Concurrent Prediction of Location, Velocity and Acceleration Profiles for Left Turning Vehicles at Signalized Intersections

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Conflicts between left-turning vehicles and road crossing pedestrians or cyclists is one major safety issue at signalized intersections. Simulation based as well as non-simulation based conflict analysis methods can be utilized to explore the characteristics of these conflicts such as time to collision, collision point and speed. Existing microscopic simulation based studies have considered vehicle paths and speed profiles separately with separate models and therefore the spatio-temporal consistency between the path (or location) and the speed of a turning vehicle is questionable. This paper explores a method that can estimate a path of a turning vehicle, corresponding speed and acceleration profiles simultaneously and with a good accuracy. Trajectory data of turning vehicles collected at signalized intersections in Nagoya city are used to develop this model. Proposed method in this study could be used in enhancing microscopic simulation models and evaluating non-simulation-based surrogate safety measures based on more realistic representation of turning vehicle trajectories.

Key Words : *Left-turning trajectories, Signalized intersections, Pedestrian safety, Intersection geometry*

1. INTRODUCTION

As highlighted in several previous studies, conflicts between turning vehicles and road crossing pedestrians or cyclists are considered as a major safety issue related with signalized intersections[1,2]. Although pedestrians and cyclists are given the right-of-way at signalized intersections during the shared green phase, drivers might attempt to compete for small gaps creating a dangerous environment for pedestrians and cyclists who are more vulnerable than drivers. Reasons like geometrical characteristics of intersections, characteristics of traffic signal phases and driver behavior have made it harder to ensure the safety of pedestrians and cyclists. Thus, it is important to understand the mechanism of crash occurrence at signalized intersections.

Among those approaches, which are available for crash prediction and safety assessment at signalized intersections, historical crash statistics based methods[3,4] are probably the most famous method. Relationship between crash occurrence and contributing factors, such as characteristics of drivers and geometrical features of intersections, are statistically modelled in these approaches. Apart from these, traffic conflict analysis methods[5,6] are also a famous method that uses empirical data. For both these methods, sufficient amount of data are required to obtain reliable estimates for crash or conflict occurrences and related measures. Collecting and processing such empirical data are time consuming and expensive. Further, with these approaches it is difficult to conduct safety assessments at planning stages. Due to these limitations, traffic microscopic simulation based surrogate safety methods, became famous in later years[7]. However, most of these existing microscopic simulation tools simplify vehicle maneuvers, for example turning movements, inside intersections. Due to these simplifications, properties of trajectories of turning vehicles, such as variations in speed and acceleration, might not have been accurately captured. Thus, it is questionable whether these simulation models are suitable for safety assessments at intersections. In order to overcome these limitations in existing simulation based approaches, recent studies have developed microscopic simulation models which are dedicated for safety evaluation at signalized intersections[8]. However, these approaches have considered vehicle paths and speed profiles separately with separate models. For example, Tan et al.[8] considered vehicle turning path model by Asano et al.[1] and speed profile models by Wolfermann et al.[9] separately in their simulation model. Combination of different models does not guarantee the spatio-temporal consistency between the location and the speed of a turning vehicle. Thus, properties of turning vehicle trajectories might not be captured accurately. Further, not only path and speed profiles but also additional information, such as jerk profiles, are also required to identify critical traffic conflicts accurately[10]. Considering these limitations in existing literature, this paper develops a modelling approach that can estimate a path and kinematic information (speed, acceleration and jerk profiles) of a turning vehicle simultaneously and with a good accuracy. Details of the modelling method and comparisons with empirical trajectories are discussed in following sections.

2. MODELLING APPROACH

The proposed modelling approach is based on minimum-jerk theory (jerk is defined as the time derivative of acceleration) which has been initially utilized to describe skilled human arm movements on a plane (2-dimensional space). Theoretical background of this approach is briefly described in this section along with the trajectory data collected at several signalized intersections for developing and testing the model.

(1) Minimum-jerk theory

Flash and Hogan[11] demonstrated that the smoothness of a planar arm movement in reaching, writing and drawing tasks can be evaluated as a function of jerk. As they described the cost function J that should be minimized can be given as:

$$J = \frac{1}{2} \int_0^{t_f} \left(\left(\frac{d^3x}{dt^3} \right)^2 + \left(\frac{d^3y}{dt^3} \right)^2 \right) dt \qquad (1)$$

Here t_f is the movement time from an initial position to a final position.

Later this theory has been utilized for complex movement planning problems in robot limbs[12,13] and motion planning and control problems in autonomous vehicles[14]. Shamir[15] utilized minimum-jerk theory to design smooth and ergonomic optimal overtaking maneuvers for autonomous vehicles. In addition to these studies, car following behavior was also described with minimum jerk theory[16]. The main assumption of this model by Hiraoka et al.[16] was that the car following behavior (towards the vehicle in front) matches with skilled reaching movements of the human arm. In the proposed approach we describe in this paper, turning maneuvers performed by drivers at signalized intersections are also treated as skilled tasks. Therefore, it is assumed that vehicle turning maneuvers can also be described with minimum jerk theory.

(2) Trajectory data

Video data was collected at several signalized intersections located in Nagoya city, Japan for model development and testing purposes. Geometric characteristics of study sites considered in this paper are summarized in Table 1.

Table 1 Geometric characteristics of considered intersections.

Intersection	Approach	Corner radius (m)	Intersection angle (°)
Suemori-dori 2	E	9.7	88.3
Kawana	W	21	106
Nishi-osu	W	17	76.9
Ueda	E	9.5	65
	S	13.5	119

These intersections display different geometrical characteristics and a shared green signal phase has been assigned for left turning vehicles and pedestrians/cyclists. Trajectories of left turning vehicles at these intersections were extracted using the video image processing system TrafficAnalyzer[17]. Out of those trajectories only free-flowing vehicles, i.e., vehicles with no impact from pedestrians and cyclists, were considered in this paper.

How the minimum-jerk theory is applied to model trajectories of turning vehicles is discussed in next section. Further, properties of model parameters (or boundary conditions) obtained from empirical data are also briefly discussed.

3. MODELLING TRAJECTORIES OF TURNING VEHICLES

As described in previous section the objective function to be minimized is the time integration of squared that is given in Equation 1. Solution for this minimization problem was obtained by Flash and Hogan[11] as a set of fifth order polynomials of time as follows:

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \quad (2a)$$

$$y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \quad (2b)$$

Where;

a_j and b_j (j = {0, ..., 5}) are constants

In order to solve this system of equations 12 boundary conditions are required. Initial and final locations of the trajectory are considered as the turn initiation point and the turn completion point (i.e. two ends of the curved path of a vehicle trajectory) respectively. These can be easily obtained from geometry data of the intersection and its periphery. Speeds and accelerations of a vehicle at entry and exit locations are dependent on the characteristics of the entry and exit links. These speed and acceleration information at entry and exit points were considered as model inputs. Thus, location, velocity and acceleration vectors (in 2-D space) at initial and final points provide 12 boundary conditions.

However, t_f is still an unknown. The value of t_f primarily depends on the initial speed as can be understood from Figure 1. As explains in this figure a vehicle with higher approaching speed negotiate a particular turn faster compared to a slower vehicle. Further, entry speed may have effects from geometrical characteristics, such as corner radius and intersection angle, of the intersection and as a result t_f may also be affected. Thus, t_f could be obtained as a function of entry conditions and intersection geometries. However, in this paper we do not intend to comprehensively model t_f. Instead we investigate the trajectory variation for different initial, final conditions and estimated t_f. Later in this paper (Section 3(2)) we compare those with trajectories generated with complete information (i.e., initial and final conditions with t_f).

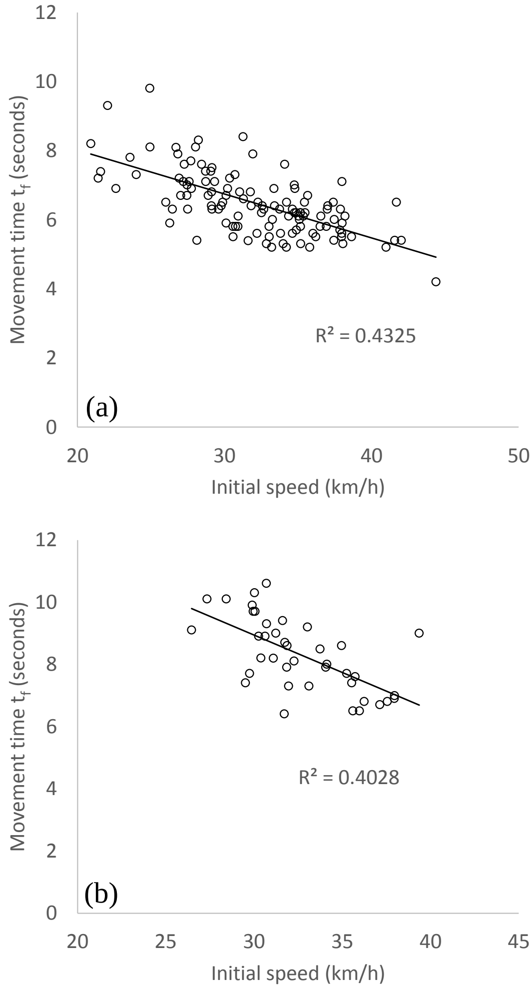


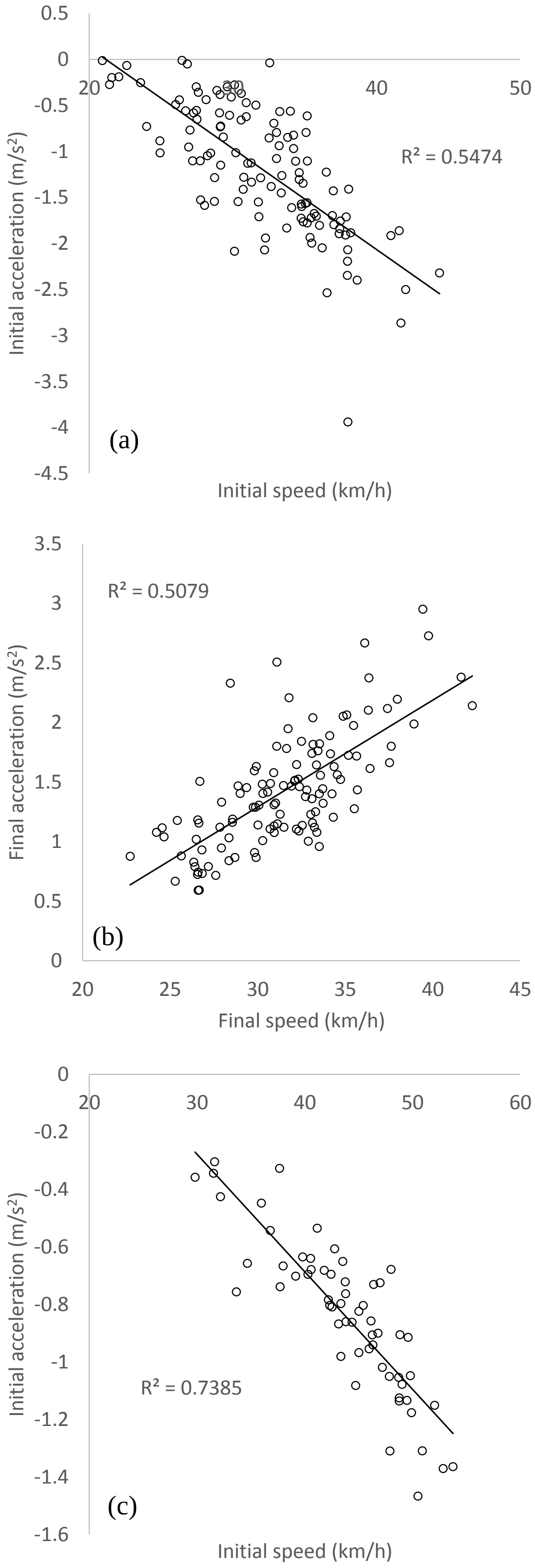
Fig.1 Variation of t_f with entry speed: (a)-Suemori-dori 2 East approach; (b)- Nishi-Osu West approach.

(1) Predicting turning trajectories and variations

t_f showed a strong correlation with initial speed for all intersections and approaches considered in this paper. Thus, initially t_f was modelled as a linear function of initial speed for each intersection (Figure 1). Initial and final speeds and accelerations were set based on distributions obtained from empirical data.

Variation of trajectories were tested for different initial and final conditions as follows:

a. Initial 85th percentile speed and final 85th percentile speed
b. Initial 85th percentile speed and final 15th percentile speed
c. Initial 15th percentile speed and final 85th percentile speed
d. Initial 15th percentile speed and final 15th percentile speed
e. Initial 50th percentile speed and final 50th percentile speed



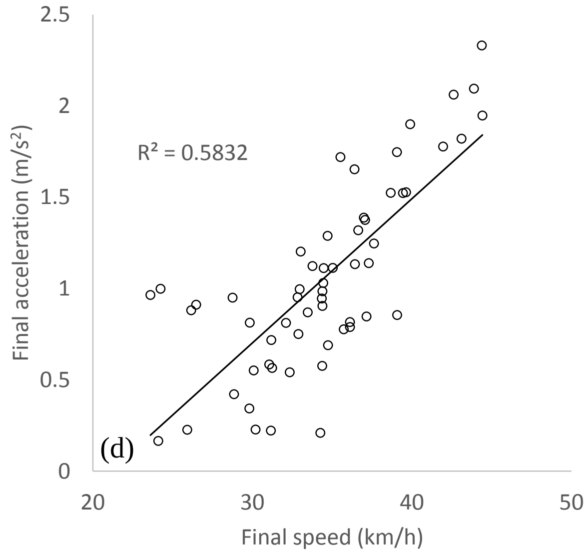


Fig.2 Variation of initial and final accelerations with initial and final speeds for: (a) and (b)-Suemori-dori 2 East approach; (c) and (d)- Ueda South approach.

Purpose of setting such initial and final conditions was to check the variation of trajectories generated for a given initial condition and a range or possible final conditions.

For each of these initial or final speeds acceleration values were set accordingly. That is because, strong correlations were observed between initial speed and initial acceleration and final speed and final acceleration. Relationships between initial speed and initial acceleration, final speed and final acceleration for two intersections are depicted in Figure 2. Thus, for 85th percentile of initial speed, 15th percentile of acceleration (or 85th percentile of deceleration) and for 85th percentile of final speed, 85th percentile of acceleration were chosen as inputs. Similarly, for 15th percentile of initial speed, 85th percentile of acceleration and for 15th percentile of final speed, 15th percentile of acceleration were chosen. For 50th percentile initial and final speeds (case e above), 50th percentile initial and final accelerations were set. For all these cases t_f were calculated based in the initial speed.

Equation 2(a) and (b) were solved for those settings (initial and final conditions for inner and outer lanes and t_f) separately. Obtained trajectories (paths, speed, acceleration profiles) are compared with observed trajectories for two intersections, which display different geometrical characteristics, in following sub sections. Modelled paths, speed, acceleration and jerk profiles have named in accordance with the tested cases (case a to e) described above. Straight arrowheads on these figures represent the approaching direc-tion of vehicles to the intersection.

a) Comparison of vehicle paths

Figure 3(a) and 4(a) compare modelled paths for 50^{th} percentile initial and final conditions (acceleration and deceleration) with observed trajectories for Suemori Dori 2 intersection East approach and Ueda intersection South approach respectively. That is, modelled 'median' path is represented.

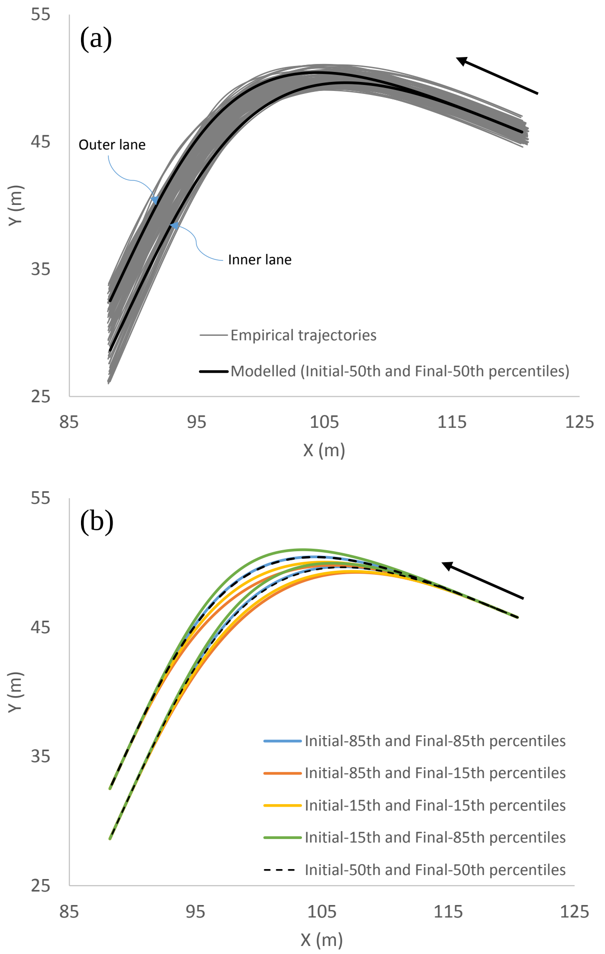


Fig.3 Variation of modelled vehicle paths for inner and outer laned of Suemori Dori 2 intersection: (a) - comparison of modelled 50^{th} percentile case with empirical paths; (b) – comparison of modelled paths for different initial and final conditions.

Figure 3(b) and 4(b) compare variation of modelled paths for different initial and final conditions (a to e described above) for same intersections. It can be observed from these figures that the proposed model can generate turning vehicle paths accurately for intersections with different geometries even with the roughly estimated t_f. Further, it is clear that the modelled paths are not considerably deviated from each other for different initial and final conditions. However, it can be observed that intersections with smaller intersection angles show larger variations. Similar observations have been made by previous studies with conducted using different path generation models[1)].

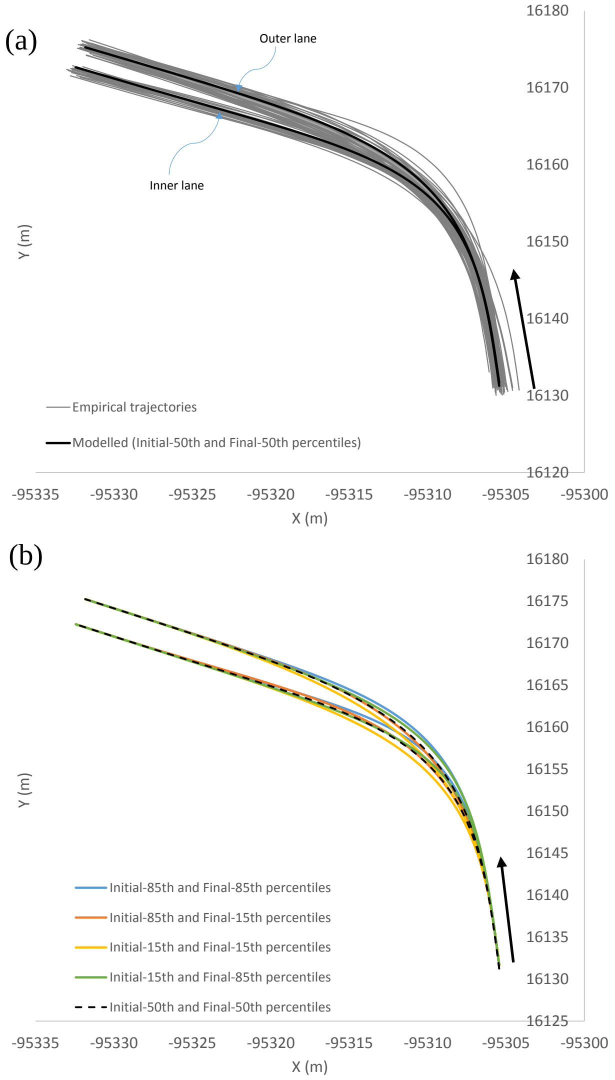


Fig.4 Variation of modelled vehicle paths for inner and outer laned of Ueda intersection: (a) - comparison of modelled 50^{th} percentile case with empirical paths; (b) – comparison of modelled paths for different initial and final conditions.

b) Comparison of speed profiles

Once constants in Equation 2(a) and 2(b) are determined based on given boundary conditions and movement times, corresponding speed profiles can be obtained by differentiating those equations.

Speed profiles corresponding to the paths (empirical and modelled) described in the previous section (Section 3.1.(b)) are compared in this section. Figure 5 and 6 compare modelled speed profiles for different initial and final conditions and actual speed profiles for inner and outer lanes for Suemori Dori intersection and Ueda intersection respectively.

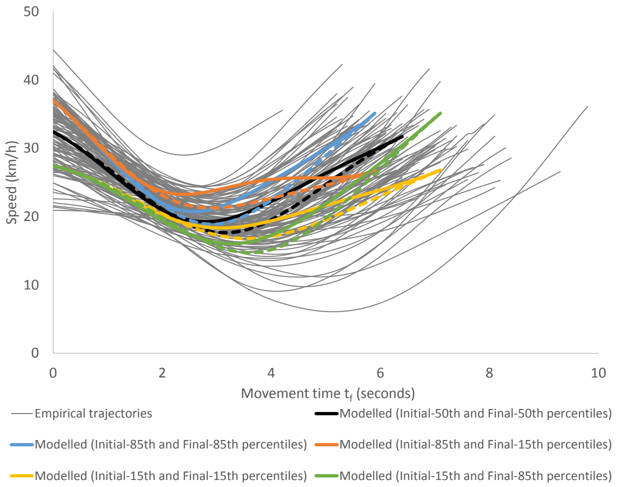


Fig.5 Comparison of modelled and actual speed profiles for Suemori Dori 2 intersection East approach (solid lines represent inner lane and broken lines represent outer lane).

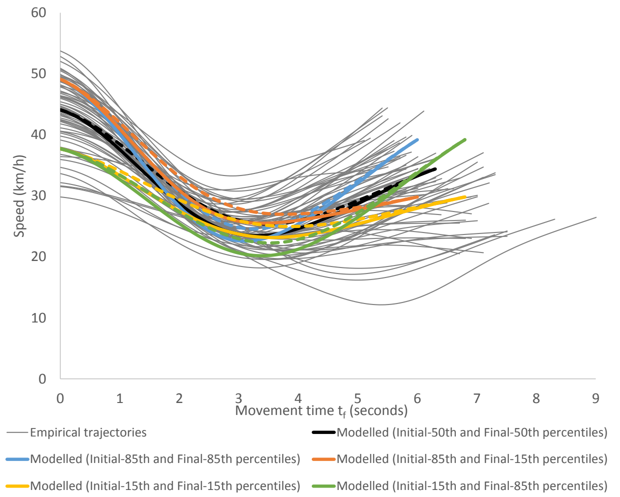


Fig.6 Comparison of modelled and actual speed profiles for Ueda intersection South approach (solid lines represent inner lane and broken lines represent outer lane).

As can be understood from Figure 5 and 6, large variations can be observed for initial and final speeds as well as for t_f in actual speed profiles. Modelled speed have adequately captured such variations for a given initial condition and various final conditions and vice versa. Further, note that the minimum speeds as well as intermediate speeds, which are model outputs, are within the range of actual values for both intersections. Similar to the variation in paths, variation of minimum speeds tend to increase for decreasing intersection angles.

c) Comparison of acceleration profiles

Acceleration patterns were also obtained for different initial and final conditions based on the second derivative of Equation 2(a) and (b). These are compared with observed empirical acceleration profiles as depicted in Figure 7 and 8 for Suemori Dori and Ueda Intersections respectively.

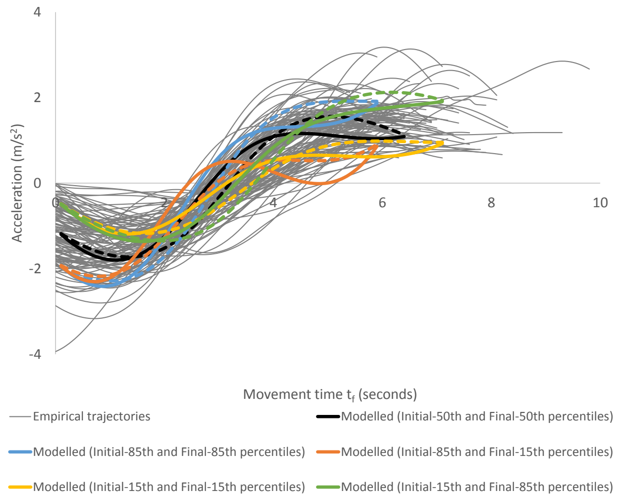


Fig.7 Comparison of modelled and actual acceleration profiles for Suemori Dori 2 intersection East approach (solid lines represent inner lane and broken lines represent outer lane).

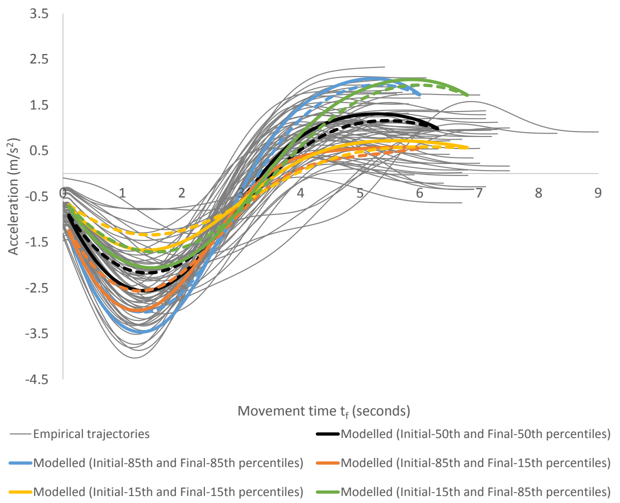


Fig.8 Comparison of modelled and actual acceleration profiles for Ueda intersection South approach (solid lines represent inner lane and broken lines represent outer lane).

Figure 7 and 8 explain that acceleration patterns and variations are also within the range of observed profiles.

d) Comparison of jerk profiles

Jerk profiles for tested cases obtained based on the third derivative of Equation 2(a) and (b) are compared with empirical profiles in Figure 9 and 10 for Suemori Dori and Ueda Intersections respectively. As can be explained through these figures, profiles with similar trends to actual profiles have been reproduced by the proposed approach with different initial and final settings. It can be stated that these modelled profiles realistically represent the actual behaviors as no pre-determined shapes were assumed (for speed, acceleration and jerk) in the model.

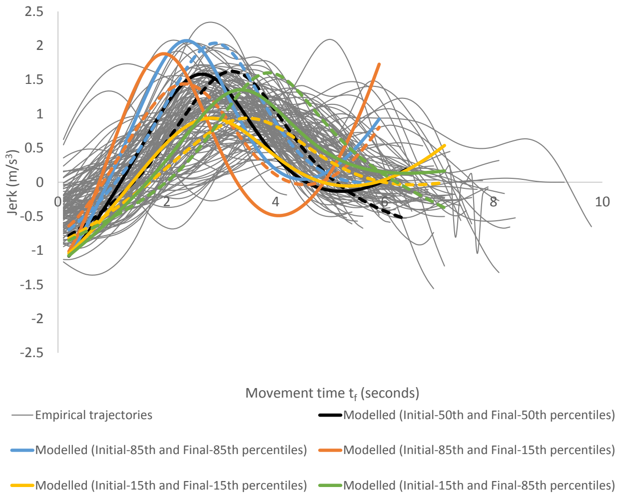


Fig.9 Comparison of modelled and actual jerk profiles for Suemori Dori 2 intersection East approach (solid lines represent inner lane and broken lines represent outer lane).

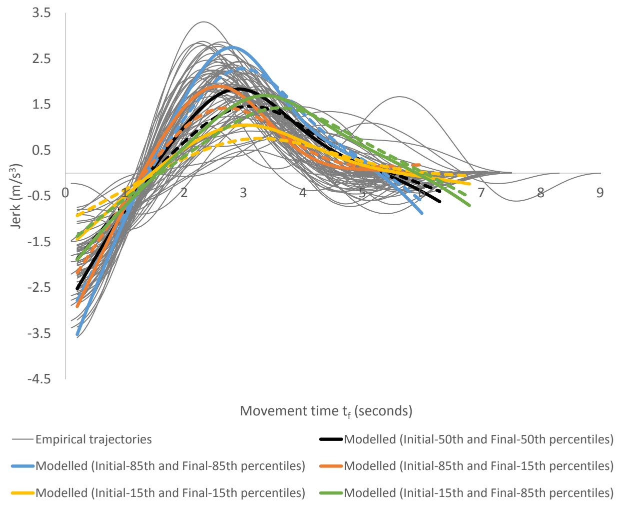


Fig.10 Comparison of modelled and actual jerk profiles for Ueda intersection South approach (solid lines represent inner lane and broken lines represent outer lane).

In Figures 5-10 it can be observed that modelled speed, acceleration and jerk profiles display different trends for different lanes regardless of the same initial and final speed and acceleration conditions. This observation indicates that such deviations can also be captured by this approach. However, these characteristics as well as sensitivity of the model predictions should further be confirmed with more empirical data for different geometrical settings.

In this section we considered that initial and final locations of a vehicle is known. Further, initial speed and acceleration were also assumed to be known. With the given initial speed t_f was estimated. Under such known or given conditions, variations of trajectories were estimated for different possible exit conditions. It was shown that trends and variations of location, speed and acceleration profiles were realistically captured. However, accuracy of a predicted trajectory will depend on the accuracy and reliability of t_f which is a crucial component of this modelling approach. This could be understood by the discussion provided in next section.

(2) Predicting turning trajectories with complete information

When all boundary conditions and t_f are known how well a trajectory of a turning vehicle can be generated by the minimum jerk theory is explored in this section. For this, two free-flow turning vehicles were selected (each from observed trajectories at Suemori Dori 2 and Ueda intersections). Boundary conditions and t_f were extracted from these 2 trajectories and the system of equations (Equation 2(a) and (b)) were solved for each of those. Resulting trajectories generated with each set of those boundary conditions and t_f are compared with actual ones in Figure 11 and 12. These figures highlight that when the complete information on boundary conditions and movement time are known, a trajectory of a vehicle can be predicted with a remarkable accuracy.

Further, these comparisons confirm that the minimum-jerk trajectory for a certain set of boundary conditions and t_f is well matched with the actual trajectory for same conditions. This means, in general, drivers actually tend to minimize the jerk (or the rate of change of acceleration) when making turning movements at intersections. Incorporating such realistic characteristics of trajectories in simulation models, which are developed for safety assessments at intersections, are beneficial in many ways. For example, the location of a potential crash or conflict and the speed at the crash point, which are important to assess the crash severity, can be obtained accurately. Not only location and speed, but also other important information, such as information on jerk that is an important indicator related with critical driving[18], can also be obtained without any additional models or simplifications. Accuracy of such indicators may be enhanced by considering higher order models, for example, minimum-snap model (snap is defined as the time derivative of jerk). However, such higher order models require additional boundary conditions as well as more computational efforts.

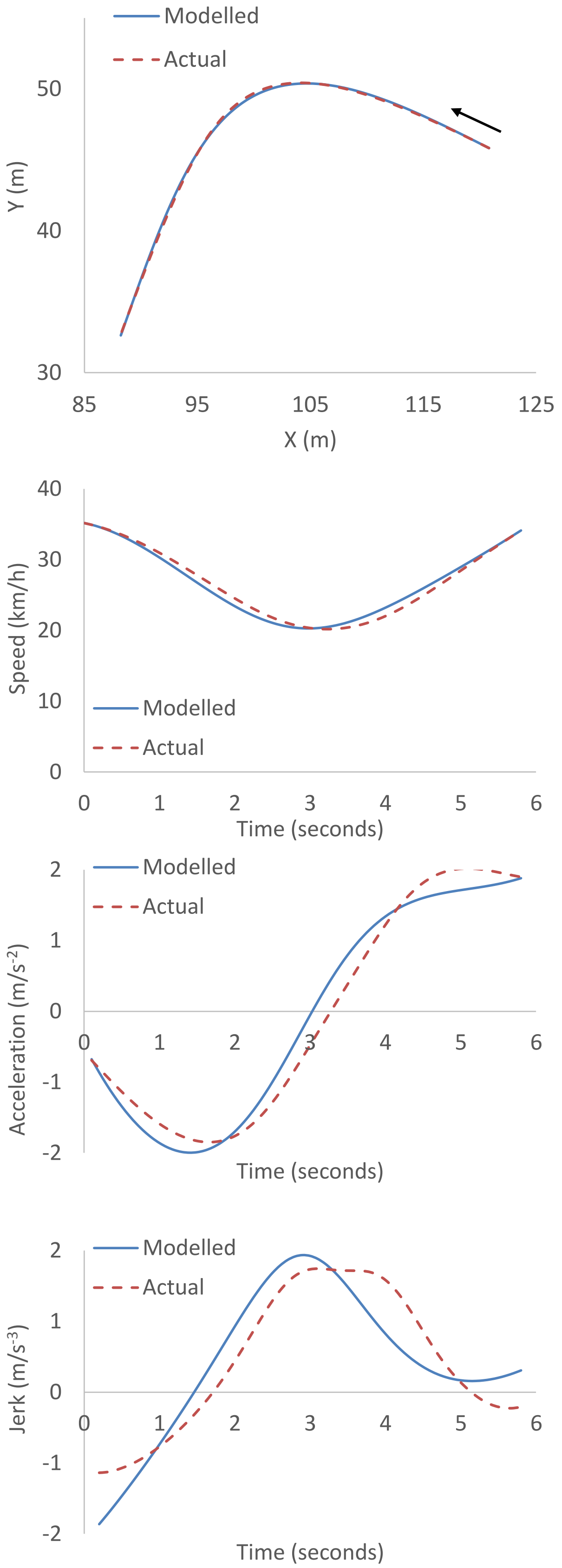


Fig.11 Comparison of a modelled and an actiual trajectory at Suemori Dori 2 intersection.

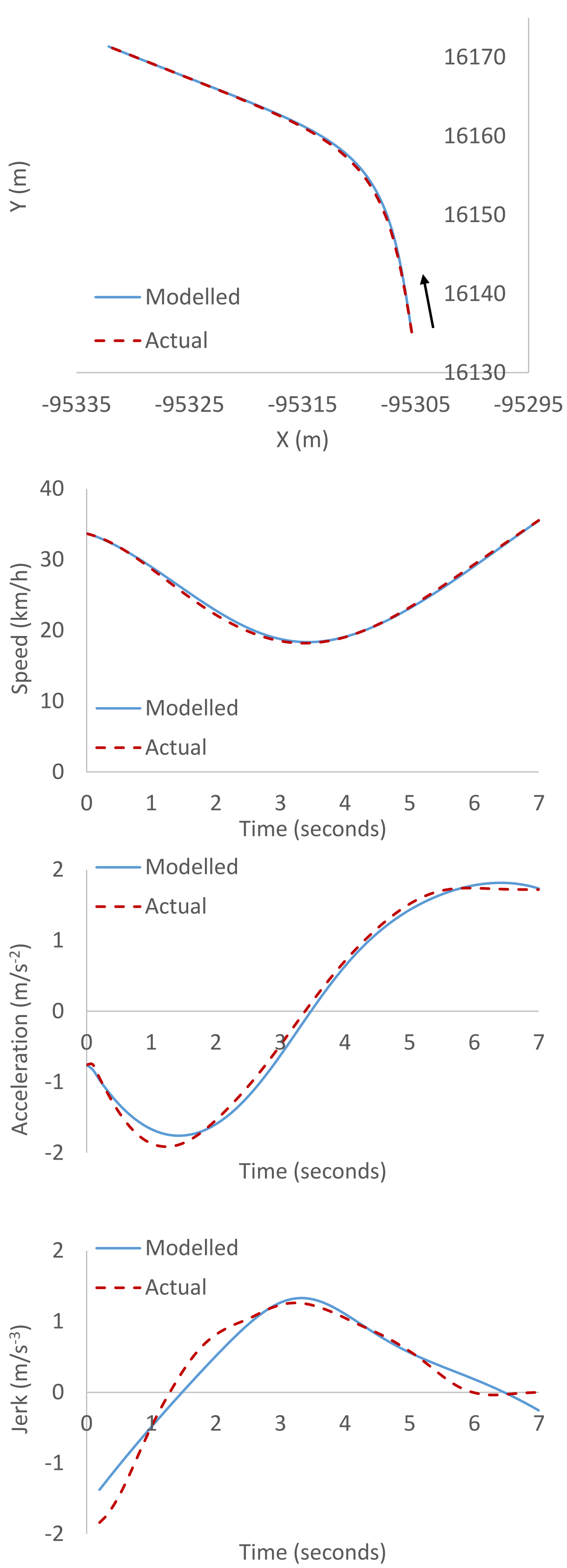


Fig.12 Comparison of a modelled and an actiual trajectory at Ueda inter-section South approach.

4. CONCLUSIONS

Realistic representation of properties of turning vehicle trajectories is important in microscopic simulation models particularly those are developed for safety assessments. Existing studies have not adequately addressed this issue. In this study minimum-jerk theory based approach was proposed to model trajectories for turning vehicles at signalized intersections. The modelling approach was initially tested for free-flow turning vehicles. Comparison of modelled and observed trajectories suggested that the proposed approach can reproduce trajectories with a remarkable accuracy. It should be noted that no pre-determined simplified trends (or shapes) were assumed before modelling speed, acceleration and jerk profiles. Thus, these modelled profiles are more realistic compared to previous studies. Variation of trajectories were also tested based on combination of different boundary conditions and shown that the model is responsive and sensitive to such different boundary conditions. A detailed sensitivity analysis should be performed as a further step to better understand the variation of turning trajectories particularly with respect to movement time (t_f).

In this paper, exit conditions and t_f, which are required to obtain the minimum jerk solution, were approximated. In applications, such as in microscopic simulation tools and in autonomous vehicles, accurate information on such conditions should be provided to obtain reliable output. Therefore, comprehensive methods should be explored under future studies to set such information in practical applications.

ACKNOWLEDGMENT: Authors are very grateful to the Japan Society for the Promotion of Science for supporting this research through JSPS KAKENHI Grant No. 15H05534 and No. 15H02270. Further, authors would like to thank Professor Hideki Nakamura and his laboratory at Nagoya University for providing required video data to conduct this study.

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(Received April 22, 2016)