A Reliable Routing for Risk-averse Navigation

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To avoid the risk of uncertain travel time, a reliable shortest path is considering the delay uncertainty and travel time reliability is of great value. Accordingly, a two-step procedure is proposed. First, a shortest path set is built by the hyperpath concept. Meanwhile the link choice probability is assigned by the maximum link delay at each node. Second, the shortest path algorithm is applied to find the reliable shortest path by penalizing the hyperpath links with low reliability. Link reliability is calculated by the link usage frequency collected from historical trips of probe vehicles. Then, an 8 by 8 grid network is used to test the path finding procedure. It is found that an optimal path with higher reliability is generated as the stochastic link delay increase, which coincides with the phenomenon that drivers tend to choose a more reliable path when the traffic condition becomes worse.

Key Words : *Travel Time Reliability, Hyperpath, Shortest Path, Navigation, Probe vehicle*

1. INTRODUCTION

As the traffic congestion becomes a critical issue in large city, the arrival time reliability is a major concern for navigation users. Comparing to the shortest path, the reliable path with accurate travel time estimation has more attractiveness. Recent studies[1-3] began to consider reliability as the optimization objective for finding a set of feasible paths. There are different definitions for reliability. The reliability was defined as the probability of completing a trip within a given travel time budget in literature[4-5]. Reliability can also be defined as the probability of not encountering congestion along a link[6-8]. Kaparias et al[9] defined the earliness and lateness reliability based on the log-normal distribution of travel time on a link, in which the shortest and longest travel times were taken into account.

In an unstable network, there will be a set of paths that may be optimal from origin to destination due to stochastic delay. In general, a common approach to find a reliable path is by penalizing the unreliable links[7-8]. Borrowing the idea from equilibrium traffic assignment, the hyperpath concept[10-11] is appropriate to the reliable path set generation process, which represents a sequence of routing strategies rather than a simple shortest path. The set of outgoing links at each intermediate node is assigned a choice probability according to the exposure to maximum delay. Bell[12-13] extended the hyperpath algorithm by adding node potentials into the link selection step, making it appropriate to risk-averse navigation. Since the hyperpath algorithm offers an efficient way to generate a set of potential shortest paths, we will apply this concept to generate a link set with reasonable travel time in the following study.

Although the above studies concerned the stochastic characteristic of travel time and made an effort to find a reliable path, they seldom considered the travel time estimation accuracy on each link of the whole network. Most of the current studies assumed that the travel time on each link can be obtained exactly at each time interval. However, not all of the link travel time can be collected in practice because most of the traffic information collection technologies such as AVI (Automatic Vehicle Identification) and loop detector cover only limited specific road sections. To date, probe vehicle with GPS equipment is known as one of the most efficient tools for traffic information collection, which enables to cover wider area than other stationary equipment. However, it is still difficult to guarantee the accuracy and instantaneity. Simulation tests[14-15)] indicate that for an absolute error in estimated average link speed to be less than 5 km/h, the network needs to have 5% of active probe vehicles within the sampling period. Furthermore, there is often a lag between the real-time estimation and the broadcast of the travel time information. As a consequence, it is inappropriate to apply the current routing algorithm to the navigation without considering the travel time estimation accuracy. To alleviate this problem, an alternative approach is to seek paths that avoid links with a history of severe congestion and to penalize those unreliable links that seldom be chosen by probe vehicles.

This study aims to find a reliable shortest path for risk-averse navigation using driving experience of probes. Assumed that the historical link usage frequency enables to reflect the driving experience and most of the local drivers with abundant driving experience prefer reliable links without severe delay, the reliability can be formulated by the link usage frequency collected from historical trips of probes. And then, a two-step procedure is proposed to find a reliable shortest path. First, a link set with reasonable travel time is built by the hyperpath concept which minimizes the expected arrival time at destination and all intermediate nodes. Second, the optimal path is obtained from the link set of hyperpath by penalizing the link travel time with low reliability. And the A* algorithm is applied to solute the one-to-one node path finding problem. To validate the proposed method, an 8 by 8 grid bi-directional network is taken to test.

2. METHODOLOGY

The model framework is based on the following assumption:

The risk-averse drivers will take a two-step procedure to determine a path before departure. First, the potential optimal path set will be determined by considering the historical delay situation. Those paths with unacceptable travel time will be excluded. And then, a path with higher reliability is preferred. They may estimate the link reliability by their historical driving experience or by referring to other drivers' experience.

Based on the above assumption, **Fig. 1** summarizes the formulation and solution procedure of the risk-averse navigation problem. In phase 1, the link travel time and link usage frequency are estimated by mining the historical trips from probe data. In phase 2, the hyperpath algorithm is applied to find a potential shortest path set, which reflects an optimal strategy that minimizes the maximum exposure to delay. Finally, a reliable shortest path is recommended by penalizing the links with low reliability from hyperpath link set.

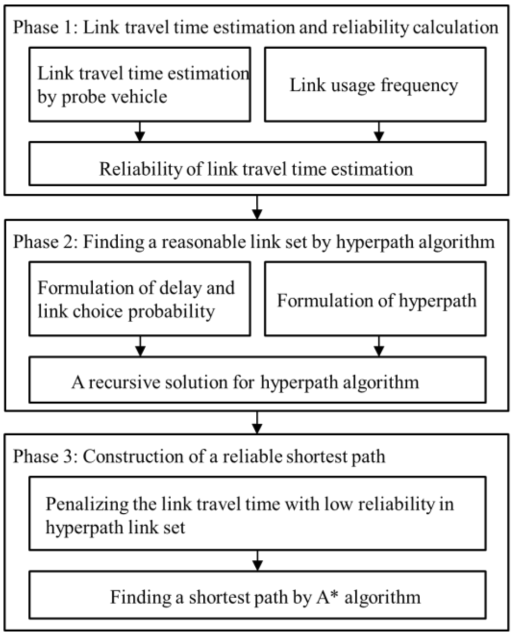

Fig. 1 Formulation and solution procedure.

(1) Formulation of reliability

The reliability of link travel time corresponds to the usage frequency of probe vehicles in the link. It means that estimation becomes more accurate as more probes passing the link during the measurement period. On the other hand, the link with high usage frequency often suggests that it is a reliable link because rational drivers will not choose an unreliable link frequently. The reliability is dependent on the error of link travel time estimation, which is strongly related to the usage frequency (or sample size of probe) in each link. More specifically, the measure of reliability is defined as the probability that the relative error (ε) is less than maximum acceptable relative error (ε_{max}). Since exact mean travel times are not known, an estimate of ε is defined by the absolute ratio of the difference between the mean travel time (T) of the probes in the current measurement period (e.g. between 2014-8-1 8:00 to 2014-8-1 8:05) and the historical overall mean travel time (μ) of all probe vehicles traversing the link (e.g. mean travel time during 8:00-8:05 in one month). Thus, the relative error is given by Eq. (1). And the reliability formulation is given by Eq. (2).

$$\varepsilon = \frac{|T-\mu|}{\mu} \tag{1}$$

$$r = Pr(\varepsilon < \varepsilon_{max}) \tag{2}$$

where $T = \frac{1}{n}\sum_{i=1}^{n} t_i$, t_i is the link travel time of probe vehicle i, n is the number of probe car in the current measurement period. $\mu = \frac{1}{m}\sum_{i=1}^{m} t_i$, m is the number of probe vehicle in the overall historical measurement period.

When it comes to measure the reliability, it is necessary to consider the distribution of link travel time. Recent studies[9,16] find that the link travel time usually follows a skewed distribution, and a log-normal distribution was suggested. Here, we follow a log-normal distribution. The distribution of the natural logarithm travel time is $t_l \sim LN(u_l, {\sigma_l}^2)$, where t_l is the natural logarithm link travel time of one probe vehicle, u_l and ${\sigma_l}^2$ are the mean and standard deviation of the natural logarithm of travel time, respectively. They can be derived from the mean (μ) and variance (σ^2) of the original link travel time as shown in Eq. (3) and (4).

$$u_l = \ln(\mu) - \frac{1}{2}\ln\left(1 + \frac{\sigma^2}{\mu^2}\right) \tag{3}$$

$$\sigma_l = \sqrt{\ln\left(1 + \frac{\sigma^2}{\mu^2}\right)} \tag{4}$$

Accordingly, the modified relative error of the natural logarithm travel time is expressed by Eq. (5).

$$\varepsilon_l = \frac{|T_l - u_l|}{u_l} \tag{5}$$

where $T_l = \frac{1}{n}\sum_{i=1}^{n} ln(t_i)$. The upper and lower bounds (ε_l^{upper} and ε_l^{lower}) of the modified relative error of logarithm travel time can be set by the travelers. Then, the reliability is expressed by Eq. (6).

$$r = Pr\left(\varepsilon_l^{lower} < \frac{T_l - u_l}{u_l} < \varepsilon_l^{upper}\right) \tag{6}$$

Since T_l is a sample mean of natural logarithm of travel times of probe, an invocation of the central limit theorem implies asymptotically,

$$\mathrm{Z} = \mathrm{N}(0,1) \sim \left[\frac{T_l - u_l}{\sigma_l/\sqrt{n}}\right] \tag{7}$$

Substituting Eq. (7) into Eq. (6),

$$r = Pr\left(\frac{\varepsilon_l^{lower} u_l}{\sigma_l/\sqrt{n}} < Z < \frac{\varepsilon_l^{upper} u_l}{\sigma_l/\sqrt{n}}\right) \tag{8}$$

Since Z follows standard normal distribution, let $\Phi(x)$ is the cumulative distribution function of Z,

$$r = \Phi\left(\frac{\varepsilon_l^{upper} u_l}{\sigma_l/\sqrt{n}}\right) - \Phi\left(\frac{\varepsilon_l^{lower} u_l}{\sigma_l/\sqrt{n}}\right) \tag{9}$$

(2) Application of hyperpath concept to establishment of potential optimal path set

a) Hyperpath concept

The hyperpath concept is originally proposed in the context of equilibrium traffic assignment[10] and public transport assignment[11]. Instead of recommending a specific shortest path, hyperpath can provide a set of paths, each of which is a potential optimum that may be attractive to travelers due to travel time uncertainty. More specifically, the strategy of hyperpath finding is as follows:

"To avoid the risk of stochastic delay at node i and pass this node as soon as possible, the attractive outgoing alternative links of node i will be considered. Since the delay is stochastic, the best way is to assign alternative links choice probabilities so as to minimize the maximum exposure to delay."

Throughout the paper, the following notations will be used:

E: Set of links of the whole network
I: Set of nodes of the whole network
r: Origin node
s: Destination node
H: Hyperpath link set
E_i^+: Set of outgoing links from node i
E_i^-: Set of incoming links to node i
c_{ij}: Undelay travel time on link(i,j)
d_{ij}: Maximum delay on link(i,j)
p_{ij}: Probability that link(i,j) is chosen
π_{ij}: Conditional probability that link(i,j) is chosen at node i
w_i: Expected maximum delay at node i
p_i: Probability that node i is chosen
M: A large number

Similar to the concept of waiting time at a bus stop, the expected maximum delay (w_i) at node i can be seen as the "combined waiting time" of link$(i,j) \in E_i^+$. Following Bell's study[12], w_i for attractive links can be interpreted as node delay exposure. It means that if there is only one attractive link$(i,j) \in E_i^+$, the traveler will be fully exposed to delay d_{ij}, whereas more alternative links will help avoid the exposure to maximum delay. It is usually assumed that the expected maximum delay to traverse link(i,j) followed exponential distribution because it is possible to derive closed-form expressions of w_i and π_{ij}[11-12]. The combined exposure to delay at node i can be given by Eq. (10). The link choice probability can be given by Eq. (11)

$$w_i = \frac{1}{\sum_{(i,j)\in E_i^+}\frac{1}{d_{ij}}} \tag{10}$$

$$\pi_{ij} = \frac{\frac{1}{d_{ij}}}{\sum_{(i,j)\in E_i^+}\frac{1}{d_{ij}}} \tag{11}$$

b) Formulation of hyperpath

The path finding problem considered here closely resembles the equilibrium traffic assignment for transit network introduced by Spiess and Florian[11]. In the traffic assignment model, the traffic demand from origin r to destination s is assigned to the network yielding link volumes v_{ij} by given a strategy. Generally, the optimal strategy E^* is the strategy that minimizes the expected total travel time including waiting time. The assignment model can be expressed by the following linear program. Let **P0** be the problem defined by Eq. (12-17)

P0: $Min\ \sum_{(i,j)\in E} c_{ij}\, v_{ij} + \sum_{i\in I} \frac{V_i}{\sum_{(i,j)\in E_i^+} \frac{1}{d_{ij}} x_{ij}}$ (12)

Subject to

$$v_{ij} = x_{ij}\pi_{ij}V_i, (i,j) \in E_i^+, i \in I \quad (13)$$

$$V_i = \sum_{(i,j)\in E_i^-} v_{ij} + g_i, i \in I \quad (14)$$

$$\sum_{(i,j)\in E_i^+} v_{ij} - \sum_{(i,j)\in E_i^-} v_{ij} = g_i, i \in I \quad (15)$$

$$V_i \geq 0, i \in I \quad (16)$$

$$x_{ij} = \begin{cases} 0, if (i,j) \notin E^* \\ 1, if\ (i,j) \in E^* \end{cases} \quad (17)$$

Eq. (14) and (15) enforce flow conservation. g_i denotes the demand at node i. V_i denotes the sum of the volumes of all incoming links and the demand at that node. v_{ij} denotes the link volume.

Considering a special case in which the demand from node r to node s is 1, and no demand at any intermediate node $i, i \in I - \{r\}$, is generated, thus **P0** can be applied to the path finding problem for individual navigation. Note that $g_i = 1$ if $i = r$, $g_i = -1$ if $i = s$, and $g_i = 0$ otherwise. In this condition, $V_i = \sum_{(i,j)\in E_i^-} v_{ij} = 1$ if $(i,j) \in E^*$, $V_i = 0$ otherwise. Because the demand from origin to destination equal to 1, the assigned link volume v_{ij} can be explained as the probability that link(i,j) is chosen. Hence, v_{ij} can be replaced by p_{ij}. In practice, the undelay travel time c_{ij} could be estimated by the average travel time and the maximum delay d_{ij} on link(i,j) could be estimated by the 95% quantile statistics of travel time. Then the hyperpath problem for risk-averse routing guidance can be formulated as follows:

P1: $Min\ f_0(\boldsymbol{p}, \boldsymbol{w}) = \sum_{(i,j)\in E} c_{ij}\, p_{ij} + \sum_{i\in I} w_i$ (18)

Subject to

$$\sum_{(i,j)\in E_i^+} p_{ij} - \sum_{(i,j)\in E_i^-} p_{ij} = g_i, i \in I \quad (19)$$

$$p_{ij} d_{ij} \leq w_i, (i,j) \in E_i^+, i \in I \quad (20)$$

$$p_{ij} \geq 0, (i,j) \in E \quad (21)$$

c) Solution algorithm of hyperpath

The recursive formulation developed by Bellman[17] was an effective way to yield the optimal path by an iterative solution procedure. A generalized Bellman's equation by incorporating the expected waiting time at stops was introduced in the public transport assignment problem[10]. Here, we apply this procedure to the solution of hyperpath algorithm. The total expected travel time of hyperpath in a stochastic network with delays can be expressed recursively by Eq. ((22).

$$u_j = \begin{cases} 0, & if\ i = s \\ \min_{T_i \subseteq I_i^+} \left\{ \frac{\left(\sum_{i\in T_i} \phi_{ij}(c_{ij}+u_i)+1\right)}{\sum_{i\in T_i} \phi_{ij}} \right\} & , if\ i \neq s \end{cases} \quad (22)$$

where $\phi_{ij} = 1/d_{ij}$, u_j is the expected travel time of the shortest hyperpath from node r to node j. The above generalized Bellman's equation suggests the following iterative procedure for computing the hyperpath:

Step 1: Initialization

Set $u_i \leftarrow \infty,\ i \in I - \{r\}, u_r \leftarrow 0$; $\phi_{ij} \leftarrow \frac{1}{d_{ij}}\ if\ d_{ij} > 0, otherwise\ f_{ij} \leftarrow M, \forall (i,j) \in E$; $\Phi_i \leftarrow 0, \forall i \in I$; $p_i \leftarrow 0,\ i \in I - \{r\}, p_r \leftarrow 1$; $H \leftarrow \emptyset; L \leftarrow E$

Step 2: Updating node labels

Select link$(i,j) \in L$ with minimum $u_i + c_{ij}$; $L \leftarrow L -$ link(i,j);

If $u_i + c_{ij} \leq u_j$ then update node label:

If $u_j \leftarrow \infty$ and $\Phi_j = 0$, then

$u_j = u_i + c_{ij} + d_{ij}$

Else

$u_j \leftarrow \frac{\Phi_j}{\Phi_j + \phi_{ij}} u_j + \frac{\phi_{ij}}{\Phi_j + \phi_{ij}} (u_i + c_{ij})$ (Eq. ((22))

$\Phi_j \leftarrow \Phi_j + \phi_{ij}$

$H \leftarrow H + \{\text{link}(i,j)\}$

Repeat step 2 until $L = \emptyset$ or $u_i + c_{ij} > u_s$

Step 3: Updating link choice probability and node choice probability

Sort link$(i,j) \in H$ in decreasing order of $u_i + c_{ij}$

$p_{ij} \leftarrow \frac{\phi_{ij} p_j}{\Phi_j}$

$p_i \leftarrow p_i + p_{ij}$

End

(3) Construction of a reliable shortest path

The hyperpath algorithm generates a set of paths and assigned a choice probability to each link according to the maximum delay. Actually, the shortest path set in the hyperpath is restricted by the lower and upper bounds of optimistic (all undelay) and pessimistic (all maximum delay) travel time[18,19]. Thus, it is reasonable to recommend the paths generated by hyperpath algorithm as the alternative paths with acceptable travel time to users. Considering that most users prefer a reliable shortest path with higher travel time estimation accuracy, it is necessary to find an optimal path from the link set of hyperpath. Here, the idea of link travel time penalty is introduced as an attempt to find a reliable shortest path. To avoid the selection of the unreliable links, the links with lower reliability will be penalized by their historical maximum delay. The penalized link travel time can be represented as Eq. (23).

$$T' = T + d_{ij}(1 - r) \quad (23)$$

where T denotes the estimated travel time by probes in the current measurement period, d_{ij} denotes the historical maximum delay, and r denotes the reliability. If the reliability equal to 1, we believe that the current estimated travel time can represent the real travel time. If the reliability equal to 0, a pessimistic estimation of travel time with maximum delay will replace the current estimated travel time, which decreases the link choice probability.

After modifying the travel time of each link in the hyperpath link set, the famous A* algorithm[20] is applied to find the reliable shortest path. The algorithm is described as follows:

Step 1: Initialization

Set $u_i \leftarrow \infty,\ i \in I - \{r\};\ u_r \leftarrow 0;\ f_i \leftarrow \infty;\ N \leftarrow \emptyset;\ N^o \leftarrow \emptyset; N^c \leftarrow \emptyset;\ L \leftarrow \emptyset$

Set $N^o \leftarrow N^o + \{r\}$

Step 2: Node selection

Select node i in N^o with minimum $u_i + h_i$, $N^o \leftarrow N^o - \{i\}$, $N^c \leftarrow N^c + \{i\}$

Step 3: Node expansion: scan the outgoing links of node i. For each link(i, j)

If $u_i + c_{ij} + h_j \leq u_j$ then update node label:

$u_j = u_i + c_{ij}$

$f_j = u_j + h_j$

$L_i = link(i, j)$

$N^o \leftarrow N^o + \{j\}$

Step 4: Stopping rule

If $u_i + c_{ij} + h_j > u_j$ or $N^o = \emptyset$ then stop

Else go to step 2

End

4. IMPLEMENTATION AND RESULTS

(1) Test in an 8 by 8 grid network

To demonstrate the proposed path finding method, a typical 8 by 8 grid bi-directional network is taken to test. The path finding procedure is tested under three congestion levels. The grid network is composed by 64 nodes and 224 directional links. The travel time of each link includes two components, i.e. undelay travel time and maximum delay. The undelay travel time is set as 1+rnd(0,1), where rnd(0,1) is a random between 0 and 1. The maximum delay is set as 0.5*rnd(0,1), rnd(0,1), and 2*rnd(0,1) for the low delay, medium delay, and high delay situation, respectively. The reliability of each link is set as rnd(0,1). The origin and destination are located in the diagonal corner of the network (from node 1 to node 64). The number on the link of the left-side network shows the link choice probability by hyperpath algorithm, and the number shows the reliability of each link on the right-side network.

As shown in **Fig. 2**, the network traffic condition is set as a lower congestion level. Because the traffic state is relatively stable, choice probabilities of several links are assigned to 1. For example, link(1,2) and link(2,3) are fully accepted as the reliable link. Though link(1,9) and link(2,10) are connected with node 1 and node 2, choice probabilities of them are zero. It indicates that link(1,9) and link(2,10) are unattractive because their undelay travel time are

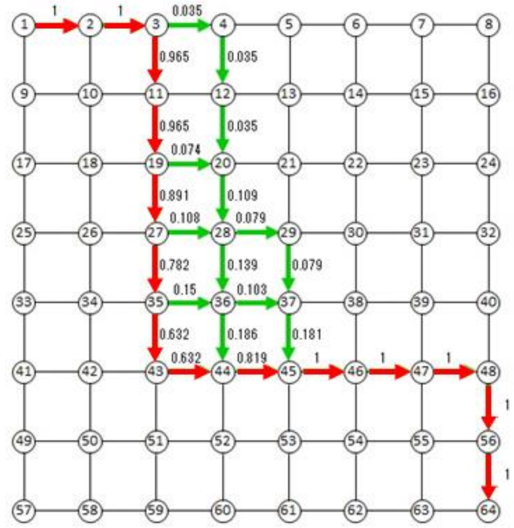

Fig. 2 Hyperpath (green) and reliable path (red) in low delay network

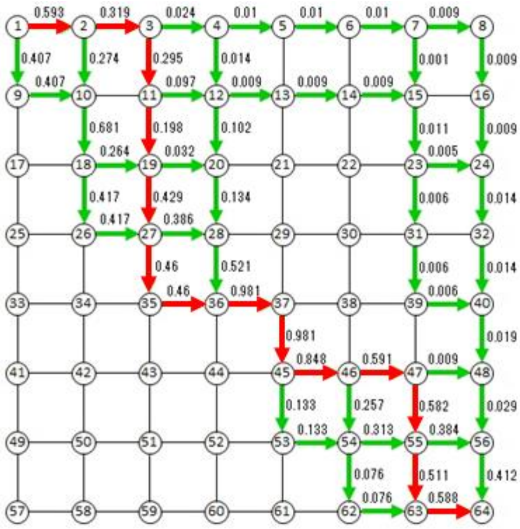

Fig. 3 Hyperpath (green) and reliable path (red) in median delay network

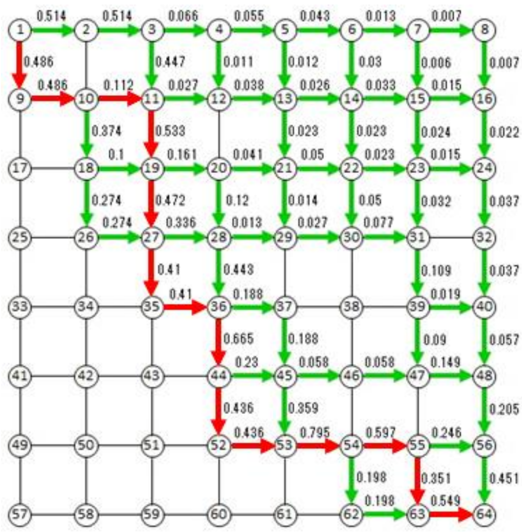

Fig. 4 Hyperpath (green) and reliable path (red) in high delay network

Table 1 Comparative performance in different delay levels

Scenario	Link number of hyperpath	Average link reliability
Low delay	27	0.59
Medium delay	57	0.62
High delay	72	0.75

relatively large that it is impossible to provide a better result even though link(1,2) and link(2,3) may suffer the maximum delay. As the delay level becomes higher, the link choice behavior become more complicated. As shown in **Fig. 3** and **Fig. 4**, as the network delay increases, more links are considered as the attractive alternatives. The driver has to make decision at each node to choose a better solution. In practice, most of the drivers do not change the route so frequently. Instead, a risk-averse driver often chooses a path with high reliability according to their experience before departure. Therefore, it is helpful to provide a reliable shortest path from the hyperpath link set.

Table 1 shows the performance of the proposed path finding method in different delay levels. It is found that the number of link set in hyperpath increases with the potential congestion become severe. It is reasonable because the link travel time becomes uncertain and more links are potential to provide a better route. On the other hand, it demonstrates that the recommended optimal path has higher reliability as the stochastic link delay increase, which coincides with the phenomenon that drivers tend to choose a more reliable path when the traffic condition becomes worse.

5. CONCLUSION AND FUTURE WORK

To better help travelers plan their trips to avoid the risk of uncertain travel time, the hyperpath concept and shortest path concept considering driving experience of probe are applied to path finding procedure. Particularly, the reliability is calculated by the link usage frequency collected from historical trips of probe. Then, a virtual network with 8 by 8 nodes is taken to test the performance. The numerical experiments indicate that the reliable shortest path generated from the hyperpath link set enables to guarantee the travel time efficiency and accuracy reasonably. The major contributions of this study are summarized as follows.

(1) A reliability function considering the link usage frequency is proposed. High usage frequency often suggests a reliable link because rational probe vehicle drivers will not choose an unreliable link frequently.

(2) A two-step path finding procedure is developed for finding a reliable shortest path. In step 1, the hyperpath concept helps to find a set of shortest paths, each of which is a potential optimum that may be attractive due to travel time uncertainty. In step 2, the attractive links in hyperpath link set are penalized by their reliability and then the A* algorithm helps to find a shortest path with least penalized travel time. There are two advantages. First, the reasonable detours are taken into account and the link choice probability of each outgoing link of the attractive node is estimated by the maximum delay, which guarantees the potential shortest paths are included. Second, not only the hyperpath with recommended link choice probability, but also a shortest path with high reliability is provided, which helps the travelers plan their trips effectively.

(3) In the numerical experiment of 8 by 8 node network, it is found that the recommended optimal path has higher reliability as the stochastic link delay increase, which coincides with the phenomenon that drivers tend to choose a more reliable path when the traffic condition becomes worse.

However, there are still several problems needing further attention in future work. For example, the turn restrictions and direction constraint should be considered in practical network. In addition, the reliability of a path instead of a link should be taken into account, and then a reliable total travel time can be provided to travelers. These limitations will be improved by a more comprehensive modeling approach and a more strict validation procedure in our future work.

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