 Flow-density diagrams and speed-density diagrams of the presented model under various penetration rates of autonomous vehicle P_{av} with T_{ACC} =0.5 s (a, b), 1.1 s (c, d).

Fig.2 shows the relationship between road capacity, the frequency of aggressive vehicle stop with regard to density under two scenarios with different T_{ACC} values respectively. Five cases under various CAV penetration rates are included in each scenario. From Fig.2 (a, c) we can directly observe that a smaller T_{ACC} value and a higher penetration rate of CAV corresponds to a higher gain in road capacity. Capacity is equal to the maximal flow rate attained in the free flow phase. It is understandable that a smaller desired net time gap attained by CAVs contributes a larger improvement in capacity. Since CAVs can drive more closely within the traffic flow, and a larger penetration rate of CAV in the mixed flow reinforces this process. Fig.2 (b, d) indicates that the introduction of CAVs in the mixed flow would be beneficial for traffic safety. Under both cases, even with a different parameter in the desired net time gap, the frequency of aggressive stop decrease with the increase in the CAV penetration rate within the mixed flow. The system will attain a considerable gain in terms of safety when CAV penetration rate reaches 25%, and this effect is much more evident when CAVs are under a more cautious strategy in the ACC process. The difference between the two cases indicates that a more cautious strategy in the ACC performance would contribute to a greater improvement in traffic safety. The simulation results indicate that in the coming future, the trade-off between capacity gain and safety improvement needs to be taken into account in the deployment of CAV technology.



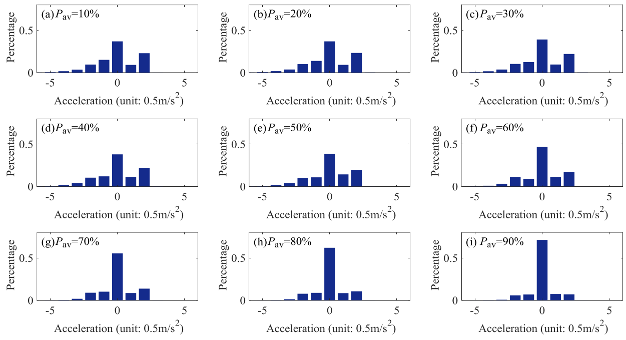
a



b

Fig.3. Time-to-collision distributions under various penetration rates of autonomous vehicle P_{av}, with T_{ACC} =0.5 s (a) and 1. 1s (b), density equals to 50 veh/km/lane.

Fig. 3 presents the time-to-collision distributions of two cases with different T_{ACC} values, with P_{av} increase from 10% to 90%. The frequency of low *TTC*, namely the most left region in the plot, represents the negative effect on traffic safety, which indicates crash likely to occur if the vehicle is not operated properly under such cases. In contrast, a higher *TTC* indicates the positive performance. In the first case, improvement is not obvious at a low CAV penetration rate. Significant improvement on safety can only be observed when CAV reached a major component in the mixed flow. While in the latter case, improvement in safety can be observed even CAVs at a relatively lower penetration rate. The difference between the aforementioned two cases indicates that the performance in CAV driving actually has a direct impact on the evaluation of safety effect on the mixed traffic flow. A more cautious strategy on the CAV driving would possess a greater benefit on traffic safety at the beginning of introducing CAVs in the current traffic system.



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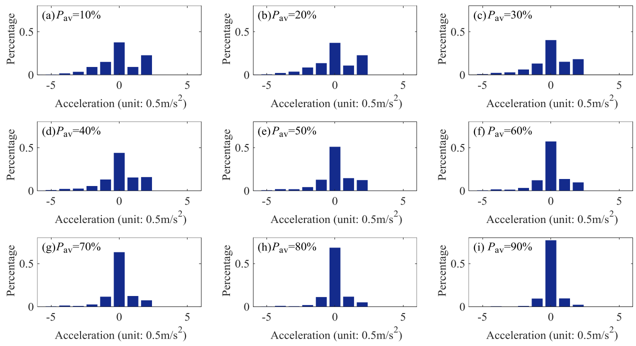
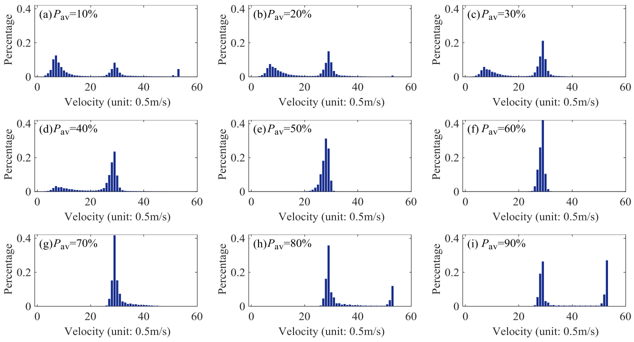


b

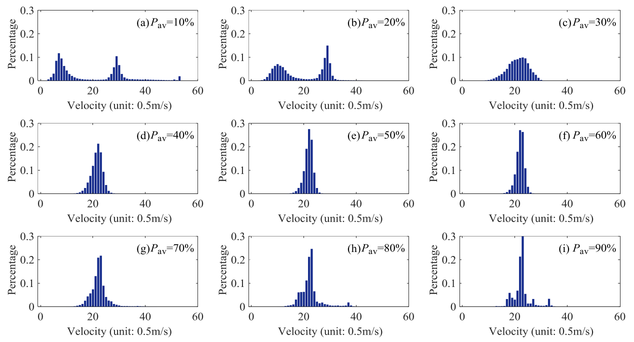
Fig.4. Acceleration rate distributions under various penetration rates of autonomous vehicle P_{av}, with T_{ACC} =0.5 s (a) and 1. 1s (b), density equals to 50 veh/km/lane.

Fig.4 presents acceleration rate distributions of two cases with different T_{ACC} values, with P_{av} increase from 10% to 90%. Under both cases, a gradual increase in the frequency of the acceleration rate 0 can be easily found, which indicates that the introduction of CAV would boost the portion of smooth driving within the mixed traffic flow. The frequency of high deceleration rate is also decreased, which can be considered as a positive sign for traffic safety. With the increase in CAV penetration rate, a smoother traffic flow can be attained. Compare the results from two cases, we can find that the results of latter case with a higher T_{ACC} value is better than the first case with a lower T_{ACC} value. This indicates that a more cautious following strategy of the CAV would contribute to a greater benefit on traffic safety.

Fig.5 presents velocity distributions of the aforementioned two cases. Under a low CAV penetration rate of 10%, the velocity distribution shows three peaks in the velocity plane. The most left branch in-



a



b

Fig.5. Velocity distributions under various penetration rates of autonomous vehicle P_{av}, with T_{ACC} =0.5 s (a) and 1. 1s (b), density equals to 50 veh/km/lane.

dicates congested flow and the most right branch indicates free flow. The medium branch indicates traffic flow in a transition state between free flow and congested flow. With the increase in CAV penetration rate, the free flow branch and congested flow branch gradually shrink, the velocity distribution gradually centralized around the medium branch. This phenomenon indicates that the introduction of CAVs in the mixed flow would boost smooth driving. In Fig.5(a), when CAV reach a dominant role in the mixed flow, the free flow branch turns out again. This phenomenon indicates that at a certain density region, with the increase in the CAV penetration rate, traffic flow state can be recovered to the free flow phase. However, such effect strongly depends on the CAV capability, in Fig.5(b) with a larger parameter in T_{ACC}, there is no such effect.

5. CONCLUSIONS

In this work, we reported a further study of the heterogeneous flow dynamics with both conventional vehicles and CAVs. Simulation results were presented which aims to provide some insights into the impact of the CAV on traffic safety and sheds light on how would the mixed traffic flow dynamics evolve with the gradual adoption of CAV under current traffic system.

The frequency of aggressive stops in the mixed flow under different CAV penetration rates indicates that the condition of traffic safety would be greatly improved with the increase in the CAV penetration rate. More cautious following strategy of the CAV would contribute to a greater benefit on traffic safety, though less gain in capacity. Acceleration rate and velocity distribution of the mixed traffic flow indicate that the introduction of CAV would contribute to a higher portion of smooth driving in the mixed traffic flow. Velocity difference between vehicles is decreased and traffic flow is greatly smoothed.

The authors hope the present work could help to foster a better understanding of the mixed traffic flow dynamics in the coming future.

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