Bus Bunching under Consideration of Passenger Choice between Bunched Buses

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Bus bunching is a well-known phenomenon on many bus routes where an initial delay to one service can disturb the whole schedule due to resulting differences in dwell times of subsequent buses at stops. This paper deals with the passenger behavior when there are more than one bus serving at the stop, focusing on their choices and possible switching actions from the queue of the bus they are waiting to board. A parameter γ is introduced to denote the percentage of passengers boarding the front bus when buses are bunched. A set of discrete state equations is implemented to obtain the departure times of a set of buses following the occurrence of an exogenous delay to one of the buses at a bus stop. Overtaking and no-overtaking cases are also distinguished from each other in this paper. Measures are introduced to evaluate the performance of the bus service along a corridor under different γ levels. We find that it is an optimal measure to keep the percentage of passengers boarding the front bus low. Beside, overtaking is a favourable counter-measure against comparatively high front-bus preference.

Key Words : *Bus Bunching, Passenger Behavior at the Stop, Queue Switching, Service Regularity*

1. INTRODUCTION

Good public transport services are an essential part of a sustainable urban transport system, and improving public transport service quality is a major challenge for the operators and government agencies. The lack of bus service reliability is a major problem for bus passengers and service operators. A key feature of an unreliable service is the irregular arrivals of buses at stops. The effect of two successive services of a single line arriving at stops with shorter than designed headways is generally defined as bus bunching. Bus bunching is undesirable for passengers because it leads to increases of waiting time at some bus stops, and unpredictability of bus arrival times. Studies have shown that passengers value their time waiting at bus stops more than they do to on-board travel time. Hollander and Liu (2008) found that the value of service reliability to bus passengers is four times higher than that of mean travel time.

Bus bunching may be caused by the first service being delayed due to unforeseen traffic congestion en-route or unplanned high demand at previous stops. The subsequent service then has fewer passengers to pick up at that stop and departs earlier than scheduled. At downstream stops the effect is emphasised as the (small) delay to the first vehicle and the (slight) early arrival of the second vehicle result in increasingly longer dwell times for the first bus and increasingly shorter dwell times for the second bus. The bus bunching effect on a single line of service was first described in a seminal work by Newell and Potts (1964). They studied an idealised corridor with evenly spaced bus stops, identical travel times between stops, and constant passenger loads at bus stops. Given a small delay of the first bus of a service at a stop, Newell and Potts provide an analytical formulation of the deviation of bus arrival time to schedule for all buses and at all subsequent stops. They show that adjacent buses alternate between being behind and ahead of schedule, leading to bus bunching. The scale of the bunching effect and the stability of the bus system is affected not only by the size of the original delay to the first bus, but also by the ratio (referred to as the k value later) between passenger arrival rate and boarding rate. They show that if $1/2 < k < 1$, instability occurs. In practice, however, one would expect the passenger arrival rate to be much smaller than the loading rate, i.e. $0 < k < 1/2$. In this case, Newell and Potts show that the system can recover from the original perturbation and return to schedule. Furthermore, bus bunching is more noted in high frequency services, where the headway between buses is small and the delay to headway ratio is easily over the threshold so that bus bunching amplifies (rather than being damped and remaining localised) further down the route.

Following on from Newell and Potts' work, there has been a significant body of literature designing operational strategies to avoid the bunching effect. In particular holding strategies of early buses as well as strategies to keep minimum distances between subsequent services have been analysed and shown to be successfully applied in literature. The holding strategies are implemented through building slacks in the schedule at key timing points and holding buses at these points to keep them to schedule (e.g. Osuna and Newell, 1971; Newell, 1974; Hickman, 2001; Eberlein, 2001; Cats et al, 2012). Due to the complexity of the problem, most of these early studies involve solving just one controlled timing point. Using a simulation approach, Hickman (2001) derived a set of static holding solutions, which do not respond to dynamical changes in the actual bus performances on the day. Eberlein et al (2001) proposed a model for dynamic bus holding which take real-time information on bus headways into consideration and strives to minimise passenger waiting time.

Daganzo (2009) explored a more systematic approach to the dynamic holding problem with real-time bus performance. Daganzo's method is able to consider holding at multiple timing points, therefore providing opportunity for return to schedule for long bus routes. In addition, the model takes into account random effects in bus travel time, bus dwell time and passenger demand, making it resemble more to real-life situations. Daganzo and Pilachowski (2011) proposed an adaptive bus control scheme based on a two-way bus-to-bus cooperation, where a bus adjusts its speed to both its front and rear headways. They show that the scheme yields significant improvements in bus headways and bus travel time. Moving away from the traditional ideal of schedule and a prior target headway, Pilachowski (2009) proposed to use GPS data to counteract directly the cause of the bunching by allowing the buses to cooperate with each other and to determine their speed based on relative position, while Bartholdi and Eisenstein (2012) proposed a self-coordinating method to equalise bus headway.

Despite these recent developments, most of the existing studies present an oversimplified model of the bus bunching phenomenon, notably with a single line of service, with fixed service frequency, uniformly distributed (in time and space) passenger flows, and no bus overtaking. They neglect important aspects of real-life bus systems, such as passenger behaviour, en-route service perturbation, transport operator policies such as holding and overtaking, and complex network features such as common lines. Newell and Potts (1964), for instance, assume fixed frequency, constant dwell times, equal-distance stops and equal-travel time between stops, and that buses cannot overtake. In real-life situations, busy urban corridors are often served by multiple lines of bus services, with different frequencies and different sequence of stops. Besides, when buses are bunched at the stop, some passengers are likely to stick to front bus, while some others are intended to get on the back bus, which will change the dwell time of buses and the order of bus departures.

To explore the effect of common section of a corridor served by multiple buses, Hernández et al (2015) proposed real-time control strategies in a corridor with multiple bus services while the common section is short. Schmöcker et al (2015) discussed the problem in a long common section with 10 stops, and found that common sections will contribute when overtaking policy is allowed, but the model they proposed is simplified by infinite capacity and no alighting process, which makes it still a little unrealistic.

In this paper, returning to the bunching problem on a corridor with one line, we focus on the passenger behaviour when there is more than one bus stopping at the stop at the same time. We investigate especially their choices and possible switching actions from the queue of the bus they are waiting to board to that of the coming one. A parameter γ is introduced to denote the percentage of passengers remaining in queue for the front bus. We presume the same percentage applies further to passengers arriving at the stop during the dwell time, γ thus could be regarded as a kind of front-bus preference. Different scenarios of different arrival and departure sequences are discussed respectively. Furthermore, we consider resulting differences in bunching depending on whether overtaking of buses at bus stops is allowed or not.

Chapter 2 of the paper sets out the basic model notations. Chapter 3 introduce the percentage of passengers who board the front bus when buses are bunched. Chapter 4 describes the formulation of bus propagation model, and without and with bus overtaking. Four evaluation indices are proposed in Chapter 5, and the performance of the model are illustrated through case studies in Chapter 6. Finally, Chapter 7 draws conclusions of the study and discusses the implications on network design.

2. NOTATION AND BASIC ASSUMPTIONS

The following notation will be used throughout the paper.

Let

m bus number according to arrival time at the bus stop (which does not necessarily have to be the dispatching order from depot if we allow for overtaking) with m=0,1,2,..,M

n bus stop number with n=0,1,2,..N

h headway of the line

The above set of variables defines the basic service characteristics. In the following we introduce variables for specific buses at specific stops.

a_m time at which bus m of line l arrives at stop n (measured from t_{00}=0)

$d_{m,n}$ time at which bus m of line l leaves at stop n (measured from t_{00}=0)

$v_{m,n}$ travel time of bus m between stops n-1 and n; taken as fixed value in this study

$\rho_{m,n}$ initial "exogenous" delay to bus m of line l before or at the nth stop

$\Delta_{m,n}$ passenger arrival period over which demand for bus m at stop n accumulates

$w_{m,n}$ dwell time of bus m at stop n

b_m passenger loading rate of bus m

q_n passenger arrival rate at stop n for passengers

$k_{m,n}$ ratio between passenger arrival and loading for bus m at stop n

We assume that bus travel time between stops is constant so that $v_{m,n}$ simplifies to v. Instead in the later case study we assume that one bus is subject to an initial delay at stops denoted by $\rho_{m,n}$. This event triggers the subsequent bunching effect. The difference between assuming random link travel times and delays at stops is that in the latter passengers arriving at the stop during the delay period can board the bus whereas in the former obviously they cannot. We also note that replacing $\rho_{m,n}$ by one (or multiple) link delay presents no methodological difficulty in the approach presented hereafter.

The passenger arrival period, $\Delta_{m,n}$, for a regular service will be equal to the service headway. In case of a bunched service, various definitions are possible, depending on bus stop layout, operational policy as well as passenger behaviour. In particular, passengers arriving while two buses are at the same time at the stop will have a choice between these. The effect of different assumptions regarding $\Delta_{m,n}$ will be discussed later.

The boarding time per passenger is primarily depending on doors and ticketing system. Sun et al (2014) report that the loading time per passenger further depends on the interaction between boarding and alighting passengers. In the following we omit this issue and instead make the simplifying assumption that all buses are identical, i.e. have the same boarding rate per passenger, so that we can assume a fixed b_m and omit the subscript m. Further, whereas Fonzone et al (in press) assume that arrival patterns are time dependent here we assume a constant q_n. With these assumptions also k becomes time and bus independent and can be defined as

$$k_n = \frac{q_n}{b} \tag{1}$$

Clearly to avoid queues at bus stops building up over the analysis period we require

$$0 \leq k_n < 1 \tag{2}$$

In this paper, passengers who arrive during the dwell time can still board the bus, which means the arrival period of passengers is not equal to headway which is the interval of two adjacent arrivals, but is defined as the interval of two adjacent departures. $\Delta_{m,n}$ can be generally obtained as

$$\Delta_{m,n} = min\{d_{m,n} - d_{m',n} | d_{m',n} \leq d_{m,n}\}, \forall m \tag{3}$$

Which simplifies to (4) if overtaking is not allowed or does not occur

$$\Delta_{m,n} = d_{m,n} - d_{m-1,n} \tag{4}$$

We further note that equation (5) to obtain dwell times does not hold if several buses are serving the stop. We elaborate on this in the following section.

$$w_{m,n} = \Delta_{m,n} k_n \tag{5}$$

3. PASSENGER CHOICE BETWEEN BUNCHED BUSES

Let us now consider the case that two buses are boarding passengers at the same time. As described in the introduction let γ denote the front-bus preference of passengers waiting to board and newly arriving during the dwell time of the buses. Therefore, with $\gamma = 1$ all passengers keep boarding the front bus, whereas with $\gamma = 0$ all passengers at the bus stop swop to board the bus that arrived later.

In line with previous notation we utilise in the following:

$a_{m-1,n}$ and $a_{m+1,n}$ as time at which the previous

and next bus to *m* arrives at stop *n*
$d_{m-1,n}$ time at which bus a_{m-1n} departs from the stop
$\xi_n(t)$ last departure time given time *t*
$x_n(t)$ queue of passengers at stop *n* at time *t* who want to board the bus

Considering departure of the previous bus and arrival of the bus subsequent to bus *m*, each column of below table shows a possible event sequence. In the top row the number stands for the number of buses at the bus stop and the letter behind the number for the position of the bus of interest at the bus stops. 2f stands for the bus being the front bus at the stop, 2b for the bus being at the back of two buses at the bus stop and 3m for the bus being the middle one of three at the bus stop. The arrow stands for the state transition of the bus.

The lower part of the table then shows which cases can occur depending on γ and depending on whether we allow for overtaking between buses or not. Case 1 denotes the non-bunched case in that bus *m* arrives after the previous bus has left and departs before the next bus has arrived. The case clearly can occur for all γ and independent of whether overtaking is considered or not. It can be solved with the equations shown in Section A.

The second case, 1->2f , denotes the case that while the bus is still boarding the subsequent bus arrives. In this case, and all subsequent cases, hence the solution depends on γ. Firstly if $\gamma = 1$ then all passengers board the bus of interest, so that the waiting time of the bus is identical to the case without considering bus *m*+1. The case also makes bunching worse compared to smaller γ values, as bus *m*+1 does not help to relieve bus *m* though it has already caught up with this bus. For cases $1 > \gamma > 0$ the relief by bus *m* needs to be taken into account as shown in Section B. With decreasing γ the dwell time of bus *m* will continuously decrease so that in the extreme case of $\gamma = 0$ bus *m* leaves immediately when bus *m*+1 is arriving. This corresponds to the case of the bus driver in the front bus trying to reduce the bunching effect by pushing all passengers to the back bus.

In case overtaking is allowed bus *m*+1 will overtake bus *m* in case $\gamma > 0.5$ as this means more than half the passengers will remain boarding bus m. In case of $\gamma = 0.5$ the buses become "twin buses" as they depart at the same time, whereas in the case of $\gamma < 0.5$ bus *m* will depart before bus *m*+1. One issue is though that obtaining the exact dwell time in case of $1 > \gamma > 0.5$ is not possible with our solution approach. Bus *m*+1 will overtake bus m so that from d_{m+1} until d_m bus *m* becomes the only bus at the stop again. Hence obtaining d_{m+1} is required in order to obtain d_m. One approximation that can be made is though to (linearly) approximate the dwell time for this case from the cases $\gamma = 1$ and $\gamma = 0.5$ as the dwell time for bus *m* must also be smaller than for $\gamma = 1$ but bigger than for $\gamma = 0.5$.

The following five columns all presume that bus *m* arrives while bus *m*-1 has not yet departed. Firstly consider a system without overtaking. For $\gamma \geq 0.5$ this hence means that bus *m* leaves together with bus *m*-1 so that the behaviour of bus *m*+1 does not have to be considered and one always obtains $w_m = d_{m-1} - a_m$ which is equivalent to $d_m = d_{m-1}$. In case of $\gamma < 0.5$ instead bus *m* will have to pick up more than half the passengers queuing at the stop and hence these passengers need to be considered in determining the dwell time of bus *m* (Cases C and D in the table). Let us for simplicity assume that only two buses can board passengers at the same time, i.e. a third (and fourth..) bus that might be at the stop at the same time can not pick up passengers until one of the front two buses has departed. Under this assumption case 2b->3m->2f simplifies to the 2b->2f case. That is, until departure of bus *m*-1, bus *m* is the latter of two buses. At departure of bus *m*-1 then bus *m* becomes the front bus of two boarding buses. This case is denoted as F and solved below. Note further that for $\gamma = 0$ again the solutions simplify. Whenever the bus transits into the 2f state, it can leave immediately, if it is in the 2b state, bus *m*-1 can leave immediately and hence it becomes identical to the 1 bus waiting time if bus *m* can leave before bus *m*+1 is arriving. Finally, note that for $\gamma < 0.5$ no overtaking occurs at the bus stop as the previous bus will always be able to leave before the subsequent bus as it will have to pick less passengers of the remaining queuing travellers at the stop.

Table 1 Possible event sequences from the view of bus m and corresponding calculation of dwell time

Possible event sequences							
Case	**1**	**1->2f**	**2b -> 1**	**2b->1->2f**	**2b**	**2b->3m->2f**	**2b->3m**
event sequence (increasing time)	d_{m-1}		a_m				
	a_m		d_{m-1}		d_m	a_{m+1}	
	d_m	a_{m+1}	d_m	a_{m+1}	a_{m+1}	d_{m-1}	d_m
	a_{m+1}	d_m	a_{m+1}	d_m	d_{m-1}	d_m	d_{m-1}
Solution to specific cases							
No overtaking							
$\gamma = 1$	A	A (m+1 waits behind m)	$w_m = d_{m-1} - a_m$ (m-1 and m bunched)	Does not occur: $d_m = a_{m+1} = d_{m-1}$	Does not occur: case presumes overtaking	$w_m = d_{m-1} - a_m$ (m-1 and m bunched)*	Does not occur: case presumes overtaking
$1 > \gamma > 0.5$		B (m and $m+1$ bunched)					
$\gamma = 0.5$							
$0.5 > \gamma > 0$		B	C	D		F*	
$\gamma = 0$		$w_m = a_{m+1} - a_m$		$w_m = a_{m+1} - a_m$		$w_m = d_{m-1} - a_m$*	
Overtaking							
$\gamma = 1$	A	A (m+1 leaves immediately)	Does not occur: $d_m < d_{m-1}$	Does not occur: $w_m = 0$ and $d_m < d_{m-1}$	$w_m = 0$ (and overtaking)	Does not occur: $w_m = 0$ and $d_m < d_{m-1}$	$w_m = 0$ (and overtaking; only if $a_{m+1} = a_m$)
$1 > \gamma > 0.5$		X (m+1 overtakes m)			E		E*
$\gamma \leq 0.5$		Overtaking does not occur, identical to no overtaking case					

A: 1bus, no bunching case

$$w_{m,n} = \frac{1}{b}\int_{d_{m-1,n}}^{d_{m,n}} q_{ln}dt = \frac{1}{b}\int_{d_{m-1,n}}^{a_{m,n}+w_{m,n}} q_{nl}dt = k_{ln}\left(a_{m,n} + w_{m,n} - d_{m-1,n}\right) \quad (6)$$

$$w_{m,n} = \frac{k_{ln}\left(a_{m,n} - d_{m-1,n}\right)}{(1 - k_{ln})} \quad (7)$$

B: 1->2f and assuming no overtaking (either because $\gamma < 0.5$ or because overtaking not allowed)

We obtain the queue x at stop n at time $a_{m+1,n}$ as

$$x_n\left(a_{m+1,n}\right) = q\left(a_{m+1,n} - \xi_n\left(a_{m+1,n}\right)\right) - b\left(a_{m+1,n} - a_{m,n}\right) \quad (8)$$

Waiting time of bus m can be obtained as

$$w_{m,n} = \left(a_{m+1,n} - a_{m,n}\right) + \frac{\gamma}{b}\left(x_n\left(a_{m+1,n}\right) + \int_{a_{m+1,n}}^{d_{m,n}} q_{ln}dt\right) \quad (9)$$

Which is equivalent to

$$w_{m,n} = \left(a_{m+1,n} - a_{m,n}\right) + \frac{\gamma x_n\left(a_{m+1,n}\right)}{b(1-\gamma k)} \quad (10)$$

C: 2b -> 1 and no overtaking because $\gamma < 0.5$

We firstly obtain the queue of passengers at the stop when bus m is arriving as

$$x_n\left(a_{m,n}\right) = q\left(a_{m,n} - \xi_n\left(a_{m,n}\right)\right) - b\left(a_{m,n} - a_{m-1,n}\right) \quad (11)$$

The waiting time can be formulated as follows

$$w_{m,n} = \frac{1-\gamma}{b}\left(x_n\left(a_{m,n}\right) + \int_{a_{m,n}}^{d_{m-1,n}} q_{ln}dt\right) + \frac{1}{b}\int_{d_{m-1,n}}^{d_{m,n}} q_{ln}dt \quad (12)$$

And hence

$$w_{m(l),n} = \frac{(1-\gamma)x_n + b\gamma k\left(a_{m,n} - d_{m-1,n}\right)}{b(1-k)} \quad (13)$$

D: 2b->1->2f and no overtaking because $\gamma < 0.5$

In this case we need to obtain the queue of passengers at the arrival of bus m (when bus m enters the 2b state) as well as at time $a_{m+1,n}$ when bus m enters the 2f state. In fact, as shown below the

$x_n(a_{m+1,n})$ obviously depends on $x_n(a_{m,n})$.

$$x_n(a_{m,n}) = q\left(a_{m,n} - \xi_n(a_{m,n})\right) - b(a_{m,n} - a_{m-1,n}) \quad (14)$$

$$x_n(a_{m+1,n}) = x_n(a_{m,n}) - 2 * b(d_{m-1,n} - a_{m,n}) - b(a_{m+1,n} - d_{m-1,n}) + q(a_{m+1,n} - a_{m,n}) \quad (15)$$

As this case implies that the bus is still at the stop at the arrival time of bus *m*+1 the waiting time can be obtained as

$$w_{m,n} = (a_{m+1,n} - a_{m,n}) + \frac{\gamma}{b}\left(x_n(a_{m+1,n}) + \int_{a_{m+1,n}}^{d_{m,n}} q_n dt\right) \quad (16)$$

Which can be solved to

$$w_{m,n} = (a_{m+1,n} - a_{m,n}) + \frac{\gamma x_n(a_{m+1,n})}{b(1-\gamma k)} \quad (17)$$

E: 2b (only for $\gamma > 0.5$, includes overtaking)

We obtain again the queue at the stop when bus *m* is entering stage 2b as

$$x_n(a_{m,n}) = q\left(a_{m,n} - \xi_n(a_{m,n})\right) - b(a_{m,n} - a_{m-1,n}) \quad (18)$$

This lead to

$$w_{m,n} = \frac{1-\gamma}{b}\left(x_n(a_{m,n}) + \int_{a_{m,n}}^{d_{m,n}} q_{ln} dt\right) \quad (19)$$

And hence

$$w_{m,n} = \frac{(1-\gamma)x_n(a_{m,n})}{b(1-k+\gamma k)} \quad (20)$$

F: 2b->(3m)->2f, no overtaking (because $\gamma < 0.5$)

Finally, with our assumption that only two buses are boarding passengers simultaneously, we obtain that in this case the bus transfer immediately from the 2b state into the 2f state. The transition occurs at time $d_{m-1,n}$ and we obtain the queue at this point in time by

$$x_n(a_{m,n}) = q\left(a_{m,n} - \xi_n(a_{m,n})\right) - b(a_{m,n} - a_{m-1,n}) \quad (21)$$

$$x_n(d_{m-1,n}) = x_n(a_{m,n}) - 2b(d_{m-1,n} - a_{m,n}) = q\left(a_{m,n} - \xi_n(a_{mn})\right) + b(a_{m,n} + a_{m-1,n} - 2d_{m-1,n}) \quad (22)$$

Then the waiting time can be obtained by

$$w_{m,n} = (d_{m-1,n} - a_{m,n}) + \frac{\gamma}{b}\left(x_n(d_{m-1,n}) + \int_{d_{m-1,n}}^{d_{m,n}} q_n dt\right) \quad (23)$$

And hence

$$w_{m,n} = (d_{m-1,n} - a_{m,n}) + \frac{\gamma x_n(d_{m-1,n})}{b(1-\gamma k)} \quad (24)$$

X: 1->2f, $\gamma > 0.5$, with overtaking

The only case that we can not solve accurately is the case denoted by X in above table. As noted the reason is that the departure time for bus *m*+1 needs to be known or solved simultaneously when we solve for the departure time of bus *m*. One could do so by a time step simulation approach similar to work described in Fonzone et al (in press). However, as the two limiting cases for $\gamma = 1$ (case A) and $\gamma = 0.5$ (case B) can be solved accurately and since we know that the waiting time is continuously decreasing for bus *m* in state 2f for decreasing γ we can approximate:

$$w_{m,n} = 2\big((\gamma - 0.5)A + (1-\gamma)B\big) \quad (25)$$

4. BUS PROPAGATION MODEL

The following algorithm then solves the problem considering the dwell time equations as in previous equations. We note that stochastic link travel times (instead of random delays at stops) could be easily implemented by adding "error terms" ρ also to link travel times.

Initialisation
- Set $a_{m,1}$ $\forall m$
- Set $\Delta_{1,n}$ $\forall n$

For each stop n in increasing order
- Sort buses according to arrival times at stop
  - For each bus m in order of increasing arrival times obtain
    - Obtain $w_{m,n}$ as in Section 3
    - $d_{m,n} = a_{m,n} + w_{m,n} + \rho_{m,n}$ (26)
    - $a_{m,n+1} = d_{m,n} + v_{m,n}$ (27)

5. EVALUATION MEASURES

To evaluate the system performance of a series of successive buses, the emphasis is put on the service regularity in this paper and the index of the service interval duration $\Delta_{m,n}$ as in equation (4) and its standard deviation. $\Delta_{m,n}$ has a direct effect on the waiting time of passengers at the stop. Assuming constant passenger arrival patterns, a constant $\Delta_{m,n}$ will also minimise passenger waiting times.

As a critical index, $\Delta_{m,n}$, the passenger arrival period over which demand for bus *m* at stop *n* accumulates is essential for the evaluation.

The mean and maximum of $\Delta_{m,n}$ of all the bus services can be obtained respectively as

$$\bar{w} = \frac{\sum_n \sum_m \Delta_{m,n}}{M \times N} \tag{28}$$

$$\hat{w} = \max_n \max_m \Delta_{m,n} \tag{29}$$

Further, the total standard deviation of $\Delta_{m,n}$ and the stop-specific maximum standard deviation of $\Delta_{m,n}$, can be obtained respectively as

$$\bar{\sigma} = \sqrt{\frac{\sum_n \sum_m (\Delta_{m,n} - \bar{\Delta}_{m,n})^2}{M \times N}} \tag{30}$$

$$\hat{\sigma} = \max_n \sqrt{\frac{\sum_m (\Delta_{m,n} - \bar{\Delta}_{m,n})^2}{M}} \tag{31}$$

In order to learn under which degree of front-bus preference the system will reach the optimum as well as to discuss the effect of overtaking or no-overtaking policy, all the evaluation indices will be calculated under different preference level and distinguished by overtaking and no-overtaking case.

6. CASE STUDY

(1) Specifications

We consider a single line with 10 stops in this case study. The bus line runs with a frequency of h=6min. Further, we assume that the travel time between two adjacent stops takes a constant value of 3min. Then we assume that an initial random delay of 2min occurs for the 2nd bus at the 2nd stop. This means that the first bus is unaffected and hence runs with the expected headways and encounter the same (expected) dwell times at the stop.

To evaluate the effect of different choice strategy of passengers, we test the bus system with different γ, (γ =0, 0.1, 0.2, …, 1). We also distinguish different overtaking policies, especially when γ >0.5, obvious differences are expected to be observed between overtaking and no-overtaking cases.

(2) Illustration of bus trajectories

Figures 1 to 3 show the bus trajectories for 3 extreme cases. The case that all the passenger choose to board the back bus is illustrated in figure 1. The case that all the passenger will stick to the front bus and the back bus hence can overtake it is shown in figure 2. Figure 3 is to distinguish from figure 2 with the case that the back bus is not allowed to overtake the front one although no one is going to get on the back one.

Comparing the figure 2 and figure 3 firstly, and it is clear to see that bus system will provide superior services with shorter maximal departure intervals which determine the maximal waiting time at the stops and smaller variation of departure intervals which can assess the service regularity of the bus system. Particularly worth mentioning is that $\hat{w}$ and $\bar{\sigma}$ are reduced by 30% and 40% respectively. We have enough evidence to conclude that overtaking policy is of significant necessity to be applied if passengers show no propensity to take the back bus or the layout of the stop make passenger unable to take the back bus.

Secondly, we can notice that the service performances of figure 1 an figure 2 are almost the same, case 1 exceeds case 2 by just a tiny advantage. When it is difficult to meet the needs of overtaking because of the stop layout or other issues, the bus operator should try to recommend passengers to take the back bus, which can also remain the bus system in a good condition.

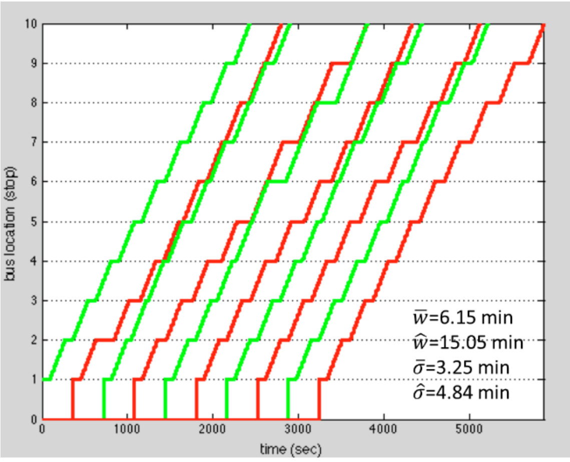

Fig.1 bus trajectory when γ = 0

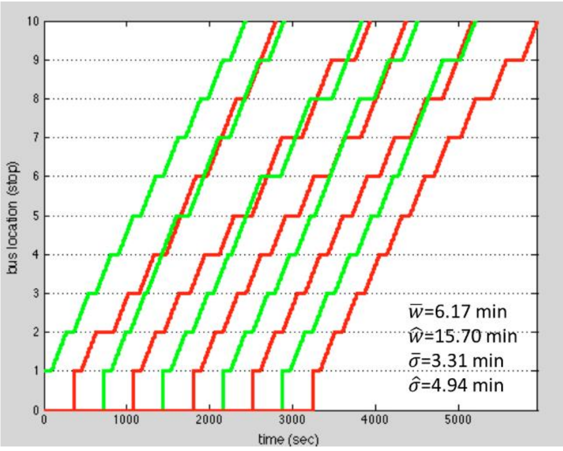

Fig.2 bus trajectory when γ = 1 with overtaking

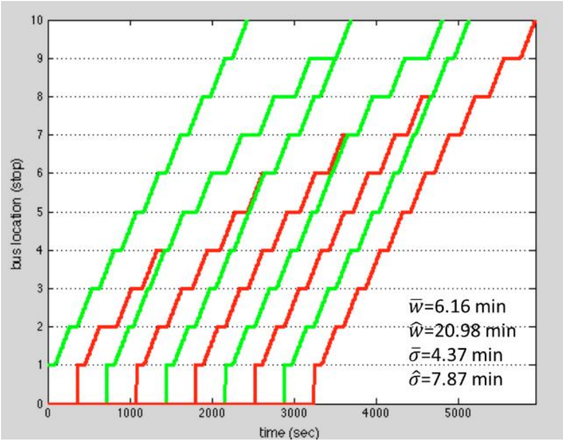

Fig.3 bus trajectory when γ = 1 without overtaking

(3) Tests with various degrees of front-bus preference

We illustrate the extreme case in previous section, and in this section, different degrees of front-bus preference are tested. Four kinds of evaluation indices are illustrated in Figures 4 and 5. Figure 4 includes the total standard deviation of Δ(m,n) of the system and max standard deviation of Δ(m,n) among all the stops, which is to show the system regularity under different conditions. Figure 5 is to show the max Δ(m,n) of the system under different queue switching degree. The analysis of the test results are divided into 2 cases to unfold: overtaking and no-overtaking.

(a) Overtaking is allowed

As is shown in Figures 4 and 6, two kinds of standard deviation of Δ(m,n) and max Δ(m,n) reach the minimum at point of γ=0, maximum at and almost symmetric with respect to the point of γ=0.5. Besides, these three evaluation indices keep stability to some extent with the increase of front-bus preference. This can be explaind as follows: Several services are getting closer to each other after the initial delay is generated and overtaking is occurring. During this period, the system is disturbed gradually with the increase of γ until it hit the threshold of 0.5, then overtaking is activated to retrieve the service regularity from the over-shortened headways. To keep the front-bus preference low is an optimal measure and overtaking is a favourable counter-measure against comparatively high front-bus preference.

(b) Overtaking is not allowed

Except for max Δ(m,n), other indices also reach the minimum when all the passengers do not prefer to board the front bus, which confirms the above conclusion that it is better to ask passengers to get on the back bus. Compared with the overtaking case, it is clear to observe the significant increase in the total and max standard deviation of Δ(m,n), which proves the superiority of overtaking policy in maintaining the regularity of the bus service. When γ >0.5, the higher the front-bus preference is, the more improvement could be obtained by overtaking policy. It is also interesting to notice that all the indices decrease to some extent when γ exceeds 0.5 and approaches 1, which probably means the no-overtaking policy also can contribute to the service under some certain circumstances.

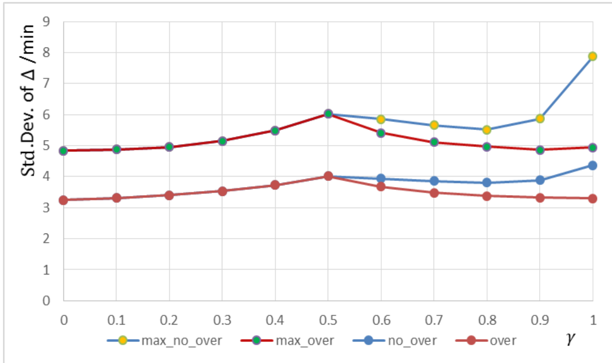

Fig.4 total standard deviation and max one by stop of Δ(m,n)

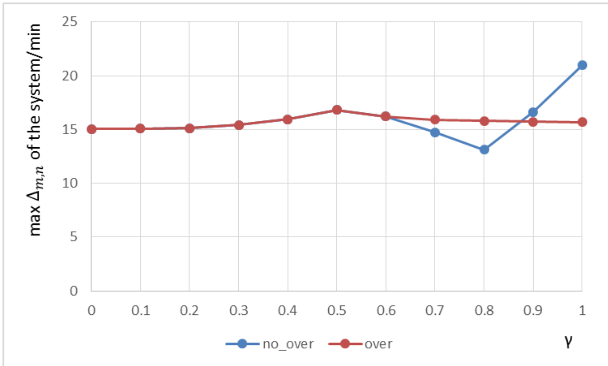

Fig.5 max Δ(m,n) of the system

7. CONCLUSIONS AND FURTHER WORK

(1) Conclusions

This paper is aiming to explain the effect of passenger behaviour at stops on bus bunching. A passenger behavior parameter to denote the preference to board the front bus is introduced. We then discuss which arrival and departure patterns can occur at a bus stop and solve the resulting problems to obtain the bus dwell times.

We evaluate the resulting service regularity given an initial disturbance to an early bus at one of the first stops along a corridor. For this we obtain the standard deviation and maximum headways between two bus depatures.

In our illustrative case study, we firstly focus on the extreme cases that all of the passengers will get on the front bus or none of them will board it. Overtaking policy is of significant necessity to be applied if passengers show no propensity to take the back bus or the layout of the stop makes passenger unable to take the back bus.

In the case that no passenger prefers to board the front bus always provides the best service, and it is an optimal measure to keep the front-bus preference under a low degree.

Finally, overtaking is a favourable counter-measure if front-bus preference is high. When it exceeds 0.5, the higher the front-bus preference is, the more improvement could be obtained by allowing for overtaking.

(2) Further work

We note that the front-bus preference degree of the passengers might differ depending on the position of passengers in the queue and whether they arrive before bus arrival or while the bus is boarding passengers. Such behaviour could be reflected with an additional parameter in our model.

More importantly though, when buses are bunched at the stop, which bus the passengers will choose to board will also depend on the remaining space of the bus. Capacity issues as well as alighting issues are though neglected in this paper for simplicity and to be able to illustrate the effect of boarding behavior better. In further work, we are considering though adding the alighting process and capacity constraints to increase realism of the model. The γ parameter would then become a function of the available bus capacity.

Finally, it would be interesting to apply this passenger choice model to a corridor served by several bus lines.

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