A Multi- hierarchy Facility Location Model Demand Uncertainty: A case study for Relief Distribution

Rattanaporn KASEMSRI[1], Kazushi SANO[2], Hiroaki NISHIUCHI[3] , Amila JAYASINGHE[3]

[1]Graduate School of Civil and Environmental Engineering, Nagaoka University of Technology
(Niigata, 940-2188, Japan)
E-mail: s125077@stn.nagaokaut.ac.jp
[2]Associate Professor of Civil and Environmental Engineering, Nagaoka University of Technology
(Niigata, 940-2188, Japan)
E-mail: sano@nagaokaut.ac.jp
[3]Assistant Professor of Civil and Environmental Engineering, Nagaoka University of Technology
(Niigata, 940-2188, Japan)
E-mail: nishiuchi@vos.nagaokaut.ac.jp
[4]Graduate School of Civil and Environmental Engineering, Nagaoka University of Technology
(Niigata, 940-2188, Japan)
E-mail: s147013@stn.nagaokaut.ac.jp

Humanitarian logistics have been indicated as significant issues in several terms of natural disaster operations and management. Indeed, a fluctuation demand as known as the demand uncertainty usually happens during a post disaster. Therefore, we proposes a few distinct network designs both single and double hierarchies of facility sites to find a most efficient network for a relief distribution under the demand uncertainty. An objective is to search the facility locations and optimize the transportation amount that can be achieved a minimum total delivery cost which includes transportation cost, facility cost and transshipment cost. The parameter uncertainty is handled by robust counterpart in Robust Optimization (RO) which has the capability to operate under lack of full information on the nature of uncertainty and increasing the popularity. The Tohoku's Earthquakes in 2011, Japan is a case study. We focus on Miyagi prefecture which is the most affected area and huge number of evacuees. A bottle of water is considered to be a requisite item for preliminary succor. The results show that the network configurations and truck sizes are significant with total delivery cost and their robustness. The uncertainty model is useful to help the planner to identify trade-offs between the inability to recover fully costs for excess link flow, and the need to manage transportation resource such as trucks, drivers and etc. to satisfy with the demand.

Key Words: *multi-hierarchy, uncertainty demand, facility locations, Robust Optimization*

1. INTRODUCTION

The post disaster logistics functions are defined for two significant issues as proving essentials to survived victims and recuing the victims. This study is considered the vital item distributions to relieve the large number of survived victims. There are sub three problems in logistics activities; location, routing and location-routing which are realized with a cost efficiency, a quick response, a satisfied demand and an environment issue. Moreover, the efficiency of planning and coordinating of logistic activities are necessary to treat them.

The location problem is one of the most important aspects in logistic activities. Some researchers have been done about the appropriate location of medical centers where the evacuees can be quickly accessed. Not only the medical centers but also the location of shelters is conducted. Details of these researches are shown next part of the literature reviews. This study intends to design the depot locations by considering the cost efficiency and also the satisfaction with the demand.

This model is to design principally the distribution network with multi-layer of facility locations by using the multi-source Capacitated Facility Location Problem (CFLP), or sometime is called the Capacitated Concentrator Location Problem (CCLP). The model is designed for single and double layers of depots to make the model more realistic and satisfied with the demand.

As we know that there are enormous impacts as both a humanitarian crisis and a massive economic aftermath of the 2011 Tohoku earthquake and tsunami. The Japan's central bank said that the economic losses of Kobe quake in 1995 were 10 trillion yen for both immediate problems with industrial production suspended in many factories, and the longer term issue of the cost of rebuilding. However, the Japanese Government, BOJ Governor Masaaki Shirakawa had estimated that this cost is much higher than the cost of just the direct material damage could exceed 25 trillion yen. Moreover, the several costs are generated to recover the situations during disaster and post disaster period, for example reconstruction cost, rescue cost, logistics cost and etc. The logistic cost was present by Nagurney *et al.*[3] as approximately 80 percent from overall of operation responding cost. Therefore, the cost efficiency should be one of many aspects that must be considered. By this reason, this study would like to play on the logistic cost efficiency. An improved supply distribution cost can reduce the expenditure of the whole of operation cost during the amelioration period. A bottle of water is considered to be a requisite item for preliminary succor. Even the total delivery cost minimization is not only one to consider in humanitarian logistics however it is a good criterion to compare the results of distinct network systems.

Furthermore, the real situations usually meet with the fluctuation of parameter uncertainty. Therefore, this study also stresses the importance of uncertainty of parameters; here is the supply and demand uncertainty. The methodology to handle with this demand fluctuation is Robust Optimization. The models that illustrate for uncertainty parameters are known as robust optimization model which are opposite with deterministic models.

This study considers robust counterpart in Robust Optimization (RO) which is provided by AIMMS software and more recently applied to handle under uncertainty of the parameters in the models. Robust optimization is designed to meet some major challenges associated with uncertainty-affected optimization problems as follows; to operate under lack of full information on the nature of uncertainty, to model the problem in a form that can be solved efficiently and to provide guarantees about the performance of the solutions. Robust Optimization is an uncertainty modeling approach suitable for a situation where the uncertainty ranges are known and not necessarily the distribution. Typically some inputs take an uncertain value anywhere between a fixed minimum and a maximum. This demand uncertainty can present how the worst case is when we consider the fluctuation of the demand. The Robust Optimization is very suitable for many problems as only simple inputs are required from the user about the data uncertainty because there are no scenarios or distribution functions need to be defined. The advantage of Robust Optimization models is that they grow only slightly when uncertainty is added. As the result, the model can be solved efficiently. Many fields of the academic study had discussed uncertainty parameter handling with robust optimization approaches, for an instance, a design and operations of chemical processes, an electrical capacity system, supply chain networks and transportation planning design.

Lin *et al.*[4] focused on logistics efficiency improvement. They said that the prioritized items for delivery and an extensive time period are importance of humanitarian logistics. They presented the location of temporary depots around the disaster-effected area between the long travel distances of demand points and the central depots.

2. OBJECTIVES

The major compositions of this study for both deterministic demand and uncertainty demand are summarized as follows:

1. To search the appropriate locations of depots to distribution the relief items in Miyagi prefectures.
2. To allocate the transportation link flow at each network configurations.
3. To minimize the total delivery cost which includes the transportation cost, the opening facility cost and the transshipment cost.
4. To compare the total delivery cost efficiency and their sensitivity.

3. MODEL STRUCTURE

The problem is designed for three different network frames. We categorize the distinct networks by the network configurations and the dispatched truck sizes. Two types of the network configurations are single hierarchy and double hierarchies of facility site candidates, defined as central depots and depots. Then, the problem is imposed that there are three network configurations with two echelons and four network configurations with three echelons. The first network configuration is the locations where the serviceable supports known as suppliers. The second network configuration is the central relief depot in case of double hierarchies. The third configuration is the relief depots for double hierarchies and the relief depot in case of a single hierarchy. These second and third network locations are unknown and need to be defined with the most efficiency. Finally, a possible area that was attacked by the natural disaster is called an affected area which can be defined known locations as demands. The transportation truck sizes are 10-ton trucks and 4-ton trucks. The specification of the networks is described below.

Network 1

This network is determined with the three network configurations which include suppliers, relief depots and shelter demands. The relief depot candidate sites are located inside the affected areas. The relief items are dispatched from suppliers to relief depots by using 10-ton trucks. Then, the 4-ton trucks are used for portage the relief items from relief depots to shelter demands.

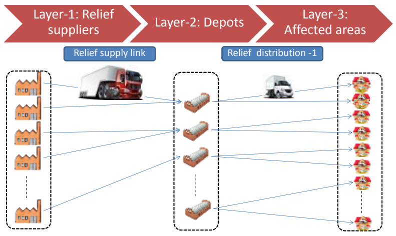


Fig.1 The single hierarchy network framework and 10-4-ton truck delivery

Network 2

This network is determined with the four network configurations which include suppliers, central relief depots, relief depots and shelter demands. The central relief depot candidates are supposed to locate inside the affected areas. The 10-ton trucks are proposed to transport the relief items from supplies to central relief depots and central relief depots to relief depots. Then, the relief items are carried from relief depots to shelter demands by using 4-ton trucks.

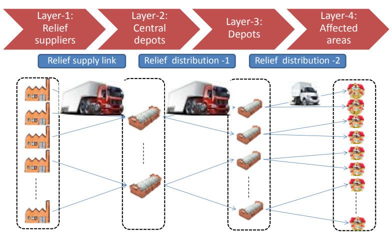


Fig.2 The two hierarchies network framework and 10-10-4-ton truck delivery

Network 3

This network is duplicate structure with the Network 2 in term of the number of network configuration and their location. However, there is the difference in term of the truck size. The 10-ton trucks are assumed to deliver the relief items from supplies to central relief depots. Then, 4-ton trucks are assigned to deliver from central relief depots to relief depots and from relief depots to shelter demands respectively.

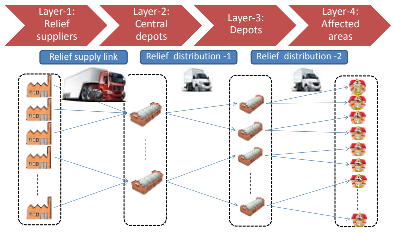


Fig.3 The two hierarchies network framework and 10-4-4-ton truck delivery

4. MATHEMATICS

Indices

M : Set of the supplier nodes (i) (i=1,2,3…M)

N : Set of the candidate central depots (j) (j=1,2,3…N)

L : Set of the candidate depots (k) (k=1,2,3…L)

P : Set of the demand nodes or shelters (l) (l=1,2,3…P)

TS : Set of the truck size (s)

Notations

$x^1_{ij}, x^2_{jk}, x^3_{kl}$: The flow of items from i to j, j to k, and k to l

C^1_j : The capacity at the candidate central depots j (j=1,2,3…N)

C^2_k : The capacity at the candidate depots k (k=1,2,3…L)

Sets of parameters

S_i : The amount of items at the supply nodes i

D_l : The demand at the affected area nodes l

$c^1_{ij}, c^2_{jk}, c^3_{kl}$: The travel cost between i to j, j to k and k to l

f^1_j, f^2_k : The opening depot cost at j and k

tc^1_j, tc^2_k : The transshipment cost at j and k

$v^1_{ij}, v^2_{jk}, v^3_{kl}$: The capacity of truck between i to j, j to k and k to l

$w^1_{ij}, w^2_{jk}, w^3_{kl}$: The maximum working time of drivers between i to j, j to k and k to l

$d^1_{ij}, d^2_{jk}, d^3_{kl}$: The distance between i to j, j to k and k to l

$t^1_{ij}, t^2_{jk}, t^3_{kl}$: The travel time between i to j, j to k and k to l

R_s : The energy consumption rate of truck size s

S_s : The driver salary of truck size s

T_s : The truck cost of truck size s

4.1 Objective function

$$min\left\{\sum_{j=1}^{N}\sum_{i=1}^{M} c_{ij}^{1}x_{ij}^{1} + \sum_{k=1}^{L}\sum_{j=1}^{N} c_{jk}^{2}x_{jk}^{2} + \sum_{l=1}^{P}\sum_{k=1}^{L} c_{kl}^{3}x_{kl}^{3} + \sum_{j=1}^{N} f_j^1 Y_j + \sum_{k=1}^{L} f_k^2 Z_k + \sum_{j=1}^{N} tc_j^1 Y_j + \sum_{k=1}^{L} tc_k^2 Z_k\right\} \quad (1)$$

When

$$c_{ij}^1 = \left(R_s d_{ij}^1\right) + \left(S_s t_{ij}^1\right) + T_s \quad (2)$$

$$c_{jk}^2 = \left(R_s d_{jk}^2\right) + \left(S_s t_{jk}^2\right) + T_s \quad (3)$$

$$c_{kl}^3 = \left(R_s d_{kl}^3\right) + \left(S_s t_{kl}^3\right) + T_s \quad (4)$$

Decision variables

$$Y_j = \begin{cases} 1, \text{ if central depots is located at } j \\ 0, \text{ otherwise} \end{cases} \text{for } j \,\varepsilon\, N \quad (5)$$

$$Z_k = \begin{cases} 1, \text{ if depots is located at } k \\ 0, \text{ otherwise} \end{cases} \text{for } k \,\varepsilon\, M \quad (6)$$

Subject to

$$\sum_{j=1}^{N} x_{ij} \le S_i \quad (7)$$

$$\sum_{i=1}^{M} x_{ij}^1 \le \sum_{k=1}^{L} x_{jk}^2\, Y_j \quad (8)$$

$$\sum_{k=1}^{L} x_{jk}^2 \le C_j^1 Y_j \quad (9)$$

$$\sum_{j=1}^{N} x_{jk}^2 \le \sum_{l=1}^{P} x_{kl}^3\, Z_k \quad (10)$$

$$\sum_{l=1}^{P} x_{kl}^3 \le C_k^2 Z_k \quad (11)$$

$$\sum_{k=1}^{L} x_{kl}^3 \le D_l \quad (12)$$

$$\sum_{j=1}^{N} x_{ij}^1 \le v_{ij}^1 \quad (13)$$

$$\sum_{k=1}^{L} x_{jk}^2 \le v_{jk}^2 \quad (14)$$

$$\sum_{l=1}^{P} x_{kl}^3 \le v_{kl}^3 \quad (15)$$

$$\sum_{j=1}^{N} t_{ij}^1 \le w_{ij}^1 \quad (16)$$

$$\sum_{k=1}^{L} t_{jk}^2 \le w_{jk}^2 \quad (17)$$

$$\sum_{l=1}^{P} t_{kl}^3 \le w_{kl}^3 \quad (18)$$

$$x_{ij}, y_{jk}, z_{kl} \ge 0 \quad (19)$$

$$Y_j,\ Z_k \in \{0,1\}\ for\ all\ j\ and\ k \quad (20)$$

Constraint (7) guarantees that the total amount flow from suppliers i to central depots j is not over than the amount of serving goods at suppliers i. Constraint (8) restricts for the summation of link flow from i to j does not exceed than the capacity of opening the central depots j. Constraint (9) limit for the total amount of link flows from j to k not exceeding than the total availability of goods at opening central depots j. Constraint (10) restricts that the summation amount of link flow from j to k must not be over than the capacity of next network configuration or depots k. Constraint (11) is ensured that the total amount from depots k to demand l is not over than the availability of goods at depots k. Constraint (12) is confirmed that the total amount serving from depots k is satisfied with the demand l. Constraint (13), (14), (15) are determined to prohibit that the amount of a commodity cannot exceed the maximum truck volume restriction. Constraint (16), (17), (18) are restricted for the total driving hours of driver which are not over than the maximum working time. Constraint (19) is confirmed that each link flow from site i to j, j to k and k to l need to define with some amount of goods. Constraint (20) is generated to specify that the both decision variables Y_j and Z_k are binary variable 0 and 1, 1 is represented, if the facility is located at site j and k and 0 is otherwise.

4.2 Mathematical with Robust Formulation by using Robust Counterpart

This study focuses on the multi-source and multi-layer of facility location problem with uncertainty demand by considering the ellipsoidal uncertainty set in robust optimization approach. *Ben-Tal and Nemirovski* consider ellipsoidal uncertainty set with linear programming[11)]. *Kouvelis and Yu* discussed the robust discrete optimization and its applications.

They proposed an approach to find a solution that minimizes the worst case performance under a set of scenarios for the data[12]. *Bertsimas and Brown* proposed a methodology for constructing uncertainty sets for robust liner optimization based on decision maker risk preferences[13]. *Josef* gave an overview on the state-of-the-art and recent advances in mixed integer optimization to solve planning and design problems in the process in industry. Stochastic programming for continuous LP problems is now part of the most of the optimization packages, and there is encouraging progress in the field of stochastic MILP and robust MILP[14]. *Ben-Tal, Bertsimas and Brown* proposed a soft robust model for optimization under ambiguity[15]. Whenever the uncertainty set of a mixed-integer robust problem is an ellipsoidal, and then the robust counterpart can be reformulated as a mixed-integer second-order cone program (SOCP).

This study focuses on the demand uncertainty parameter which deviates from the nominal value of the uncertain parameters. The demand uncertainty is expanded followed by the region of the ellipsoidal uncertainty set. The demand is defined as parameter D and $(\overline{D})$ is the demand that deviate from historical or nominal values. The uncertainty demand $\overline{D} \epsilon R^d$, we consider the sets around the nominal value $D \epsilon R^d$. Then, we use the ρ^2 to restrict the region around the nominal value, here is equal to 1. We determine the interval range of demand $(D - \overline{D})$ is equivalent as maximum truck capacity.

Ellipsoidal :

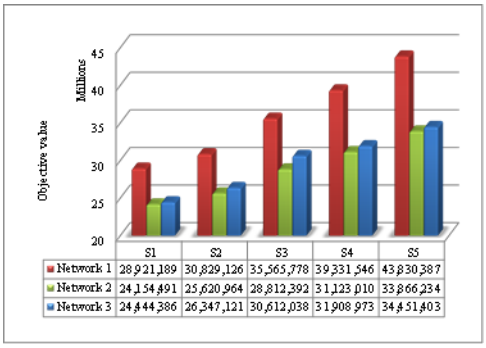
$$U = \left(\frac{(D-\overline{D})^t}{\delta \times (D-\overline{D})}\right)^2 \leq \rho^2 \tag{21}$$

$$min\left\{\sum_{j=1}^{N}\sum_{i=1}^{M} c_{ij}^1 \overline{x_{ij}^1} + \sum_{k=1}^{L}\sum_{j=1}^{N} c_{jk}^2 \overline{x_{jk}^2} + \sum_{l=1}^{P}\sum_{k=1}^{L} c_{kl}^3 \overline{x_{kl}^3} + \sum_{j=1}^{N} f_j^1 \overline{Y_j} + \sum_{k=1}^{L} f_k^2 \overline{Z_k} + \sum_{j=1}^{N} tc_j^1 \overline{Y_j} + \sum_{k=1}^{L} tc_k^2 \overline{Z_k}\right\} \tag{22}$$

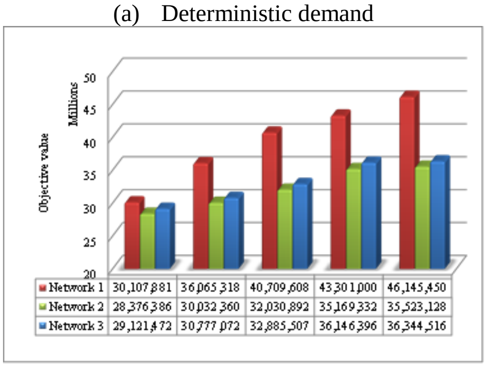
$$\sum_{k=1}^{L} \overline{x_{kl}^3} \leq \overline{D_l} \tag{23}$$

5. RESULTS

We would like to illustrate both circumstance outcomes deterministic and uncertainty model. The expectation results for both circumstances are the total delivery cost of the three different network frames. As mention before, each network frame includes five demand scenarios thus we prefer to report five expectation results for each. In order to identify the network efficiency by total delivery cost minimization and network robustness, hence we compare the total delivery cost of the three networks and indicate the best network structures. Then, we present the sensitivity analysis and compare the robustness of the three networks. Therefore, this study can be helping the decision maker to plan for post disaster distribution network and their systems when the circumstance of demand uncertainty occurs.



(a) Deterministic demand



(b) Uncertainty demand

Fig.4 The total delivery cost

From the results, we found that the network configurations and their systems are affected with the total delivery cost both deterministic demand and uncertainty demand as shown in the figure 4. It can be seen that the network 2 and network 3 as defined for two hierarchies of facility are obviously preferable cost performance when comparing with the network 1 which is single hierarchy. The total delivery cost of network 2 and network 3 lessened by 17.96 percent and 16.78 percent respectively. The total delivery cost is mostly generated by travel which is more than about 90 percent and its rapid increase depends on the amount of transportation.

When comparing network 2 and network 3, all demand scenarios in network 2 can be reduced by 1.19 percent, 2.79 percent, 6.06 percent, 2.49 percent and 1.71 percent respectively. These results demonstrate that not only network configurations but together with truck size operations are significant with total delivery cost function. By using 10-ton truck to deliver from suppliers to central depots and from central depots to depots can have a benefit of cost reduction.

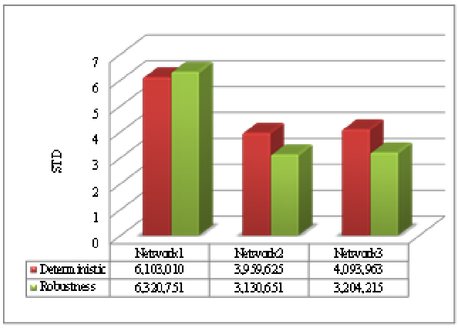


Fig.5 The Standard deviation

Figure 5 illustrates the standard deviation of objective function for each network. The standard deviation network 1 is higher than the others; means that there are much fluctuates. The standard deviation of deterministic demand, network 1 is approximately 6 million while there is around 4 million for network 2 and network 3.

Comparison the deterministic demand and uncertainty demand, network 2 and network 3 are similar that by using robust optimization to handle the uncertainty demand illustrates more robustness than deterministic demand. In addition, the fluctuation between deterministic demand and uncertainty demand of network 2 is less than network 3. Meaning that network 2 is robust than the other networks.

6. CONCLUSION AND FUTURE WORKS

This study principally analyzes the multi-facility location problems under both deterministic demand and uncertainty demand issues. We diagnose the uncertainty demand by the reasons that it is quite difficult to predict the post disaster demand. We consider a whole distribution network starting from the beginning, suppliers until the end, demands. We apply the model with the case study in order to evaluate the total delivery cost during post disaster in Miyagi prefecture. The objective function is to minimize the total delivery cost which includes the travel cost, the opening facility cost and the transshipment cost by selecting the facility sites and optimized transportation flow. We propose the three network structures which are the one network of single hierarchy facility and two network of two hierarchies with distinct truck size (large trucks and small trucks), to handle both demand known and unknown circumstances. We determine the region of uncertainty demand as ellipsoid uncertainty set. Therefore, this study can help the decision makers to prepare the appropriate network with robustness for relief distribution.

First of all, the calculation results both deterministic demand and uncertainty demand demonstrate that the network configurations are significant with total delivery cost. It can be seen clearly that the total delivery cost of network 2 and network 3 can reduce about 18 percent because the travel cost much reduces even though it requires more facility cost and transshipment cost. The results show that the travel cost has more significance than the opening facility cost. Moreover, the truck size operation is significant when the demand is high enough. This study found that large truck is appropriate to deliver both inbound and outbound at the central depots. To apply model, we suggest establishing the central depots and using large truck to deliver both inbound and outbound.

Furthermore, we also would prove that the networks are robust when the demand becomes uncertain or unknown. Here, we assume five different demand scenarios in each network based on the actual number of evacuees during post disaster. After solving the uncertainty demand by using robust optimization, the results prove that the structural networks affect on the model robustness. The two hierarchies of facility provide an extra robustness than the single hierarchy of facility. Moreover, the uncertainty demand model is robust than deterministic demand model.

Finally, we discuss the interrelated aspects to improve the future work as follows: (1) we have not considered the other parameters that can be possible to fluctuate during humanitarian logistics, for example the supply amount, the unit transportation cost, the opening facility cost and etc. Therefore, not only the uncertainty demand but also such kind of parameters should be considered simultaneously. (2) A new research can be improved with more efficiency by considering the vehicle routing problem together with our facility location problem simultaneously. This model can be referred to Location-Routing problem. The model might give more interesting results because the travel cost would be reduced by route detour.

The future work is considered the two-objective facility location problem. The model should be more reasonable by investigation both cost and time indicators simultaneously. After that the uncertainty of the demand is assigned to use with the model and then evaluates the robustness of the model.

REFERENCES

1) Van Wassenhove, L.N. Blackett Memorial Lecture: Humanitarian aid logistics: supply chain management in high gear, *Journal of the Operational Research Society*, 57, 475-489, 2006.
2) Thomas, A. Mizushima M. Logistics training: necessity or luxury? Forced Migration Review, 22, 60-61, 2005.
3) Nagurney, A. SCH-MGMT 597LG Humanitarian Logistics and Healthcare Spring 2012, Presentation, 2012
4) Lin, Y.-H., Batta, R., Rogerson, P., Blatt, A., Flanigan, M. Location of temporary depots to facilitate relief operations after an earthquake, *Socio-Economic Planning Sciences*, 46, 112-123, 2012
5) Miyagi Prefectural Government. Earthquake Damage Information, Report (in Japanese), 2012.
6) Hosoya, R., Sano, K., Ieda, H., Kato, H., Fukuda, A. Evaluation of Logistic Policies in the Tokyo Metropolitan Area Using A Micro-Simulation Model for Urban Goods Movement, *Journal of the Eastern Asia Society for Transportation Studies*, Vol.5, October, 2003
7) Bramel, J., Simchi-Levi, D. The Logic of Logistics: Theory, Algorithms, and Applications for Logitics Managemen, New York, USA, 1999
8) Indra-payoong, N. Discrete Optimization in Transport and Logistics. Bangkok, Thailand (in Thai), 2005
9) Caunhye, A.M., Nie, X., Pokharel, S. Optimization models in emergency logistics: A literature review, *Socio-Economic Planning Sciences*, 46, 4-13, 2012.
10) Mete, HO., Zabinky, ZB. Stochactic optimization of medical supply location and distribution in disaster management, *International Journal of Production Economics*, 126(1), 76-84, 2010.
11) Ben-Tal, A., Nemirovski, A. (2000) Robust solution of Linear Programming problems contaminated with uncertain data. *Math. Program.* 88, 411-424
12) Kouvelis, P., & Yu, G. (1997) Robust discrete optimization and its applications. Kluwer Academic Publishers, Norwell, MA.
13) Bertsimas, D., Brown, D.B. (2009) Constructing Uncertainty Sets for Robust Linear Optimization. *Operations Research*, 57(6), p. 1483-1495.
14) Josef, K. (2004) Solving Planning and Design Problems in the Process Industry Using Mixed Integer and Global Optimization. *Special Edition of Annals of Operations Research*, State-of-the-Art IP and MIP, p. 31-61
15) Ben-Tal, A., Bertsimas, D., Brown, D.B. (2010) A Soft Robust Model for Optimization under Ambiguity. *Operations Research*, 58(4), 2(2), 1220-1234.