

System-optimal dynamic traffic assignment in a corridor network: An analytical approach and regularities of traffic flow patterns

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This paper studies system-optimal dynamic traffic assignment for morning commute in a corridor network with multiple bottlenecks. We reveal regularities of traffic flow patterns and provide analytical solution to this problem. Our results elucidate the precise relationship between bottleneck capacities and traffic flow patterns. By further comparing traffic flow patterns of dynamic system-optimum solution and dynamic user-equilibrium in the problem, we investigate similarities between them.

Key Words: *dynamic traffic assignment; departure-time choice; corridor network; analytical solution; DSO vs. DUE*

1. INTRODUCTION

Traffic congestion caused by morning commute is a common challenge in most metropolises. Great loss due to it has drawn much attention of researchers. Previous studies on traffic flow behaviors in morning commute can be traced back to Vickrey (1969). Vickrey's bottleneck model has been further involved in a lot of literatures (e.g., Smith, 1984; Daganzo, 1985; Arnott et al., 1990; Kuwahara, 1990; Lindsey, 2004). The model takes advantage of simple network and focus on departure-time choices of commuters in traffic peaks. Recently, researchers started to study this problem in corridor networks with multiple bottlenecks (Arnott and De Palma, 2011; Akamatsu et al., 2015), which naturally includes spatial dynamics of congestions. Besides these studies on the dynamic user-equilibrium (DUE), many focus on system-optimum dynamic traffic assignment in the problem as benchmark of the most efficient allocation of traffic flow in morning commute (Daganzo and Garcia, 2000; Muñoz and Laval, 2006; Shen and Zhang, 2009).

Nevertheless, previous studies hardly revealed analytical properties of traffic flow patterns in a corridor network. Consequently, relationship between the dynamic system-optimum (DSO) solution and dynamic user-equilibrium (DUE) have not been clarified.

In this paper, we first investigate the DSO solution

for morning commute in a corridor network with multiple bottlenecks. Based on the analytical approach in this paper, we are able to provide an explicit solution and investigate regularities of the traffic flow pattern in the solution for the first time. Second, we compare the DSO and DUE problems and provide insights into the relationship between them. Similarity of flow patterns in the two problems are revealed.

In the rest of this paper, Section 2 models the DSO problem. Section 3 presents analytical properties of solutions to the problem. Section 4 considers the DUE problem and reveals similarities between flow patterns in the two problems. Section 5 confirms our findings by numerical examples. Finally, Section 6 concludes the paper.

2. MODEL OF THE DSO PROBLEM

(1) Model settings

Consider a freeway corridor that connects several residential locations to a central business district (CBD) (see Fig.1). The residential locations are indexed sequentially from the CBD. We denote the set of locations by $\mathbf{I}=\{1,2,\dots,I\}$. There is a single bottleneck with capacity μ_i just downstream of the on-ramp from the location $i \in \mathbf{I}$. The bottlenecks are modeled by the point queue model; at each

bottleneck, a queue is formed vertically when the inflow exceeds the capacity. The traffic demand at each location $i \in \mathbf{I}$ is a given constant Q_i .

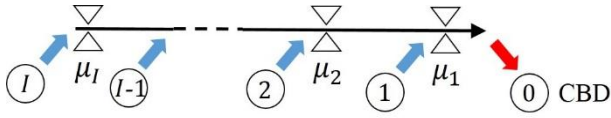


Fig.1 The corridor network

To be specific, we assume that tradable network permits (TNP) scheme (Wada and Akamatsu, 2013) is implemented as a first-best traffic policy. In this scheme, a permit is a right that allows a permit holder to pass through a pre-specified bottleneck during a pre-specified time period, and a perfectly competitive trading market for the network permits is launched. By limiting the number of permits to be either equal to or less than the capacity of the bottleneck, the queue at each bottleneck is eliminated. This scheme internalizes congestion externalities and is mathematically equivalent to an optimal dynamic road pricing scheme that eliminates congestion.

Each commuter chooses a departure-time $t \in \mathbf{T}$ to minimize his/her own travel cost $T_i(t)$, where \mathbf{T} denotes a sufficiently large time window to allow all commuters to arrive at the CBD. For the time index t , we take the arrival time at the CBD rather than the departure time at the residential location (Akamatsu et al., 2015). The travel cost is defined as

$$T_i(t) = s(t) + c_i + \sum_{j=1}^i p_j(t) \quad (1)$$

where $c_i(t)$ is the free-flow travel cost from location i to the CBD, $s(t)$ is the quasi-convex schedule-delay function with unique minimum at the desired arrival time $t=t_d$, $p_i(t)$ denotes the price of a permit at bottleneck i with the arrival time t . The third term in $T_i(t)$ is thus the total amount of permit costs a commuter has to pay to arrive at t .

(2) Dynamic equilibrium under first-best traffic control scheme

We formulate the dynamic equilibrium under TNP scheme. Let $q_i(t)$ be the arrival-flow rate at time t of commuters residing at i and thus $\{q_i(t)\}$ represents the arrival-flow pattern. Then, the equilibrium under TNP scheme can be formulated by three conditions. Firstly, the no-arbitrage condition of commuters' arrival-time choices asks that no commuter has incentive to alter his/her choice of arrival time:

$$0 \leq [T_i(t) - \rho_i] \perp q_i(t) \geq 0 \quad (2)$$

where ρ_i states the equilibrium commuting cost of commuters residing at location i . Secondly, the demand-supply equilibrium condition at permit

market asks that

$$0 \leq \left[\mu_i - \sum_{j=i}^I q_j(t) \right] \perp p_i(t) \geq 0 \quad (3)$$

Finally, the conservation constraint of commuters requires that

$$0 \leq \int_{\mathbf{T}} q_i(t) dt - Q_i \perp \rho_i \geq 0 \quad (4)$$

An equilibrium under TNP scheme is a set of variables $\{q_i(t), \rho_i, p_i(t)\}$ that satisfies the above conditions. The following theorem gives an equivalent linear programming to this formulation.

Theorem 2.1. *The equilibrium under TNP scheme is the solution to the following linear programming (LP):*

$$\min_{q \geq 0} \sum_{i \in \mathbf{I}} \int_{t \in \mathbf{T}} s(t) \cdot q_i(t) dt \quad (5)$$

$$s.t. \sum_{j=i}^I q_j(t) \leq \mu_i \quad \forall i \in \mathbf{I}, t \in \mathbf{T} \quad (6)$$

$$\int_{\mathbf{T}} q_i(t) dt = Q_i \quad \forall i \in \mathbf{I} \quad (7)$$

By this theorem, numerical solution to the equilibrium under TNP scheme can be easily obtained by solving this LP. Notice that the free-flow travel cost c_i is insignificant in the formulation and thus omitted.

3. ANALYTICAL APPROACH TO THE DSO PROBLEM

(1) Reducibility of tandem bottlenecks

In this subsection, we will expound that which of the bottlenecks in our model deserve concerns and others are dispensable. By doing this, we actually investigate the reducibility of tandem bottlenecks and simplify the network without loss of generality.

By Eq.(3) we know that if the permit price $p_i(t)$ equals zero for all $t \in \mathbf{T}$, then the capacity constraint of bottleneck i is not binding and we call it a “false bottleneck”. A “false bottleneck” is actually not a bottleneck in the DSO solution because traffics get through it as free-flow at all times. We define a “false bottleneck” formally in the following.

Definition 3.1. *A bottleneck i with the permit price $p_i(t)=0, \forall t \in \mathbf{T}$ in the DSO solution is called a **false bottleneck**, and a network with no false bottleneck is called a **reduced network**.*

The concept of reduced network is the key to

derive the analytical solution as we will show later. The case of interest here is how to determine the underlying reduced network of an arbitrary corridor network. For convenience of expression, we first define normalized demand at each bottleneck with respect to its capacity as

$$\psi_i = \sum_{j=i}^I Q_j / \mu_i \quad (8)$$

To construct the reduced network from the original corridor network, we provide the criterion to screen out false bottlenecks in a corridor network by the following lemma.

Lemma 3.1. *In a corridor network with I tandem bottlenecks, bottleneck $i(i \leq I)$ is not a false bottleneck if and only if*

$$\psi_i > \psi_k \quad \forall k < i \quad (9)$$

This lemma enables us to detect false bottlenecks in a corridor network. We can then construct the reduced network by the following algorithm.

Algorithm 3.1. (construct reduced network from a corridor network $\{(\mu_i, Q_i) | i=1, 2, \dots, I\}$):

Step 0. Let $n := 0$;

Step 1. **For** i from 2 to I ,

If $\psi_{i-n} \leq \psi_{i-n-1}$,
 $Q_{i-n-1} := Q_{i-n-1} + Q_{i-n}$;
For k from $i-n$ to $I-n$,
 $Q_k := Q_{k+1}$;
 $\mu_k := \mu_{k+1}$;

$n := n + 1$;

Step 2. Return $\{(\mu_i, Q_i) | i=1, 2, \dots, I-n\}$ as the reduced corridor network.

(2) Analytical solution

To present properties of flow patterns in a reduced network conveniently, let t_i^e and t_i^l be the earliest arrival time and the latest arrival time of commuters residing at location i respectively. Consequently, the arrival-time window of location i is $\mathbf{T}_i \equiv [t_i^e, t_i^l]$. The following lemma provides regularities of the DSO arrival-flow pattern in a reduced corridor network.

Lemma 3.2. *In a reduced corridor network, arrival-time windows of the DSO solution satisfy*

$$\mathbf{T}_i \subset \mathbf{T}_{i+1} \quad \forall i < I \quad (10)$$

By this lemma, we can show the relationship between the equilibrium commuting cost ρ_i and arrival-time window \mathbf{T}_i as Fig.2: for each location i , the difference between ρ_i and $s(t)$ equals the total permit-price cost from i to the CBD. The permit price

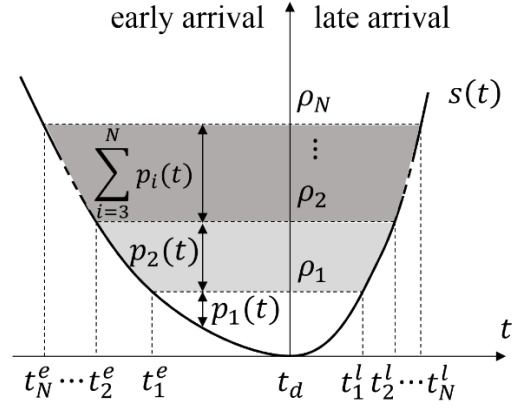


Fig.2 Relationships between variables of the DSO solution in a reduced corridor network

of each bottleneck can be determined respectively and exhibits a layered structure as shown in the figure. The arrival-time window $[t_i^e, t_i^l]$ is graphically determined by the equilibrium commuting cost ρ_i and the schedule-delay function $s(t)$. This result further implies a relationship between bottleneck capacities and arrival-time windows. To summarize this relationship formally in the following theorem, let

$$T_i \equiv |\mathbf{T}_i| = t_i^l - t_i^e \quad \forall i \in \mathbf{I} \quad (11)$$

Theorem 3.1. *In a reduced corridor network, the DSO solution satisfies*

$$T_i \cdot \hat{\mu}_i = Q_i \quad \forall i \in \mathbf{I} \quad (12)$$

where

$$\hat{\mu}_i = \begin{cases} \mu_i - \mu_{i+1} & \text{if } i < I \\ \mu_i & \text{if } i = I \end{cases} \quad (13)$$

This theorem show that the difference between capacities of adjacent bottlenecks $\hat{\mu}_i$ provides more meaning than capacity itself. The length of arrival-time window $R(\rho_i)$ is the ratio between local demand Q_i and $\hat{\mu}_i$. This implies regularities of the DSO flow pattern: (1) Upstream demands (with larger index i) have the priority to pass bottlenecks in the sense that only the extra capacity of bottleneck i over bottleneck $i+1$ is occupied by the local demand Q_i . (2) The assignment of commuters with respect to arrival-times has an ‘‘all-or-nothing’’ pattern: the arrival-flow rate $q_i(t)$ equals $\hat{\mu}_i$ for arrival time in the arrival-time window \mathbf{T}_i and equals zero for arrival time outside. By this theorem, we can further derive the analytical solution. For convenience of expression, define

$$R(\rho_i) = t_i^l - t_i^e \quad \forall i \in \mathbf{I} \quad (14)$$

Corollary 3.1. *As the analytical solution to the DSO problem in a reduced corridor network, the arrival-flow pattern of each location is*

$$q_i(t) = \begin{cases} \hat{\mu}_i & \text{if } t \in \mathbf{T}_i \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathbf{I} \quad (15)$$

The equilibrium commuting cost of each location is

$$\rho_i = R^{-1}(Q_i / \hat{\mu}_i) \quad \forall i \in \mathbf{I} \quad (16)$$

The permit price at each bottleneck is

$$p_i(t) = \begin{cases} \max\{\rho_i - s(t), 0\} & \text{if } i = 1 \\ \max\{\rho_i - s(t) - \sum_{j=1}^{i-1} p_j(t), 0\} & \text{if } i > 1 \end{cases} \quad \forall t \in \mathbf{T} \quad (17)$$

4. DSO VS. DUE

In this section, solutions to the DSO and DUE problems are compared to elucidate similarities in traffic flow patterns of the two problems. Before this, we consider the DUE problem based on the same approach as in the DSO problem.

(1) The DUE problem

As proposed in Akamatsu et al. (2015), the formulation for the DUE problem in a corridor network is

$$0 \leq [T_i(t) - \rho_i] \perp q_i(t) \geq 0 \quad (18)$$

$$0 \leq \left[\mu_i \cdot \sigma'_i(t) - \sum_{j=i}^I q_j(t) \right] \perp w_i(t) \geq 0 \quad (19)$$

$$0 \leq \int_{\mathbf{T}} q_i(t) dt - Q_i \perp \rho_i \geq 0 \quad (20)$$

where

$$\sigma'_i(t) = 1 - \sum_{j=1}^{i-1} w'_j(t) \quad (21)$$

Comparing with that for the DSO problem, formulation for the DUE problem substitutes permit price $p_i(t)$ and bottleneck capacity $\hat{\mu}_i$ with queuing delay $w_i(t)$ and “virtual” bottleneck capacity $\hat{\mu}_i \cdot \sigma'_i(t)$. The difficulty of the DUE problem is how to determine $\sigma'_i(t)$. Here, we consider a DUE with $w_i(t)$ having the same value of $p_i(t)$ for each bottleneck i at all $t \in \mathbf{T}$. By the same approach in the DSO problem, we can obtain the same equilibrium commuting cost $\{\rho_i(t)\}$ as that in the DSO problem but a different arrival-flow pattern $\{q_i^{DUE}(t)\}$:

$$q_i^{DUE}(t) = \begin{cases} \hat{\mu}_i - \mu_{i+1} \cdot s'(t) & \text{if } t \in \mathbf{T}_i - \mathbf{T}_{i-1} \\ \hat{\mu}_i \cdot [1 + s'(t)] & \text{if } t \in \mathbf{T}_{i-1} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathbf{I} \quad (22)$$

where $t_0^e = t_0^l = t_d$, $\mu_{I+1} = 0$.

One more thing that needs to be concerned here is the condition for the coincidence between cost variables $\{w_i(t)\}$ and $\{p_i(t)\}$, which is provided by the following theorem.

Theorem 4.1. *The queuing delay $\{w_i(t)\}$ in the DUE coincides with permit price $\{p_i(t)\}$ in the DSO solution in a reduced corridor network if and only if*

$$-1 \leq s'(t) \leq \mu_i / \mu_{i+1} - 1 \quad \forall i < I, t \in \mathbf{T}_i \quad (23)$$

Notice that the left-hand side of Eq.(23) is actually the existence condition of DUE by Akamatsu et al. (2015) but in a reduced corridor network. The intuitive interpretation of this part is clear: commuters must prefer early-arrival schedule to queuing delay, i.e. the cost of early-arrival schedule must be less than that of the same amount of queuing delay. The right-hand side of Eq.(23) is the condition for coincidence. This condition actually asks that no commuter arrives later than t_i where $s'(t_i) = \mu_i / \mu_{i+1} - 1$ and $t_i > t_d$, because late-arrival schedule delay increases faster than the dissipation of queuing delay at any arrival time later than t_i in this solution.

(2) Comparison of traffic flow patterns

The important characteristic of the coincidence introduced in the previous subsection is the similarity between DSO and DUE traffic flow patterns. Consider the aggregate outflow

$$y_i(t) = \sum_{j=i}^I q_j(t) \quad (24)$$

of each bottleneck i in the two problems, or equivalently the link flow between adjacent bottlenecks (superscript for distinguishing DSO and DUE patterns):

$$y_i^{DSO}(t) = \begin{cases} \mu_i & \text{if } t \in \mathbf{T}_i \\ \mu_j & \text{if } t \in \mathbf{T}_j - \mathbf{T}_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathbf{I}, j > i \quad (25)$$

$$y_1^{DUE}(t) = \begin{cases} \mu_1 & \text{if } t \in \mathbf{T}_i \\ \mu_j & \text{if } t \in \mathbf{T}_j - \mathbf{T}_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad \forall j > 1 \quad (26)$$

$$y_i^{DUE}(t) = \begin{cases} \mu_i \cdot [1 + s'(t)] & \text{if } t \in \mathbf{T}_i \\ \mu_j & \text{if } t \in \mathbf{T}_j - \mathbf{T}_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad \forall i > 1, j > i \quad (27)$$

As there is no queue in the DSO solution, the DSO flow pattern $y_i^{DSO}(t)$ with respect to arrival-time t is the same as $f_i^{DSO}(\sigma_i(t))$ with respect to departure-time $\sigma_i(t)$ from bottleneck i . As for the DUE, we have

$$f_i^{DUE}(\sigma_i(t)) = y_i^{DUE}(t) \cdot \sigma_i'(t) \quad (28)$$

By this, it is derived that

$$f_i^{DUE}(\sigma_i(t)) = \begin{cases} \mu_i & \text{if } t \in \mathbf{T}_i \\ \mu_j & \text{if } t \in \mathbf{T}_j - \mathbf{T}_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathbf{I}, j > i \quad (29)$$

which is the same as $f_i^{DSO}(\sigma_i(t))$. This result reveals that the link-flow pattern of the DSO solution and DUE with respect to the departure-time are the same.

5. NUMERICAL EXAMPLES

Numerical examples in this section present our findings on regularities of the DSO and DUE flow patterns. A reduced corridor network with three bottlenecks is considered and the existence condition of DUE is satisfied in all examples. Schedule delay functions are linear for both early and late arrivals. In each figure of the examples, solid curves stand for the DSO flow pattern and dashed curves stand for the DUE flow pattern. The first row in each figure shows aggregate cumulative flow of each bottleneck and the second to the fourth rows are disaggregate cumulative flow with respect to each location (from top to bottom is location 1 to 3 in turn). Blue curves stands for cumulative departure-flow from bottlenecks and the space between blue and red curves stands for permits price in the DSO solution and queuing delay in the DUE respectively. The desired arrival-time $t_d=35$ is marked for each subfigure.

Fig.4 shows flow patterns with the coincidence

condition satisfied, meaning that the DUE is coincident with DSO solution. As the numerical result shows, flow rate of the DSO arrival-flow from each location (disaggregate flow) is constant, which is presented by linear arrival-flow curves in the figure. The permit price at each bottleneck fits the layered structure of a triangle for linear early-arrival and late-arrival penalty. As for the DUE pattern, the link-flow patterns (blue curves in the first row) is the same as that in the DSO pattern, though disaggregate flow patterns are quite different.

In examples shown by Fig.5,6,7, we gradually increase the late-arrival penalty (slope of the linear late-arrival schedule delay function) to show flow patterns with the consistency condition unsatisfied. The example in Fig.7 almost has no late arrivals due to high late-arrival penalty. DSO patterns in these examples show identical regularities as those in Fig.4. But DUE patterns do not exhibit the consistency with DSO patterns in the aggregate departure flow as in Fig.4. As a whole, aggregate flow of the DUE pattern in these examples take more cost than those of the DSO pattern due to earlier arrivals (more schedule delay cost). However, disaggregate flow patterns show that only the flow of location 3 suffers this extra cost and flow of location 2 and 1 is even more concentrated around the desired arrival-time. This phenomenon verifies the priority of commuters residing at further locations in the DSO pattern as previously mentioned.

6. CONCLUDING REMARKS

This study explored properties of DSO solution under TNP scheme for morning commute in a corridor network with multiple bottlenecks. Based on reducibility of tandem bottlenecks, we developed an analytical approach that gains insights into system-optimal traffic controls and traffic flow patterns in a reduced corridor network. In the DSO solution, (1) arrival-time (departure-time) windows of upstream bottlenecks always cover those of downstream bottlenecks in the DSO solution (Lemma 3.2 and Fig.2); (2) traffic flow from each location follows the “all-or-nothing” pattern, which means the arrival-flow (departure-flow) rate of each location is a positive constant in the arrival-time window and is zero outside the arrival-time window (Theorem 3.2); (3) the flow rate in arrival-time windows equals the extra capacity of the bottlenecks at just downstream of the location over that of the bottleneck at just upstream (Eq.(14)). In the DUE problem, (1) we derived the condition for coincidence between queuing delay in DUE and permit price in DSO solution (Theorem 4.1); (2) the DUE that coincides with DSO solution has the same link-flow pattern as that of the DSO solution (Eq.(29)). Explicit results in this paper extended analytical approach in traffic

congestion problems, especially those involve with departure-time choices. Our results have a potential to be extended in various aspects (e.g. heterogeneity of commuters) of these problems and will be explored in future.

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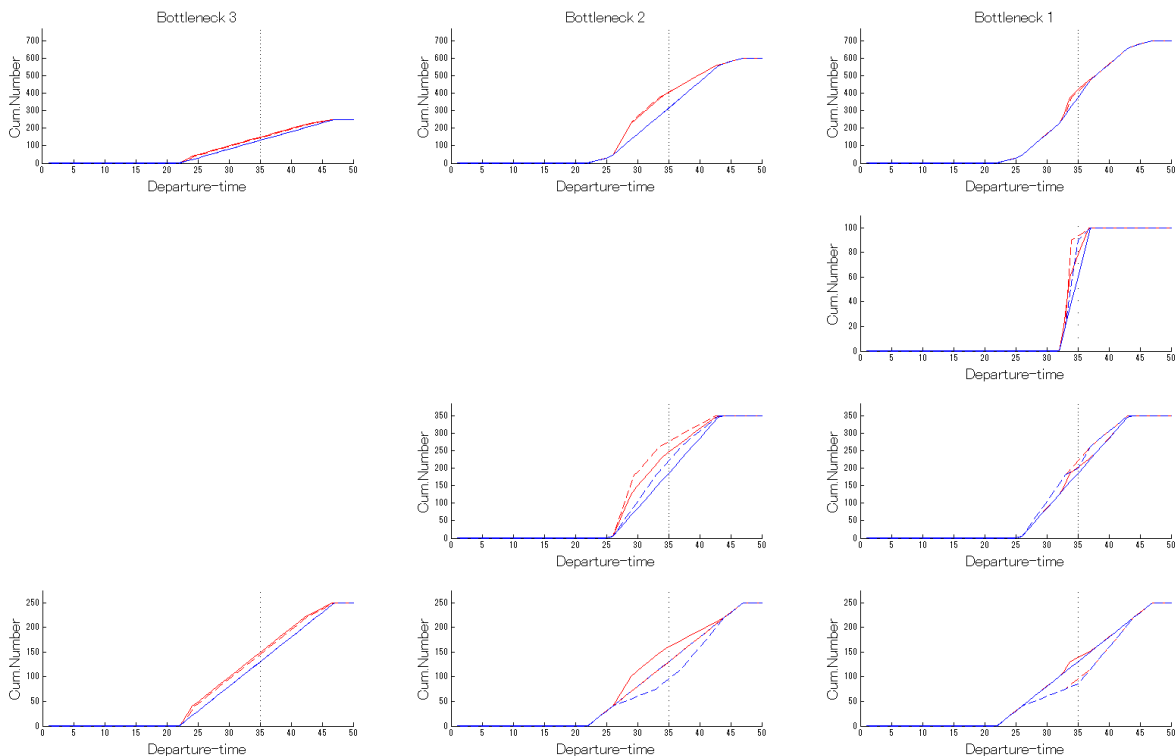


Fig.4 Flow patterns with $s(t)=\max\{0.5(35-t),0.5(t-35)\}$. DUE (dashed) is consistent with DSO (solid) in link flows (blue curves in the first row).

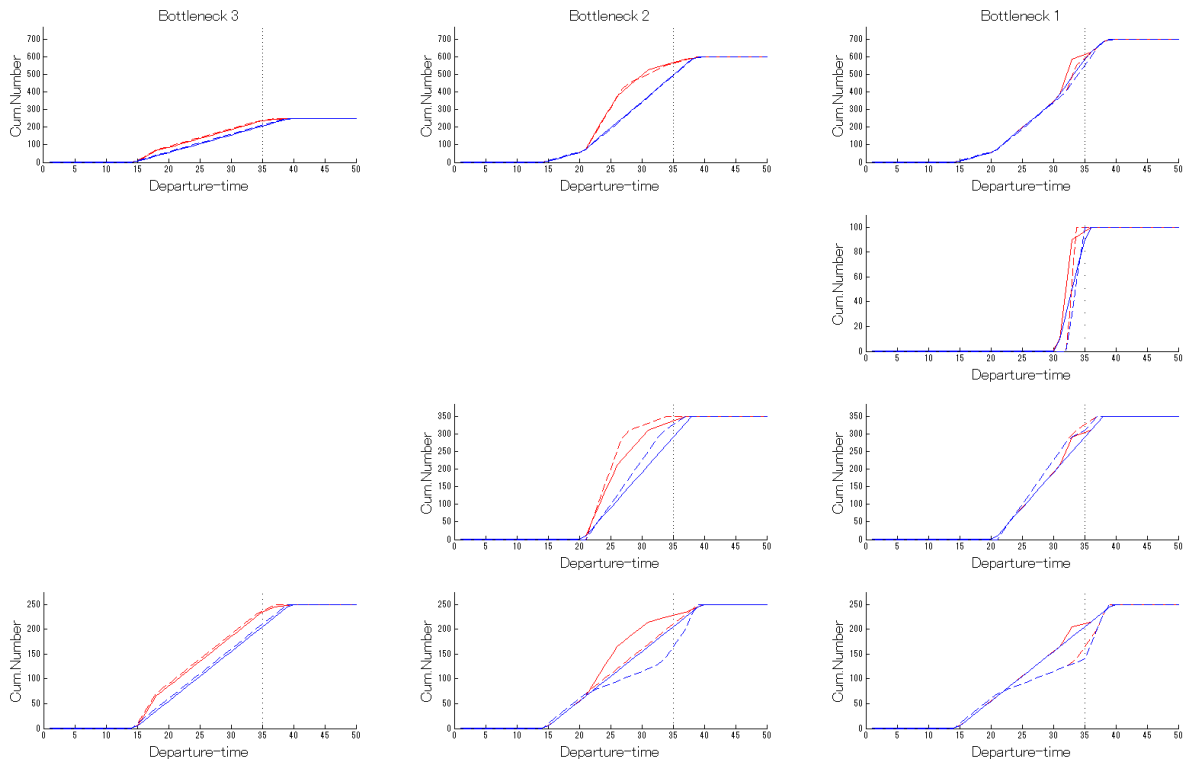


Fig.5 Flow patterns with $s(t)=\max\{0.5(35-t),2(t-35)\}$. DUE (dashed) starts to deviate from DSO (solid) in link flows.

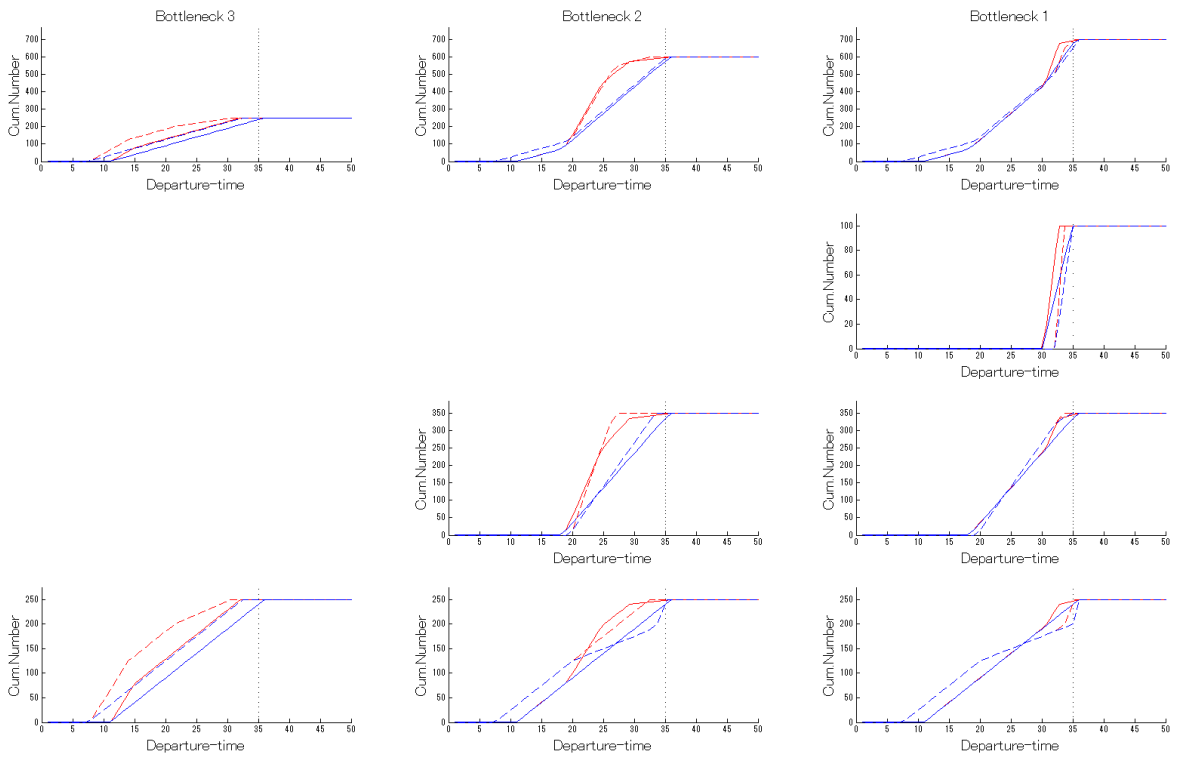


Fig.6 Flow patterns with $s(t)=\max\{0.5(35-t),8(t-35)\}$. DUE (dashed) deviates from DSO (solid) in link flows.

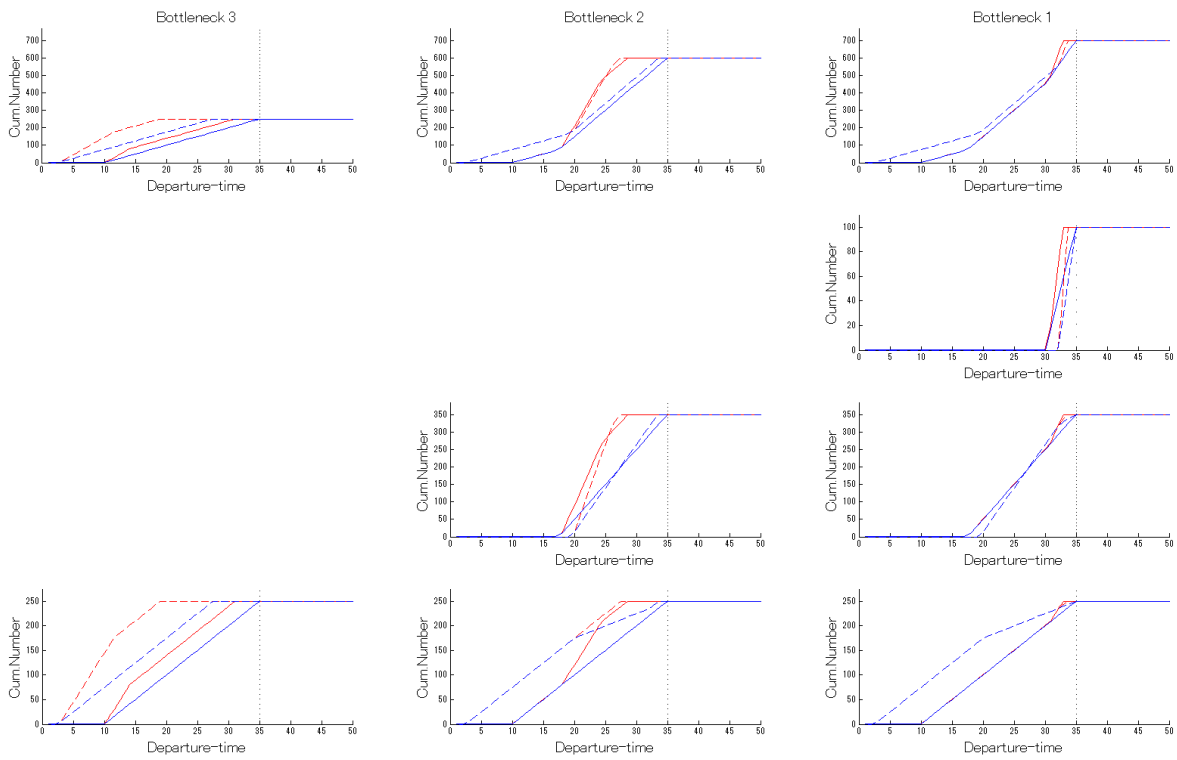


Fig.7 Flow patterns with $s(t)=\max\{0.5(35-t),32(t-35)\}$. DUE (dashed) significantly deviates from DSO (solid) in link flows.