Public Transportation Fare Level Optimization of Model Cities

Zhihua JIN[1], Jan-Dirk SCHMÖCKER[2], Saeed MAADI[3], Tomoki NAKAMURA[4] and Nobuhiro UNO[5]

[1]Student Member of JSCE, Dept. of Urban Management, Kyoto University
(C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan)
E-mail:kin@trans.kuciv.kyoto-u.ac.jp

[2]Member of JSCE, Dept. of Urban Management, Kyoto University
(C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan)
E-mail:schmoecker@trans.kuciv.kyoto-u.ac.jp

[3]Student Member of JSCE, Dept. of Urban Management, Kyoto University
(C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan)
E-mail:maadi@trans.kuciv.kyoto-u.ac.jp

[4]Member of JSCE, Dept. of Urban Management, Kyoto University
(C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan)
E-mail:nakamura@trans.kuciv.kyoto-u.ac.jp

[5]Member of JSCE, Dept. of Civil and Earth Resources Eng.
(C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan)
E-mail: uno.nobuhiro.2v@kyoto-u.ac.jp

This paper analyses the interaction between fares and public transport service quality. The rationale is that with higher fares the operator has more resources to provide a better service. Demand in turn will depend on both service quality and demand leading therefore to the question as to whether there is an optimal fare for different types of cities. The model developed in this paper builds on the work of Daganzo (2010). The model determines optimal network headway, stop spacing as well as the ratio of a central dense PT service area compared to the whole city size. The model input is kept at its minimum considering city size, average speed of services, population, the quality of an alternative service as well as fare sensitivity. In contrast to Danganzo we include fare and demand elasticity. With this it is possible to find some general rules for what city types what type of fare structures are favorable. We focus on a flat fare structure. It is found that in such a fare structure, from the viewpoint of maximizing social welfare, a minimum, low fare would be the best. However, if the operator cost coverage ratio is considered as objective function then there exists an optimal fare above the minimum fare. We discuss further for what type of cities acceptable cost coverage ratios are considered as well as illustrate a fairly complex interaction between the decision variables.

Key Words : *fare structure, city characteristics, public transportation network optimization,*

1. INTRODUCTION

(1) Background

Prices for public transport vary significantly across different parts of the world. Fare increases are often a complex political issue as the prices are in many cases at least to some degree regulated. Providing affordable public transport is associated with a very wide range of issues, including urban development, congestion regulation, environmental issues as well as general ideas of fairness and providing a basic mobility level for all. Whereas in many developing countries the fare is kept low in order to allow people with a diverse financial background being able to afford it, in other countries the idea that public transport users should pay for the service they consume leads to significantly higher prices. Furthermore, higher fares are often justified with the argument that it allows providing a better service.

Due to these issues and different objectives in general there appears to be little agreement as to what fares are appropriate or optimal. In a recent report the authors also describe that the definition of "fair fares" varies across cities and transport authorities[2)].

The objective of this paper is to contribute to this discussion by showing how fares, at least under a number of simplifying assumptions, would lead to different service quality levels and with it different demand levels. We consider that the answer to this will to a large part depend on the city parameters. We aim to provide some guidance as to what fares, PT service quality and demand levels can be derived in a range of cities. More specifically as input and city parameters we vary size, population density and the demand level. Building on the work of Daganzo[1] the variables that we vary are optimal network headway, stop spacing as well as the ratio of a central dense PT service area compared to the whole city size. These variables together with the fare level and the demand which we presume to be elastic interact and provide us with indices of social welfare as well as subsidy needed for the operator.

(2) Structure

In Section 2, existing literature is reviewed and in Section 3, the equations used in the model are explained. Section 4 compares the sensitivities and conditions including optimum in base and various scenarios to see how the difference in the characteristics of city and the other optimized variables are related with each other. Finally, Section 5, will conclude the results ubiquitously, and further expected work is stated.

2. LITERATURE REVIEW

Previous research on fares and pricing is mostly either qualitative or on congestion pricing for traffic, take e.g. Verhoef's[4] study on congestion pricing in general networks, for instance. Though they are related yet they do not quantitatively give the characteristics of PT fare structures. In particular road pricing literature does not have to address the complex interaction between fare and different service quality aspects such as service frequency, or attributes such as cleanliness of vehicles which are key concerns in modelling public transport fare structures. Existing quantitative PT research on fares are mostly case studies, or emphasizing more operators' aspects. For example, Gabriella et al.[5] conducted a qualitative study on attitudes towards public transport and private car. The study of Li et al.[6] compared the efficiency of different transit market regimes in transit services that are not exactly reliable. Here in this paper, we base our research on the model proposed by Daganzo[1], where he mainly discussed about the interaction between three parameters, network structure, distance between stops and headway, optimizing the service quality of PT. In his paper, the city was divided into two major parts, a center with grid network and a periphery with hub and spoke network, as in Figs. 1 and 2.

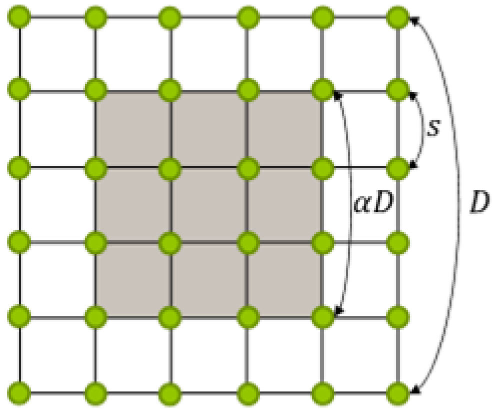

Fig. 1. City being divided into central and peripheral part with different PT network structure as in Fig. 2.

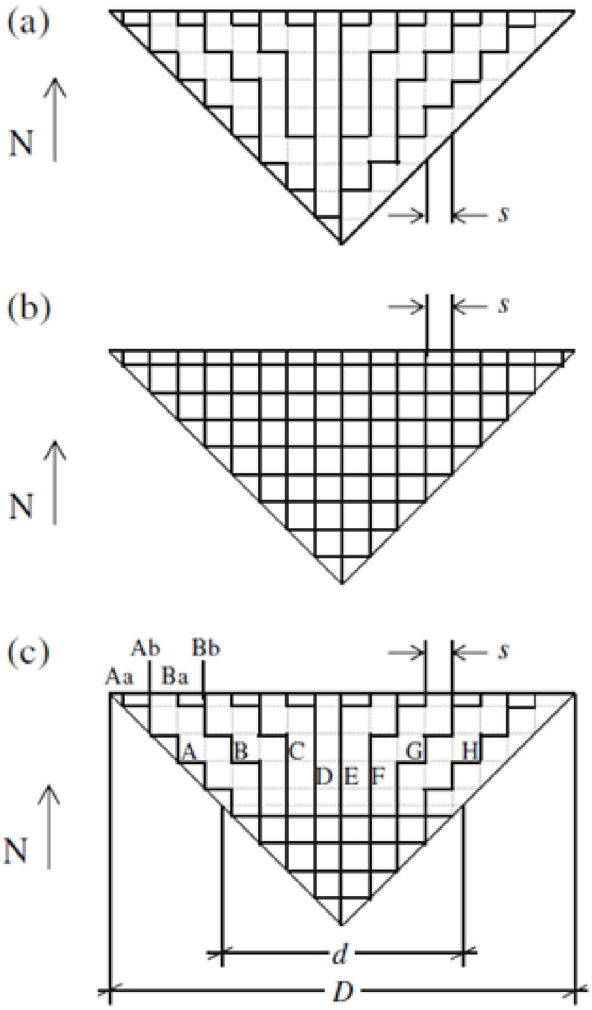

Fig. 2 The Hybrid Model Concept (taken from Daganzo (2010); a) Hub Hub and spoke network, b) Grid network and c) Hybrid network.

3. METHODOLOGY

(1) Model Overview

Daganzo's model is considering fixed demand, looking at the competence of various public transportation travel modes in the prospect of society. Thus his model did not consider about the possibility of demand change due to the change in the disutility, nor did it include fare in the model. He minimized the sum of operators' and users' disutility and obtained the value of the service characteristics. In contrast we consider the summation of all the disutility including those who are using not public transport. We focus on one PT mode, bus, and consider an additional alternative mode which passengers would choose instead of PT in case its disutility (including fare) is too high. A potential "proxy" for the demand that is rejecting the public transport option could be taxi, private vehicles, car sharing or, for short distance trips, active public transport. We aim to find the flat fare f that minimizes the total social disutility of a PT operator and the total travelling population on road. This is expressed as

$$\mathrm{z} = \lambda(\tilde{z}_o + \tilde{z}_p) + (\lambda_{total} - \lambda)(\tilde{z}_x + \frac{f_x}{\mu}) \tag{1}$$

In the above equation z denotes the total social disutility for the whole potential public transport population λ_{total} of concern. $\tilde{z}_o$ stands for the disutility of the public transport operator per passenger obtained from Daganzo's model, while $\tilde{z}_p$stands for the disutility of a public transport user also obtained from Daganzo's model. $\tilde{z}_x$ is the disutility of those who chose to use another mode of transport, here assumed as taxi. We separate in the above formulation the public transport fare term from the total operator and passenger disutility as this is our main variable of concern. λ denotes the actual demand for public transportation, and f_x, the fare for the proxy mode. μ is the time value, to convert all the monetary parameters, including operator costs, into a single time dimension.
The values of disutility are obtained by following equations for public transportation operators and users respectively:

$$\tilde{z}_o = \pi_V V + \pi_M M + \pi_L L \tag{2}$$

$$\tilde{z}_p = A + W + T + \delta/v_w e_T \tag{3}$$

$$\tilde{z}_x = \frac{E_x}{v_x} \tag{4}$$

Here Equations (2) and (3) are based on Daganzo's model, while Equation (4) obtains the disutility of proxy mode and is derived as an additional term. It is assumed that the proxy disutility does not include walking access time, waiting time and the transfer penalty. Therefore, it is only the time travelled in the vehicle, which is distance travelled by the velocity of travel via proxy. This clearly favours the proxy mode compared to public transport which is in line with our general model formulation where we take demand distribution and other characteristics as "lower limiting case" for public transport. We assume a uniform where demand distribution where all OD pairs are equally likely which clearly is the worst case for public transport as Daganzo also notes.
All of the values are determined by an estimation in parameters as below following Daganzo. L is the summation of the infrastructure length in the periphery and in the center, being fixed capital costs; and parameters V and M being the operational cost. For users' disutility, A, W and T denote walking access time, waiting time and travel time, respectively. For the additional term of transfer penalty for public transport in Eq. (3), $\frac{\delta}{\mathrm{v_w}}\ e_T$, δ stands for the weight of transfer time, while v_w indicates walking speed and e_T the expected number of transfers. For the model we further need to obtain the proportion of passengers that travel within the city center, within the periphery and those travelling within both parts of the city in order to obtain the expected distance travelled in the network. Since Daganzo's split of the travelling groups does not appear fitting to us for some fare scenarios, we derived all of the expected value for vehicular travelling distance in the appendix in proof 2 and proof 3.

$$L = \frac{D^2(1+\alpha^2)}{s} \tag{5}$$

$$V = \frac{2D^2(3\alpha - \alpha^2)}{sH} \tag{6}$$

$$M = \frac{V}{v_c} \tag{7}$$

$$\frac{1}{v_c} = \frac{1}{v} + \frac{\tau}{s} + \frac{2.5(1+e_T)\tau'\lambda sH}{(3\alpha - \alpha^2)D^2} \tag{8}$$

$$e_T = 1 + \frac{1}{2}(1-\alpha^2)^2 \tag{9}$$

$$A = \frac{s}{v_w} \tag{10}$$

$$W = \left[\frac{2+\alpha^3}{3\alpha} + \frac{(1-\alpha^2)^2}{4}\right]\mathrm{H} \tag{11}$$

$$T = \frac{E}{v_c} \tag{12}$$

$$E_{PT} = \frac{\mathrm{D}(12 - 9\alpha - 9\alpha^2 + 23\alpha^3 - 5\alpha^4 - 5\alpha}{12} \tag{13}$$

(2) Parameter Settings for Barcelona Base Case

For the alternative mode, the velocity is calculated as in Equation (14). The coefficients are based on the case of free flow, where velocity is 40km/h and flow is 0; and a “current” or base case, where the velocity is 25 km/h and the flow is the original demand of private vehicle transportation. The velocity and the flow to obtain their relationship are based on the 2013 modal share in Barcelona, which is 12.28% for bus[7] and 26.47% for private vehicles [8]. Taking Barcelona is in line with Daganzo’s paper which uses the same city as base case. Similar, we obtain taxi fares from Barcelona assuming a base fare f_{xb} and a distance depending fare f_{xd} as in (15).

$$\mathrm{v_x} = \frac{6.96\lambda}{\lambda_0} + 18.04 \tag{14}$$

$$f_x = f_{xb} + f_{xd}E_x \tag{15}$$

Optimization is done regarding four aspects as decision variables: α denoting the proportion of the square city center with a grid PT network as in Figs 1 and 2, s for grid size determining the distance between stops, H for headway as well as f and λ. The further input parameters that are utilized in above equations are D denoting the length of the square city, v_c denoting commercial speed of vehicles, τ, the time lost per stop due to the required door operation, deceleration and acceleration; and the time added per boarding passenger, τ' (hr/p). (If the effect of alighting is significant, it can be usually subsumed into τ'.) In users’ disutility calculation, v_w represents the walking access speed, v_x the speed of taxi derived assuming a liner speed-flow relationship, and the modal share data from Barcelona City Council homepage about the modal share in 2013[8].

For the model, in the base case, apart from most parameters assumed to be similar to the Daganzo’s paper, for f, the PT fare, in case of flat fare, we use f_{f0} = \$2.29 for the original fare which is equivalent of the current flat fare for buses in Barcelona. Taxi fare is first determined by base fare of \$7.46 and the distance term coefficient, \$1.28/km, which is the current taxi fare system in dollar in Barcelona.

λ_0 is the original PT demand in the model, taken as the value of Barcelona’s case here as 20,000pax/hr. λ, the demand is a variable decided by the disutility of a single user with the scenario of utilizing public transportation and the scenario of utilizing taxi by logit model or linear model, as the choice model of passengers. Equation (16) denotes the linear and (17) the logit case, with their proofs described in the appendix.

$$\lambda = \lambda_0\left(1 - \frac{\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}}{2\left(\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}\right)}\right) \tag{16}$$

$$\lambda = \frac{\lambda_0}{exp\left(\theta\left(\tilde{z}_P + \frac{f_f}{\mu} - \tilde{z}_x - \frac{f_x}{\mu}\right)\right) + 1} \tag{17}$$

Additional terms for explanations are: λ_0 denotes the original public transportation demand; while $\tilde{z}_{p0}$ indicates the base case scenario values that equals to the base case provided in Daganzo’s paper, and θ, the weight of disutility in the logit model.

In additional to fare which directly reflects the payment of PT users, here another indicator is focused on, from urban planners to estimate the amount or ratio of the subsidy they would need to provide operators to ensure that the PT fare as well as network would be at its best condition. This indicator, marked as ϕ, is calculated by:

$$\phi = \frac{f_{PT}}{\tilde{z}_o} \tag{18}$$

(3) Other Scenarios

Other scenarios mentioned in Daganzo’s paper wehre also replicated, to confirm the relationship between fare structures with city characteristics, including demand and the size of the city. These scenarios consist f: PT favored, sprawl and big city scenario. In addition, a scenario of a small town and a rough model of Kyoto was optimized.

In the case of a PT favored scenario, the base demand λ_0 is multiplied by factor 4. In the sprawl scenario, reflecting a range of US cities, the demand remains the same but the size of the city is doubled to D=20 km. The third additional scenario is a big city scenario, similar to Paris, where both population and the city size is increased from the base scenario, as $\lambda_0 = 80{,}000\text{pax/hr}$, while D=20 km.
The additional case of small town is assumed as where the demand is decreased to 1/4 and the size of the city gets halved, to confirm the case that the scale of the city is smaller while the demand density is kept the same.
In the case of Kyoto, the proxy cost is roughly estimated based on the case of small taxis, with a low base fare of \$0.66 and \$2.78 per km travelled via taxi. The modal share of bus transportations according to the person trip survey in 2010, is 5.9% and private vehicle being 24.3%. the λ is estimated as for bus transportation, according to the same survey, being approximately140,000 pax in 3 hours, but here we assume that the daily traffic volume compared to the peak hour would be larger than Barcelona, here, assumed as twice of that ratio, as this appears to be more reasonable to us given, among others, tourist demand during off-peak periods. Thus λ_{PT0} would be 18,667 pax/hr. With these value and the same free and current flow speed, the proxy velocity of Kyoto can be derived as:

$$v_{x\,\mathrm{Kyoto}} = 21.36 + \frac{3.64\lambda}{\lambda_{0\,\mathrm{Kyoto}}} \quad (19)$$

$$(\lambda_{0\,\mathrm{Kyoto}} = 18667\ \mathrm{pax/hr})$$

The minimum flat fare is set as \$2.01, as a base for a rough estimation reflecting the current bus fare in the main operation area in Kyoto, while the size of the city is assumed as 12km which is roughly resembling the area covered by the Kyoto City Bus.(Kyoto City Bus, 2016)

Table 1 City scenarios

Scenario Name	PT Demand [pax/hr]	Size of the City (edge length D) [km]	Real City Example
Base	20,000	10	Barcelona
PT Favored	80,000	10	-
Sprawl	20,000	20	US Cities
Big City	80,000	20	Paris
Small Town	5,000	5	-
Kyoto	18,667	12	-

4. RESULTS

(1) Sensitivities in linear model for flexible demand

We vary in the following the minimum fare level the operator has to charge. We find in all scenarios that the minimum fare is also equivalent to the optimal fare if Eq. (1) is taken as the objective function. Figure 3 shows the resulting change in the service quality. Note that for a fare of \$2.29, we obtain the base case. With lower fare the operator does not have sufficient resources to provide a very frequent service and also the spacing between the stops increases. Figure 4 illustrates the resulting demand. It stays constant for lower fares as well as for slight fare cases. From a customer perspective fhe fare increase is compensated by better service quality.
However, when the fare is around \$3, there is a critical point that service starts to get worse and demand decreases as shown also in Fig. 3 and 5. This is also an inflection point for the total, operators' users' and proxy disutility as Figs. 4 and 6 illustrate. To note is that the value of α remains constant, especially after the critical point, while the value of s and H increases, making the service worse. It could be explained by the theory that with a slight fare increase, operators would try to attract passengers from changing their modal choices by improving the service and slightly reducing the area of center with the best service, and passengers are paying more but are "paid back" by the improved service quality, which as well can be confirmed in Fig. 7. But when it comes to a point where PT is too expensive, the operator has to "give up" aiming to attract the same amount of passengers by making improvements. Instead passengers would tend to take the taxi regardless of the high fare which then increase the disutility of both passengers and the operator.

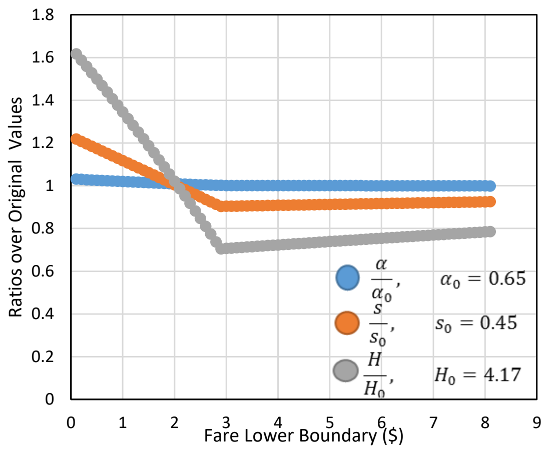

Fig. 3 Decision Variables Changes in Base Scenario (Linear)

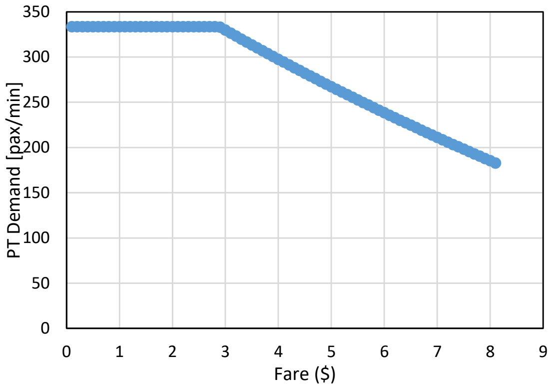

Fig. 4 PT Demand Changes in Base Scenario (Linear)

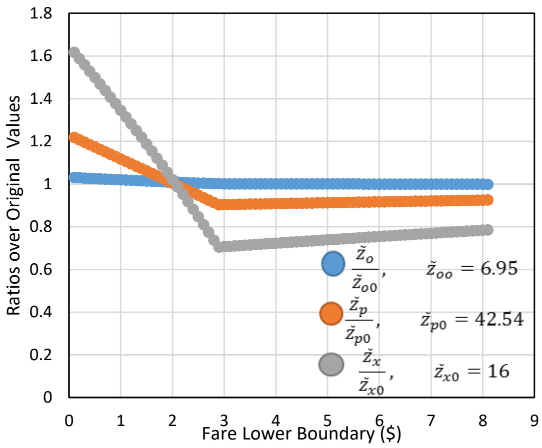

Fig. 5 Disutility Changes in Base Scenario (Linear)

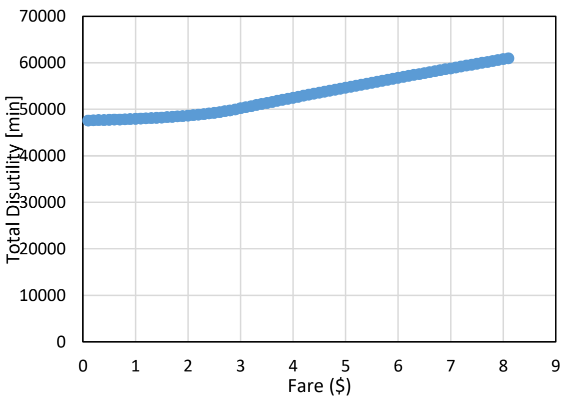

Fig. 6 Total Disutility Changes in Base Scenario (Linear)

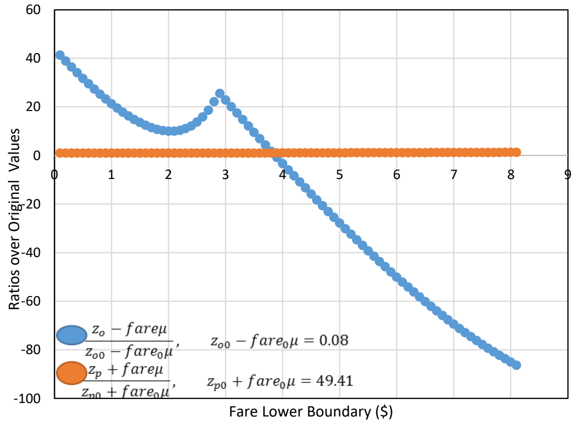

Fig. 7 Disutility taking fare in in Base Scenario (Linear)

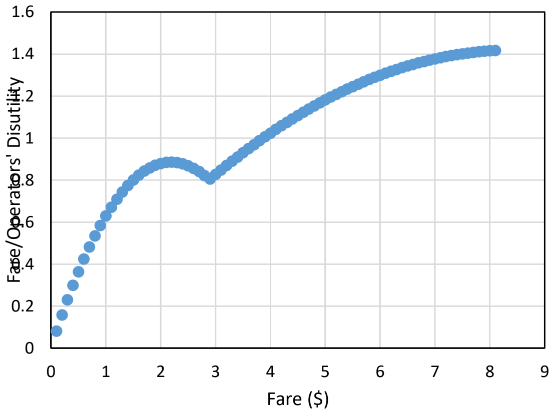

Fig. 8 ϕ Changes in Base Scenario (Linear)

We therefore conclude that though a higher fare would pressure the operators to provide a better service, it might not pay back since the increase in fare is not proportionate with the increase in the disutility. Thus particularly considering the profits that operators would make, the ratio of revenue to the disutility, in other words, how much the fare income would cover the cost of operators, should be emphasized. Looking at ϕ as defined in Eq. (18), there are two different smooth curves with an inflection point, as in Fig. 8. The inflection point reflects the start of demand decrease, and though it might be feasible to obtain more percentage of the disutility via fare revenue, from urban planners' perspective, it might not be the preferable case, since the total disutility also increases rapidly after demand starts to decrease, as shown in Fig. 3.

(2) Sensitivities in logit model in flexible demand with proxy

Assuming instead a logit model, the tendency of parameters in relation with the sensitivities regarding fare are the same with linear model, though the acceleration depends on the value of θ, thus the critical point depends on it as well. Here three examples of θ are given as 0.01, 0.2, and 0.5, each representing the case that θ being too small, appropriate and too big, respectively.

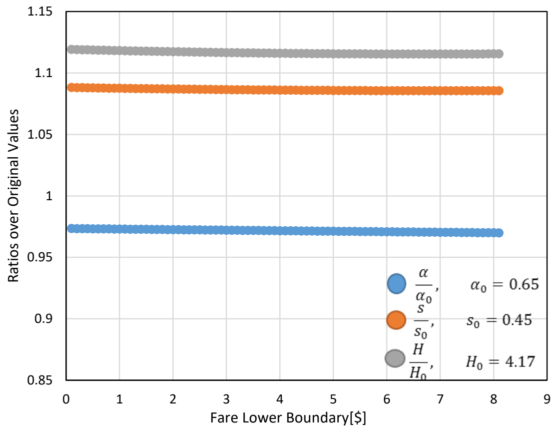

Fig. 9 Decision Variables Changes in Base Scenario (Logit: $\mathbf{\theta = 0.01}$)

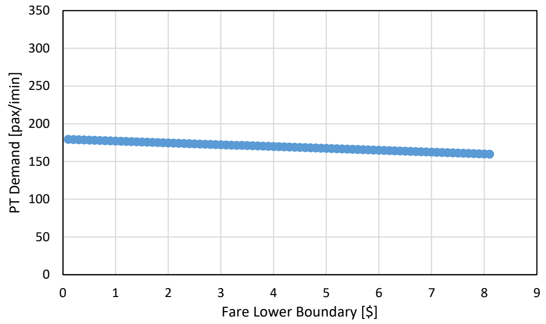

Fig. 10 PT demand Changes in Base Scenario (Logit: $\mathbf{\theta = 0.01}$)

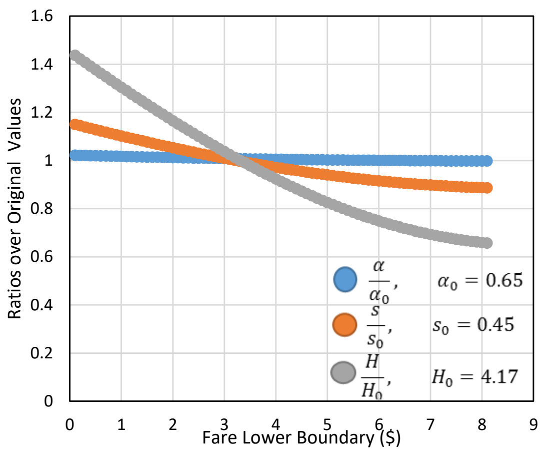

Fig. 11 Decision Variables Changes in Base Scenario (Logit: $\mathbf{\theta = 0.2}$)

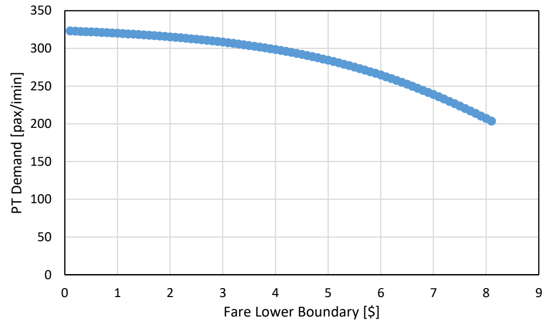

Fig. 12 PT demand Changes in Base Scenario (Logit: $\mathbf{\theta = 0.2}$)

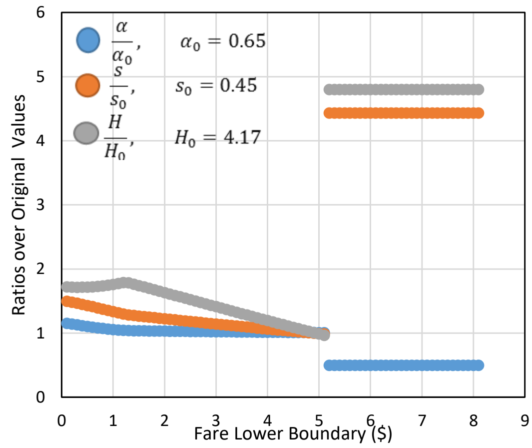

Fig. 13 Decision Variables Changes in Base Scenario (Logit: $\mathbf{\theta = 0.5}$)

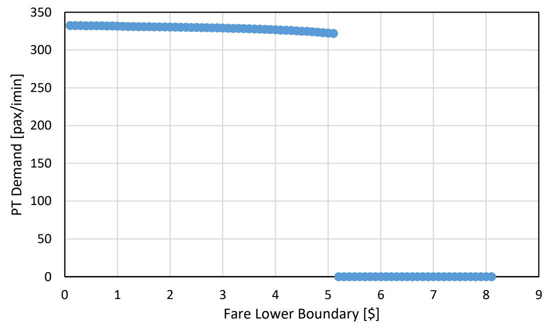

Fig. 14 PT demand Changes in Base Scenario (Logit: $\mathbf{\theta = 0.5}$)

When the value of θ appraoches 0, the demand gets indifferent to the disutility, while in the case that it gets higher, it indicates that the demand is more sensitive to disutility changes. Therefore, in the figure where $\theta = 0.01$, Figs. 9 and 10**Error! Reference source not found.**, all the parameters hardly change, while demand drops suddenly, resulting in the jump in all the other parameters at the point where the fare is around 5, in the case of $\theta =$

0.5 as in Figs. 13 and 14. At $\theta = 0.2$, the curves are smooth, resembling linear demand. As all the tendencies are the same but only the slope differs between the two models, in the reminder of this paper, for simplification, we will use the linear model.

(3) Comparisons between the Scenarios in Flat Fare with Linear Model

As mentioned in Section 4 (1), there exists a maximum ratio of fare over operators' disutility. We first compared all the scenarios at its own maximum ratio, whose variables are all listed on the next page. Here at the maximum coverage, one can observe that PT favored cities can get most of their operational costs back via fares, while the fare stays low. When the demand density is similar, the maximum ratio is similar as well. The city center ratio does not vary greatly between cases, staying stable at around 0.65.

Table 1 Comparison of Maximum ϕ Case in Various Scenarios at Flat Fare(Linear)

	Base	PT Favor-ed	Sprawl	Big City	Small Town	Kyoto
α	0.66	0.66	0.65	0.65	0.67	0.65
s [km]	0.44	0.39	0.62	0.55	0.39	0.49
H [min]	3.97	2.33	5.38	3.39	5.01	4.62
Fare [$]	2.20	1.80	2.90	2.20	2.20	2.50
$\tilde{z}_o$ [min/pax]	7.46	3.67	15.12	6.66	6.97	8.75
$\tilde{z}_p$ [min/pax]	42.07	40.55	66.85	64.88	30.91	47.92
$\tilde{z}_x$ [min/pax]	17.06	16.95	33.48	33.33	8.95	20.39
Z [min]	48846	190210	81316	313792	8454	51579
ϕ	0.89	1.47	0.58	0.91	0.95	0.86

Observing that all of the scenarios make it possible to make up to 50% of the operational cost from the fare income, all of the ratios are set at 50% to see when the coverages are the same, what would the scenarios be like, and the results are listed as in the table below. The results that can be obtained as rather similar with the previous comparison, while it is clear with a more horizontal comparison that to obtain the same ratio of operational cost by fare, with the same population density, *s* should be smaller and *H* should be bigger as the scales gets smaller. The values of disutility depend greatly on the size of the city as well as, to some degree, on the overall demand level.

Table 2 Comparison of Maximum ϕ Case in Various Scenarios at Flat Fare(Linear)

	Base	PT Favor-ed	Sprawl	Big City	Small Town	Kyoto
α	0.67	0.68	0.65	0.66	0.68	0.66
s [km]	0.52	0.45	0.68	0.61	0.48	0.58
H [min]	5.99	3.89	6.96	4.94	7.02	6.78
Fare [$]	0.70	0.30	1.80	0.70	0.70	0.90
$\tilde{z}_o$ [min/pax]	4.36	2.05	10.87	4.31	4.26	5.27
$\tilde{z}_p$ [min/pax]	46.57	45.04	70.15	68.78	35.41	52.72
$\tilde{z}_x$ [min/pax]	17.25	17.08	33.62	33.47	9.14	20.59
Z [min]	47817	188024	79902	310645	8231	50502

5. Conclusion

In this paper the model proposed by Daganzo (2010) was extended to look at the relationship between fare level, structure and the operation of PT. The ratio of the central grid network area, the distance between stops, the headway of PT and the fare level was optimized considering social welfare consisting of user and operator disutility. Following are the main conclusions drawn with the results.

It was found that the optimum value for center ratio of the city, α, does not depend much on the city characteristics, but stays around a stable value of 0.65. On the other hand, *s* and *H*, the distance between stops and headway are the major decision variables that decide operators' disutility and these variables are fluctuating greatly. Generally as fare increases, *s* and *H* would decrease to improve the service quality, while the value of α decreases to reduce the center ratio, where the service quality is the best.

The disutility of operator per passenger gets smaller when both the demand and the scale of the city is large, while the passengers' would get smaller only when the scale of the city is small and is less relevant to the demand density of public transportation.

The increase in fare level from none would in the beginning pressure the operators to provide a better service, to keep the demand, in other words, when fare level is low, passengers paying more and gets correspondingly improved service. However, there exists a critical point at which passengers would start to utilize alternative modes due to the high PT fare. In further work we aim to characterize this critical point further, as we suggest, considering also subsidy needed for PT, this fare might be useful benchmark for cities to aim at, but not to exceed.

APPENDIX A DERIVATION OF λ

Considering that with the original fare and disutility, all λ_0 of the total travelling demand is utilizing the PT, while when the sum of proxy fare and disutility equals to that of original PT, half of it would turn to proxy, then there is:

$$\lambda = \frac{\left(\lambda_0 - \frac{\lambda_0}{2}\right)}{\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}}\left(\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}\right) + \lambda_0 = \lambda_0\left(1 - \frac{\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}}{2\left(\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_{p0} - \frac{f_0}{\mu}\right)}\right)$$

While when we consider the logit model, similarly, we have:

$$\lambda = \frac{\lambda_0 exp\left(-\theta\left(\tilde{z}_p + \frac{f}{\mu}\right)\right)}{exp\left(-\theta\left(\tilde{z}_x + \frac{f_x}{\mu}\right)\right) + exp\left(-\theta\left(\tilde{z}_p + \frac{f}{\mu}\right)\right)} = \frac{\lambda_0}{exp\left(\theta\left(-\tilde{z}_x - \frac{f_x}{\mu} + \tilde{z}_p + \frac{f}{\mu}\right)\right) + 1}$$

APPENDIX B DERIVATION OF E_{PT}

First for E_{PT} we consider the distance travelled in the periphery and in the center separately. For E_{PT} in periphery, we mark it as R_P. Consider the distance between a random point on the boundary and a random point in the periphery on the same quadrant. Distance of a random point from cordon with side n on the periphery to a random point in the boundary is $\frac{3}{4}(n-d)$, and the PDF of it is $\frac{2n}{D^2}$. Thus, $E(R'_P) = \frac{D}{4}(2 - 3\alpha + \alpha^3)$. For C-P or P-C case, there is $E(R_P|C-P, P-C) = E(R'_P)$. For $P-P$ case, it becomes $E(R_P|P-P) = 2E(R'_P)$. Thus, with one of PDF of one of the OD in periphery already considered with $\frac{2n}{D^2}$, we only need to multiply the possibility of the other end, as $2\alpha^2(1-\alpha^2)$ for C-P or P-C case and $(1-\alpha^2)^2$ for P-P case when we are calculating the overall $E(R_P)$, as:

$$E(R_P) = \frac{D}{2}(1-\alpha^2)(2 - 3\alpha + \alpha^3)$$

Similarly marking the distance travelling in the center by PT as R_C, we divide 3 cases for calculation, as: A) Both OD are in the center with probability of α^4, B) Both OD are in the periphery with probability of $(1-\alpha^2)^2$, and C) One of the OD in the center and the other in the periphery with probability of $2(1-\alpha^2)\alpha^2$.

For case A, it is clear that:

$$E(R_C|A) = \frac{2}{3}d = \frac{2}{3}\alpha D$$

For case B, three cases are separated again, as a) Both OD are at the middle point with probability of $(1-\alpha)^2$; b) Neither OD are at the middle point with probability of α^2; and c) Only one of the OD are at the middle point with probability of $2\alpha(1-\alpha)$. Figuring out the expected values in each cases then adding them up, we can get:

$$\begin{aligned} E(R_C|B) &= E(R_C|Ba) * (1-\alpha)^2 + E(R_C|Bb) * \alpha^2 + E(R_C|Bb) * 2\alpha(1-\alpha) \\ &= \frac{7}{8}d * 2\alpha(1-\alpha) + \frac{3}{4}d(1-\alpha)^2 + \frac{11}{12}d\alpha^2 \\ &= \frac{\alpha D}{12}(9 + 3\alpha - \alpha^2) \end{aligned}$$

For case C, three cases are separated again, as a) The points on the same side with probability of 1/4; b) The points on opposite sides with probability of 1/4; c) The points on adjacent sides with probability of 1/2. Again by deriving them separately and summing up, there is:

$$\begin{aligned} E(R_C|C) &= E(R_C|Ca) * \frac{1}{4} + E(R_C|Cb) * \frac{1}{4} + E(R_C|Cc) * \frac{1}{2} \\ &= \alpha\left(\frac{\alpha}{12} + \frac{3}{4}\right)D \end{aligned}$$

Thus, for PT travels, the value of E_{PT} is the summation of R_C and R_P of each cases, being:

$$E_{PT}^{CP} = \frac{D}{12}(6 + 3\alpha^2 + 2\alpha^3 - 5\alpha^4 + \alpha^5 + 2\alpha^6 + \alpha^7)$$

$$E_{PT}^{CC} = \frac{2}{3}\alpha D$$

$$E_{PT}^{PP} = \frac{D}{12}(12 - 9\alpha + 3\alpha^2 + 5\alpha^3)$$

And total would be:

$$E_{PT} = E(R_P) + E(R_C) = \frac{D(12 - 9\alpha - 9\alpha^2 + 23\alpha^3 - 5\alpha^4 - 5\alpha^5 + 2\alpha^6 + \alpha^7)}{12}$$

APPENDIX C DERIVATION OF E_x

First of all, for CC case, it is the same as the PT travels in the center, since both of them are grid networks as:

$$E_x^{CC} = \frac{2}{3}\alpha D$$

While in the other cases, CP and PP we have to derive the expected value of travelling distance separately.

For CP, first we can divide into 2 cases, when the point in the periphery in at the corner or not.

When it is at the corner, the expected value would be:

$$E_x^{CP}(a) = D - \frac{1-\alpha}{2}D = \frac{1+\alpha}{2}D$$

And the possibility this case happens among CP case would be

$$\Pr(a|CP) = \frac{1-\alpha}{1+\alpha}$$

REFERENCES

1) Daganzo, C.F.: Structure of Competitive Transit Networks. *Transportation Research Part B*, Vol. 44, No 4,, pp. 434–446, 2010.
2) EMTA: Determining Fare Structures: Evidence and Recommendations from a Qualitative Survey among Transport Authorities, pp. 5, 2016.
3) Gwee, E. and Currie, G.: Review of Time-Based Public *Transport Fare Pricing. Land Transport Authorities*, Journey. Special Issue Sep. 2013, pp. 59, 63, 2013. Available at: https://www.lta.gov.sg/ltaacademy/doc/13Sep059-Gwee_Time-BasedPTFarePricing.pdf (Accessed by 2017.01.30)
4) Verhoef, E.T.: Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal tool levels and tool points *Transportation Research Part B*, Vol. 36. No. 8, pp. 707-729, 2012.
5) Beirao, G. et al.: Understanding attitudes towards public transport and private car: A qualitative study, *Transport policy*. Vol. 12, No. 6, pp. 278-489, 2007.
6) Li, Z.-C. et al. : The Optimal Transit Fare Structure under Different Market Regimes with Uncertainty in the Network, *Networks Spatial Economics*, Vol. 6, No. 2, pp. 191–216, 2009.
7) Pan, D.: Key Transport Statistics of World Cities, pp. 107, 2013. Available at: https://www.lta.gov.sg/ltaacademy/doc/13Sep105-Pan_KeyTransportStatistics.pdf (Accessed by 2017.01.31)
8) Barcelona City Council: Dades bàsiques de mobilitat 2013, pp. 20, 2014. Available at: http://prod-mobilitat.s3.amazonaws.com/DADESBASIQUES2013_p1.pdf (Accessed by 2017.01.31)
9) Kyoto City: 平成 22 年の京都市における人の動きについて（速報）～第 5 回
近畿圏パーソントリップ調査結果～. 平成２２年の京都市における人の動きについて（速報）, pp. 8, 2012 Available at: http://www.city.kyoto.lg.jp/tokei/page/0000164981.html (Accessed by 2017.01.31)
10) Kyoto City: 市バス旅客流動調査及び市バスお客様アンケート調査の結果について, 市バス旅客流動調査の結果, pp. 3, 2012. Available at: http://www.city.kyoto.lg.jp/kotsu/page/0000133511.html (Accessed by 2017.01.31)
11) Kyoto City Bus: 京都市バス・地下鉄路線図, 2016. http://www.city.kyoto.lg.jp/kotsu/page/0000019770.html (Accessed by 2017.01.31)
12) Kyoto Taxi Fare: http://www.taxisite.com/far/info/26.aspx (Accessed on 2017.01.20).
13) Miles, J. W. : On the generation of surface waves by shear flows, *J. Fluid Mech*., Vol. 3, Pt. 2, pp. 185-204, 1957.
14) Miche, M. : Amortissement des houles dans le do-maine de l'eau peu profonde, *La Houile Blanche*, No. 5, pp. 726-745, 1956.
15) Gresho, P. M., Chan, S. T., Lee, R. L. and Upson, C. D. : A modified finite element method for solving the time-dependent incompressible Navier-Stokes equations, part 1, *Int. J. Numer. Meth. Fluids*, Vol. 4, pp. 557-598, 1984.
16) Shepard, F. P. and Inman, D. L. : Nearshore water circulation related to bottom topogrphy and wave refraction, *Trans. AGU*., Vol. 31, No. 2, 1950.
17) Smith, W. : Cellular phone positioning and travel times estimates, *Proc. of 8th ITS World Congress*, CD-ROM, 2000.

(Received April 28, 2017)